Fuzzy g*-closed sets

S.S. Thakur and Manoj Mishra
Deptt. of Applied Mathematics, Jabalpur Engineering College Jabalpur (M.P.) India

ABSTRACT: In 2000 Veera Kumar [9] introduced the concepts of g*-closed sets in general topology. The present paper extends the concepts of g*-closed sets in fuzzy topology and explores their study.

I. PRELIMINARIES

Let X be a nonempty set and I = [0, 1]. A fuzzy set on X is a mapping from X to I. The null fuzzy set is a mapping 0 from X to I which assumes only the values 0 and 1 belongs to [7].

A family τ of fuzzy set X is called the fuzzy topology [4] on X if 0 and 1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The member of τ are called fuzzy open sets and their complement are fuzzy closed sets. For a fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed superset of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A.

Definition 1.1: A fuzzy set A of a fuzzy topological space (X, τ) is called:
(a) Fuzzy g-closed if cl(A) ≤ G whenever A ≤ G and G ∈ τ [7].
(b) Fuzzy g-open if 1-A is fuzzy g*-closed [7].

Definition 1.2: A mapping f : (X, τ)→(Y, I) is said to be gc - irresolute if f(O) is fuzzy g-closed in X for every fuzzy g-closed set O in Y [8].

2. FUZZY g*-CLOSED SETS

Definition 2.1: A fuzzy set A of a fuzzy topological space (X, τ) is called a fuzzy g*-closed if cl(A) ≤ O whenever A ≤ O and O is fuzzy g-open.

Remark 2.1: Every fuzzy closed set is fuzzy g*-closed and every fuzzy g*-closed set is fuzzy g-closed but the converses may not be true.

Example 2.1: Let X = {a, b} and τ = {0, 1, U} be an fuzzy topology on X, where U(a) = 0.5, U(b) = 0.6. Then the fuzzy set defined by A(a) = 0.3, A(b) = 0.4 is fuzzy g*-closed and the fuzzy set B defined by B(a) = 0.6, B(b) = 0.6 is fuzzy g-closed but it is not fuzzy g*-closed.

Theorem 2.1: Let (X, τ) be a fuzzy topological space and A is a fuzzy set of X. Then A is fuzzy g*-closed if and only if (AqF) ⇒ (cl(AqF)) for every fuzzy g-closed set F of X.

Proof: Necessity: Let F be an fuzzy g-closed subset of X, and (AqF). Then by Lemma (1.1.A), A ≤ 1-F and 1-F fuzzy g-open in X. Therefore cl(A) ≤ 1- F because A is fuzzy g*-closed. Hence by cl(AqF).

Sufficiency: Let O be a fuzzy g-open set of X such that A ≤ O. Then (AqF) ⇒ (cl(AqF)). Therefore cl(A) ≤ O. Hence A is fuzzy g*-closed in X.

Theorem 2.2: Let A and B be two fuzzy g*-closed sets in a fuzzy topological space (X, τ), then A ∪ B is fuzzy g*-closed.

Proof: Let O be an fuzzy g-open set in X, such that A ∪ B ≤ O. Then cl(A) ≤ O and cl(A) ≤ cl(B) ≤ O. Therefore cl(A) ∪ cl(B) = cl(A ∪ B) ≤ O. Hence A ∪ B is fuzzy g*-closed.

Remark 2.2: The intersection of two fuzzy g*-closed sets in a fuzzy topological space (X, τ) may not be fuzzy g*-closed.

For Example 2.2: Let X = {a, b} and U, A and B be the fuzzy subsets of X defined as follows:

U(a) = 0.7, U(b) = 0.6
A(a) = 0.6, A(b) = 0.7
B(a) = 0.8, B(b) = 0.5

Let τ = {0, 1, U} be an fuzzy topology on X. Then A and B are fuzzy g*-closed in (X, τ) but A ∩ B is not fuzzy g-closed.
Theorem 2.3: Let and A be a fuzzy g*-closed set in fuzzy topological space (X, S) and A \subseteq B \subseteq \text{cl}(B). Then B is fuzzy g*-closed in X.

Proof: Let O be fuzzy g-open set such that B \subseteq O. Then A \subseteq O. Since A is fuzzy g*-closed, cl(A) \subseteq O. Now B \subseteq \text{cl}(A) \Rightarrow \text{cl}(B) \subseteq \text{cl}(A) \subseteq O. Consequently B is fuzzy g*-closed.

Theorem 2.4: Let (Y, S_Y) be a subspace of a fuzzy topological space (X, S) and A be fuzzy set in Y. If A is fuzzy g*-closed in X then A is fuzzy g*-closed in Y.

