Biological Forum – An International Journal **14(2): 1086-1090(2022)**

ISSN No. (Print): 0975-1130
ISSN No. (Online): 2249-3239

Recommendation of Nitrogen and Sulfur for Optimum Yield in the Intermediate Stage of Cultivation using a Mathematical Model

Jyotiranjan Behera[1*] and M.K. Mahanti[2]
[1]*College of Basic Science and Humanities, O.U.A.T., Bhubaneswar (Orissa), India.*
[2]*Department of Mathematics, Orissa University of Agriculture and Technology, Bhubaneswar (Orissa), India.*

(Corresponding author: Jyotiranjan Behera)*
(Received 14 March 2022, Accepted 23 May, 2022)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: With the increase in population around the world, there is a shortage of land for agriculture. Increasing production on less land has now become the focus of research. Even though sufficient fertilizer may have been applied at the beginning of cultivation it may so happen that yield is not optimum due to some natural and man-made reasons. To address to such aspects of inadequate availability of fertilizers that hinders optimal yield, a mathematical model has been proposed in this work by which a corrective fertilizer dose can be applied in the intermediate stage of cultivation. The proposed method first determines the amount of nitrogen N and sulphur S fertilizers necessary to optimize yield by a mathematical optimization technique, namely, Marquardt's method. Measurements of plant nutrient uptakes of applied fertilizers in a scheduled intermediate stage of cultivation are used in both trial and commercial plantations to determine whether the commercial plantation enjoys adequate supply of fertilizers to produce optimal yield. If it is found in the intermediate stage of cultivation that the fertilizers available to the crop will not eventually result in optimal yield, a numerical technique for solution of simultaneous non-linear equations is proposed to be used to decide how much more fertilizer doses should be supplied to the crop to ensure optimal yield.

Keywords: Fertilizer response, onion plant, Fertilizer nitrogen, Fertilizer sulfur, Newton's Method, Mathematical optimization, Marquardt's Method.

INTRODUCTION

Fertilizer is an essential source of nutrients for better growth of plants. Farmers can improve nutrient management of plants by applying fertilizers in the right amount and at the right time. But, due to several factors, plants may not have proper nutrients for maximum yield. It may be possible that a farmer knows how much fertilizer should be applied for maximum agronomic yield, but due to several reasons, fertilizer available to crop is not adequate. For instance, applied fertilizers in the soil can be washed away from farm fields into water ways during excess rain (Yu, *et al.*, 2021). Nitrogen can be lost from farm fields in the form of gaseous nitrogen based compounds like ammonia and nitrogen oxides (Velthof & Rietra 2019). Sometimes, quality of fertilizers applied by the farmers is questionable. There are several instances where cultivation suffers due to low quality of fertilizers used (Bold *et al.*, 2015). The amount of fertilizers purchased by the farmers and applied in the field depends on weights and measurements which may not be perfect (Bonilla *et al.*, 2020). Thus, a farmer may have the idea that the desired amounts of fertilizers have been used in the field, but the crop doesn't get enough fertilizers from soil in order to produce optimum yield.

One of the ways by which the abovementioned problems can be addressed is to measure the amount of fertilizer that are available to the crop in an intermediate stage of cultivation and, based on the measurement, to adopt corrective measures by supplying more fertilizers, if it is found that fertilizer available to the crop in the intermediate stage, will not lead to optimum yield. These solution processes can be achieved in the following two stages. The first stage involves experiment carried out on trial plantation. In the second stage, the outcome of trial plantation is used in commercial plantation.

We have demonstrated the applicability of the two stages by considered onion cultivation (Tilahun *et al.*, 2021) where the fertilizers used are S(Sulfur) and N (Nitrogen). College of basic science and humanities, O.U.A.T, Bhubaneswar.

Satage-1

Using trial plantation data, a yield response surface to fertilizer applications is formulated by multiple linear regressions (Gomez and Gomez 1984) with yield (Y) is the dependent variable and N and S are the independent variables.

(i) Using the yield response surface, the amount of fertilizers S and N that are needed to be applied to maximize yield is calculated by applying a mathematical optimization technique for unconstrained nonlinear programming problem.

From the trial plantation data, response surfaces of plant uptake of N and S, expressed as functions of fertilizers applied at the time of sowing, are determined using multiple linear regressions (Weisberg, 1947).

(ii) The fertilizer response surfaces are used to determine the nutrient levels of N and S fertilizers that the plant should possess, in order that it can have maximum yield.