Proof: Let A \subseteq O_Y where O_Y is fuzzy g-open in Y. Then there exists fuzzy g-open set O in X such that O_Y = O \cap Y. Therefore A \subseteq O and since A is fuzzy g*-closed in X, cl(A) \subseteq O. It follows that cl_y(A) = cl(A) \cap Y \subseteq O \cap Y = O_Y. Hence A is fuzzy g*-closed in Y.

Definition 2.2: A fuzzy set A of fuzzy topological space (X, S) is called fuzzy g*-open if its complement 1 - A is fuzzy g*-closed.

Remark 2.2: Every fuzzy open set is fuzzy g*-open and every fuzzy g*-open set is fuzzy g-open. But the converses may not be true.

Theorem 2.5: An fuzzy set A of a fuzzy topological space (X, S) is fuzzy g*-open if and only if and if F \subseteq \text{int}(A) whenever F is fuzzy g-closed and F \subseteq A.

Proof: Obvious.

Theorem 2.6: Let A and B are q-separated fuzzy g*-open subsets of a fuzzy topological space (X, S), then A \cup B is fuzzy g*-open.

Proof: Let F be a fuzzy g-closed subset of X and F \subseteq A \cup B. Then F \cap \text{cl}(A) \subseteq A \cup B \Rightarrow F \cap \text{cl}(A) = (A \cap \text{cl}(A)) \cup (B \cap \text{cl}(A)) \subseteq \text{Int}(A). Similarly F \cap \text{cl}(B) \subseteq \text{Int}(B).

Now F = F \cap (A \cup B) \subseteq (F \cap \text{cl}(A)) \cup (F \cap \text{cl}(B)) \subseteq \text{Int}(A) \cup \text{Int}(B) \subseteq \text{Int}(A \cup B). Hence F \subseteq \text{Int}(A \cup B) and by theorem 2.5, A \cup B is fuzzy g*-open.

Theorem 2.7: Let A and B be two fuzzy g*-closed sets of a fuzzy topological space (X, S) and suppose that 1 - A and 1 - B are q-separated, then A \cap B is fuzzy g*-closed.

Proof: Since A^c and B^c are q-separated fuzzy g*-open sets, by Theorem 2.6, 1 - (A \cap B) = 1 - A \cup (1 - B) is fuzzy g*-open. Hence A \cap B is fuzzy g*-closed.

Theorem 2.8: Let A be fuzzy g*-open set of fuzzy topological space (X, S) and Int(A) \subseteq B \subseteq A. Then B is fuzzy g*-open.

Proof: Since 1-A \subseteq 1-B \subseteq cl(1-A) and 1-A is fuzzy g*-closed it follows from theorem 2.3 that 1-B is fuzzy g-closed. Hence B is fuzzy g*-open.

Definition 2.3: A fuzzy topological space (X, S) is said to be fuzzy g*-compact if every fuzzy g*-open cover of X has a finite sub cover.

Theorem 2.9: Let (X, S) be a fuzzy g-compact space and suppose that Y is fuzzy g*-closed crisp subset of X, then (Y, S_Y) is g-compact.

Proof: Let V be a fuzzy g*-open covering of X and let G = \{V \subseteq \mathcal{S} : B \cap V \neq \emptyset\}. Then Y \subseteq \bigcup G. Since Y is fuzzy g*-closed, cl(Y) \subseteq \bigcup G. Therefore G \subseteq \bigcup \{\text{cl}(Y)\} is a fuzzy g*-open cover of X. Since X is fuzzy g-compact, G \cup 1-(\text{cl}(Y)) has a finite sub cover \{V_1, V_2, \ldots, V_n\}. But then, \{V_1 \cap Y, V_2 \cap Y, \ldots, V_n \cap Y\} is a finite sub cover of Y.

Theorem 2.10: Let A be a fuzzy g-closed set in fuzzy topological space (X, S), and f: (X, S) \to (Y, S_Y) is fuzzy g-irresolute and fuzzy closed mapping then f(A) is a g-closed set in Y.

Proof: If f(A) \subseteq G where G is fuzzy g-open in Y then A \subseteq f^{-1}(G) and hence cl(A) \subseteq f^{-1}(G). Thus f(cl(A)) \subseteq G and f(cl(A)) is fuzzy closed set. It follows that cl(A) \subseteq cl(f(cl(A))) = f(cl(A)) \subseteq G. Hence cl(f(A)) \subseteq G and f(A) is fuzzy g*-closed.

REFERENCES