Stage-2

(a) Plant uptake of N and S in commercial planting is determined in an intermediate stage of cultivation.

(b) The uptake levels, so calculated, are compared with the uptake levels that the crop should possess for maximum yield, which was found out in the first stage.

(c) If it is found that the uptake level of commercial plantation is less than the uptake levels required for optimizing yield, it is concluded that there are deficit in fertilizers and corrective measures are to be adopted by supplying additional doses of fertilizers in commercial plantation. The following steps are followed to determine the additional doses of fertilizers.

(iii) The amount of fertilizers used in the past in the neighboring plantation is determined by solution of a system of two nonlinear equations by Newton's method (Burden and Faires, 9th edn.).

(iv) The differences between the amount of fertilizers used in the past and the amount of fertilizers that should have been used for maximum yield, which was calculated in the first stage, are the corrective fertilizer doses for optimum yield.

The proposed mathematical framework can be used in KVKs (Krisi Vikash Kendras) to advise farmers who produce a particular crop in the locality of KVKs. Commercial plants, which are cultivated in the locality of the KVKs, grow under same environmental and soil conditions. Therefore, results obtained by factorial fertilisation trials (Gomez and Gomez 1984) at KVKs can be used in neighbouring commercial plantations.

In the onion plantation, factorial trials with four rates of N and five rates of S and a total of 20 treatments (i.e. all combinations of four rates of nitrogen and five rates of sulphur), have been used for developing the response surface of yield and plant nutrient uptake in trial plantation.

The effect of different amount of applied N and S fertilizers on the crop yield, plant uptake of N and plant uptake of S, measured at a scheduled intermediate stage of cultivation, have been presented in functional form in the section "Statistical analysis and Experimental Outcome". A step by step computational procedure of the proposed mathematical framework has been provided in "Computational Steps" section. Execution of our proposed mathematical model has been illustrated taking into account the statistical data further.

A lot of experiments have been attempted to study the effect of N and S fertilizers on yield of onion in past. Optimization of onion using drying process parameter of sliced onions was by Kholikov *et al.,* (2021) used a quadratic response surface involving temperature, drying factor and thickness as decision variables. In the work of Koech *et al.,* (2017), the 3D response surface plots were employed to evaluate the effect of NPK fertilizers on yield of potato tuber by varying two variables within the experimental range under investigation and holding the other variable at its central level.

Niedbała *et al.* (2022) developed a linear model based on multiple linear regression analysis and a nonlinear model based on artificial neural network to predict potato cultivation before harvest.

In most of the works in the past, the function representing the response surface is assumed to be quadratic. It is easier to optimize a quadratic function of several variables. However, it is not true that the response surfaces will be a quadratic function always, since such a surface may not fit the data well. Here we have allowed the functional expressions for response surfaces to be of higher degree in order to be statistically significant. If the response surfaces are not quadratic, the mathematical optimization techniques used to maximize function of several variables in most of studies earlier, are no longer applicable, and we need to apply more advanced techniques such as the one we have used in this paper. Soil test crop response (STCR) has used a targeted yield approach to develop relationship between crop yield on the one hand, and soil test estimates fertilizer inputs, on the other. In this targeted yield approach, it is assumed that there is a linear relationship between crop yield and nutrient uptake. In our proposed method Marquardt's optimization is capable of handling implicit nonlinearity between crop yield and nutrient uptake.

METHODOLOGY

Description of trial. The current study is based on data generated in an investigation considering onion plantation under field condition during winter season, from January 2018 to May 2019 (Tilahun *et al.,* 2021).

The investigation was conducted at the shewa Robit integrated project of Debre Berhan University (DBU), of North shewa zone, Ethopia. It is geographically situated at 37° 20′ East longitude and 11° 55′ north latitude at an altitude of 1380 meters above the sea level. The area has a short rainy season between March and April and a long rainy season between June and September (DBRC, 2018). Annual mean minimum and maximum temperatures were 14.0°C and 30.4°Crespectively. Mean rainfall was about 77.0mm during the cropping season.

"*Bombay Red*" onion variety was used for the study. The land was ploughed with a depth of 25 cm, pulverized and levelled by oxen. Ridges and furrows were prepared using hand tools manually. The size of each plot was 3m × 2m. Seeding were transplanted 45 days after sowing in the trial field. The planting of onion seeding was done with a double row planting system at the spacing of 20cm between rows and 10 cm between plants. Each trial plot was $8.4m^2$ in size. The trial used a randomized complete block design of three replicates with 20 treatments.

The 20 treatments are a factorial combination of 4 rates of nitrogen (100, 150, 200, 250 kg/ha) and five rates of sulphur (0, 15, 30, 45, 60 kg/ha).

For plant sample analysis 0.50g dry bulb samples were taken and digested with a 2:1 mixture of nitric (HNO_3) and per chloric acids ($HClO_4$). The nitrogen content in matured bulbs was determined by Micro Kjeldahl's Method (Tandon, 1993). The concentration of S in the extract was determined turbid metrically using a spectrophotometer (Tandon, 1993). Total uptake of N and S was determined by multiplying the concentration with its yield weights.

STATISTICA ANALYSIS AND EXPERIMENTAL OUTCOME

In this section we present the statistical analysis of the data generated from the field trial (Tilahun *et al.*, 2021). For our mathematical framework, we need three response surfaces depicting the effect of different levels N and S fertilisers used in the experiment. They are

(i) Response surface of yield of onion,

(ii) Response surface of plant uptake of nitrogen,

(iii) Response surface of plant uptake of sulphur.

The effect of different amount of N and S fertilizers that were applied at sowing stage, on yield of tubers is provided in table.

The response surface for yield, after application of multiple linear regressions using data in table, is given by,

$y(N,S)=a+bN+cNS+dS^2+eN^2S$ (3.1)

where, y(N,S) is the yield of onion, a,b,c,d and e are regression coefficients having values

$a=12.75331(P=.06)$, $b=0.096327(P=.01)$, $c=0.011032(P<.001)$, $d=-0.0103(P=.002)$, $e=-4e-05(P<.001)$.

The yield response surface has coefficient of determination $(R^2=0.65)$ with significance F< .001.The effect of different level of N and S on plant uptake of N in the trial onion plantation is provided in table 1.

The response surface for uptake of nitrogen, after application of multiple linear regressions using data in table, is given by

$y_N(N,S)=a+bNS+cN^2+dN^3$ (3.2)

where , $y_N(N,S)$ represents uptake of N by plant and a,b,c and d are regression coefficients having values $a=93.10258(P<.001)$, $b=0.002329(P<.001)$, $c=0.005468(P<.001)$, $d=-1.5e-05(P<.001)$.

The nitrogen uptake response surface (3.2) has coefficient of determination (R^2) of 0.91 and adjusted (R^2) of 0.89.

The response surface for uptake of sulfur, after application of multiple linear regressions using data in table, is given by

$y_S(N,S)=a+bN^2+cN^2S+dN^3$ (3.3)

where, $y_S(N,S)$ represents uptake of S by plant and a,b,c and d are regression coefficients having values :a = 8.465147(P<.001), b=0.000835(P<.001), c=1.47e-06(P = .002), d = -2.3e-06(P=.002).

The sulfur uptake response surface has coefficient of determination $(R^2=.91)$ and $(adjusted\,R^2=.90)$

Table 1:Effects of different level of N and S on yield of tubers, plant uptake of N and uptake of S (kg/ha).

Nitrogen levels(N) (kg/ha)	Sulfur levels(S) (kg/ha)	Bulb yield (t/ha)	plant uptake of N $[N]$(kg/ha)	plant uptake of S$[S]$(kg/ha)
100	0	28.07	127.66	13.20
100	15	29.57	131.67	14.00
100	30	29.63	150.33	15.27
100	45	30.40	175.35	15.70
100	60	30.53	151.00	16.25
150	0	30.73	157.66	19.85
150	15	33.62	165.67	20.55
150	30	36.93	171.33	20.95
150	45	38.97	185.33	21.65
150	60	36.20	187.00	22.35
200	0	33.10	187.67	22.60
200	15	37.17	196.33	23.60
200	30	38.70	198.67	24.00
200	45	42.60	243.33	31.90
200	60	37.03	200.67	24.30
250	0	32.20	201.67	25.80
250	15	37.82	208.66	26.50
250	30	41.37	214.67	27.60
250	45	36.06	218.67	29.15
250	60	34.4	225.33	31.75

COMPUTATIONAL STEPS

We describe here the step by step procedure of the mathematical model adopted by us

Step-1: Find the following three response surfaces by multiple linear regressions from data provided in trial plantation

(i) At the beginning of cultivation:

Crop yield y(N,S) (see(3.1)) .

(i) At a scheduled intermediate stage of cultivation (after a time interval T from beginning of cultivation):
Plant uptake of Nitrogen $y_N(N,S)$ ((3.2)) in trial plantation,
Plant uptake of sulfur $y_S(N,S)$ ((3.3)) in trial plantation.
Step-2: Solve the following unconstrained optimization problem
Maximize y(N,S) .
By the marquardt's method (Joshi & Moudgalya 2011).
Letthe value of N and S which optimizes y(N,S) be denoted as N_{opt} and S_{opt} respectively.
Step-3: Compute $y_N(N_{opt},S_{opt})$ and $y_S(N_{opt},S_{opt})$ from equations (3.2) and (3.3). Let

$$[N]_{opt} = y_N(N_{opt},S_{opt}) \text{ and } [S]_{opt} = y_S(N_{opt},S_{opt}).$$

Step-4: Let $[N]_{app}$ and $[S]_{app}$ be the Nitrogen uptake and sulfur uptake respectively in the commercial
Plantation after time interval T from beginning of cultivation
Step-5: (i) If $[N]_{app} \geq [N]_{opt}$ and $[S]_{app} \geq [S]_{opt}$,
we conclude that the fertilizers used in the commercial plantation are sufficient to produce optimum yield. Go to step-8
(ii) If $[N]_{app} < [N]_{opt}$ or $[S]_{app} < [S]_{opt}$, go to step 6
Step-6: Solve the following system of nonlinear equations by the Newton's method (Burden & Faires, 1993)

$$y_N(N,S) = [N]_{app},$$
$$y_S(N,S) = [S]_{app}.$$

Let the solution of the system of equations be N_{app} and S_{app}, which are the amount of fertilizers applied at the commercial plantation at the time of sowing.
Step-7: The additional amount of N and S fertilizer required to be applied to the commercial plantation
are $N_{opt} - N_{app}$ and $S_{opt} - S_{app}$ depending on whether $N_{app} < [N]_{opt}$ or $S_{app} < [S]_{opt}$.
Step-8 :Stop.

RESULT AND DISCUSSION

Now we will discuss how we can compute the corrective doses of fertilizers that needed in a onion plantation cultivated in the neighborhood of the trial plantation with the help of response surfaces corresponding to yield, plant uptake of nitrogen and plant uptake of sulfur that we have obtained
First, the yield function (3.1) is optimized by Marquardt's method. We find that
$N_{opt} = 172.7310(kg/ha)$ and $S_{opt} = 34.5693(kg/ha)$.
Uptake level of nitrogen and sulfur in the onion plant are obtained by putting the values N_{opt} and S_{opt} in the right hand side of the equations (3.2) and (3.3). Thus, we obtain that
$[N]_{opt} = 192.8487(kg/ha)$ and $[S]_{opt} = 23.3196(kg/ha)$.
It follows that onion plants having plant nutrient uptake $[N] < 192.8487(kg/ha)$ and $[S] < 23.3196(kg/ha)$ can't be expected to produce maximum yield.
At this stage, it is possible to examine whether sufficient N and S are available to the commercial plantation to enable it to produce optimum yield. To this end, we need to compute the plant uptake of N and S in the commercial plantation in the laboratory. One of the two cases mentioned in Step-5 of computational steps is possible.
If the conditions mentioned in Step-5(i) occur, we conclude that we need not supply any more fertilizers and the applied fertilizers in the past are sufficient for maximum yield.
Let us assume that $[N]_{app} = 184(kg/ha)$ and $[S]_{app} = 22(kg/ha)$. As discussed earlier, it can be concluded that not enough fertilizer has been used in the past and the yield is not likely to be maximum possible. In what follows we describe the procedure for obtaining remedial fertilizer doses that will be prescribed to farms in order that yield can be optimized.
Step-1
We put the values of $[N]_{app} = 184(kg/ha)$ and $[S]_{app} = 22(kg/ha)$ in left hand side of equation (3.2) and (3.3) to obtain a system of two nonlinear equations as follows

$$93.10258 + 0.002329NS + 0.005468N^2 - 1.5e-05N^3 = 184 \quad (6.1)$$

and

$$8.465147 + 0.000835N^2 + 1.74e-06N^2S - 2.3e-06N^3 = 22 \quad (6.2)$$

Step-2
We Solve the system of equations (6.1) and (6.2) by Newton's method. The solution obtained is
$N_{app} = 163.89(kg/ha)$ and $S_{app} = 26.35(kg/ha)$.
Step-3
We conclude that the remedies fertilizers prescription for optimum yield in commercial plantation:
$N_{opt} - N_{app} = 8.84(kg/ha)$,
$S_{opt} - S_{app} = 8.21(kg/ha)$.

CONCLUSION

We have presented here a mathematical framework by means of which crop yield can be maximized by effective application of fertilizers. The model presented here involves detecting shortfall of fertilizers available to the plant in an intermediate stage of cultivation, and then recommending necessary remedial fertilizer doses to be used in the commercial plantation, if it is found that the fertilizers available to the commercial plantation is not adequate for optimum yield. Although we have considered onion cultivation as an example, the model presented by us can be easily extended to any commercial plantation. Here we have considered effect of Nitrogen and sulfur fertilizer on yield. The model can be extended easily to cases where fertilizers other than which have been mentioned here, are used. In the present model, we have proposed to test the nutrients uptake of the plant once during span of cultivation. We shall obtain a more accurate prediction about remedial fertilizers requirement if plant intake is measured at various time intervals. We have not taken the economical aspect of the fertilizer use. It may so happen that the objective of optimization of yield will conflict with the objective of economic use of fertilizer. In that case we shall have to solve a multi objective optimization problem instead of a single objective optimization problem that we have solved here.

REFERENCES

Bold, T., Kaizzi, K., Svensson, J., & Yanagizawa-Drott, D. (2015). *Low quality, low returns, low adoption: evidence from the market for fertilizer and hybrid seed in Uganda*. London, England: Centre for Economic Policy Research.

Bonilla Cedrez, C., Chamberlin, J., Guo, Z., & Hijmans, R. J. (2020). Spatial variation in fertilizer prices in Sub-Saharan Africa. *PloS one*, *15*(1), e0227764.

Burden, R. L., & Faires, J. D. (1993). Numerical Analysis, PWS-Kent Pub. *Co., Boston*.

DBARC (Debre Berhan Agricultural Research Center), (2018) Metrological data of Shewa Robit area, annual report.

Gomez, K.A. and Gomez, A.A. (1984). Statistical Procedures for Agricultural Research (2nd Edn.)

Joshi, M. C. and Moudgalya, K. M. (2011). Optimization Theory and Practice, 4th ed. Narosa publication.

Kholikov, A., Jumaev, J., Hikmatov, D., & Kuvvatov, K. (2021, September). Optimization of onion drying process parameters using the full factorial experiment method. In *IOP Conference Series: Earth and Environmental Science* (Vol. *848*, No. 1, p. 012010). IOP Publishing.

Koech, J. K., Mutiso, M. J. and Koskei, J. K. (2017). Response surface methodology approach to the optimization of potato (*Solanum tuberosum*) tuber yield using second-order rotatable design. *J. Biom. Biostat.*, 2017, *8*: 3.

Niedbała, G., Wróbel, B., Piekutowska, M., Zielewicz, W., Paszkiewicz-Jasińska, A., Wojciechowski, T., & Niazian, M. (2022). Application of Artificial Neural Networks Sensitivity Analysis for the Pre-Identification of Highly Significant Factors Influencing the Yield and Digestibility of Grassland Sward in the Climatic Conditions of Central Poland. *Agronomy*, *12*(5), 1133.

Sanford Weisberg (1947). Applied linear regression, 4thed., school of statistics, University of Minnesota Minneapolis, M N, Wiley series in probability and statistics.

Tilahun, M., Tena, W., & Desta, B. (2021). Effects of Different Nitrogen and Sulfur Fertilizer Rates on Growth, Yield, Quality and Nutrient Uptake of Onion (L.) at Shewa Robit, North Shewa, Ethiopia. *The Open Biotechnology Journal*, *15*(1).

Tandon, H. L. S. (1989). Sulphur deficiencies in soils and crops: their significance and management. *Sulphur fertilizers for Indian agriculture—a guide book'.(Ed. HLS Tandon) p, 6*.

Velthof, G. L., & Rietra, R. P. J. J. (2019). Nitrogen use efficiency and gaseous nitrogen losses from the concentrated liquid fraction of pig slurries. *International Journal of Agronomy*, *2019*.

Yu, Y., Xu, J., Zhang, P., Meng, Y., & Xiong, Y. (2021). Controlled Irrigation and Drainage Reduce Rainfall Runoff and Nitrogen Loss in Paddy Fields. *International Journal of Environmental Research and Public Health*, *18*(7), 3348.

How to cite this article: Jyotiranjan Behera and M.K. Mahanti (2022). Recommendation of Nitrogen and Sulfur for Optimum Yield in the Intermediate Stage of Cultivation using a Mathematical Model. *Biological Forum – An International Journal*, *14*(2): 1086-1090.