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Adaptation of Golden Codes with a Correlated Rayleigh Frequency-Selective Channel in OFDM System with Imperfect Channel Estimation

Kamna Mishra, Prof. Divya Jain and Prof. Vikas Gupta

Department of Electronics and Communication Engineering,
TIT College, Bhopal, (MP)

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ABSTRACT: The Golden Code (GC) is an Algebraic Space-Time Code (ASTC) for MIMO systems, based on quaternion algebras. Thanks to the algebraic construction, the Golden codes are full-rank, full-rate and have the non-vanishing determinant property. These codes have been proposed for MIMO flat fading channels in order to increase the spectral efficiency and to maximize the coding gain. The purpose of this work is to analyze the performance of the GC in a frequency selective Rayleigh channel. To deal with the frequency selectivity, we use the OFDM modulation. The BER performance of an ASTC-MIMOOFDM system, under several propagation conditions has been evaluated.

I. INTRODUCTION

By exploiting the spatial diversity, MIMO systems are able to provide high data rate over wireless channels and improve the system capacity [1]. Furthermore, the space-time codes are used to improve MIMO performances by providing a temporal and a spatial multiplexing modulation. The most known and used Space-Time Block codes (STBCs) are the Alamouti code and the Golden code [1]–[4]. The golden code, which has been proposed in 2004 by [3] for 2 x 2 MIMO system, is a full-rate and full-diversity space-time code that has a maximal coding gain. Thanks to its algebraic construction, the GC outperforms the Alamouti
codes in flat fading channels [4]. In a frequency-selective channel, the GC codes lose their proprieties due the inter-symbol interference (ISI). To overcome this drawback, the orthogonal frequency division multiplexing (OFDM) modulation can be used.

The OFDM modulation is a powerful technique usually used to reduce the ISI in a high-bit-rate transmission systems [5]. The OFDM principle is to split the information stream to be transmitted into a large number of low-bit-rate sub streams modulating distinct carriers [5]–[7]. The advantage of this technique is the transformation of a frequency-selective fading channel with a large bandwidth into a number of flat fading sub channels.

This work proposes to analyze the MIMO-OFDM system using the Golden code in Rayleigh selective-channels. The coherent MIMO-OFDM detector discussed in this work needs the knowledge of the channel transfer function. Thus, a data aided estimation channel method based on the pilot symbol insertion is discussed. This paper is organized as follows. In section II, the MIMOOFDM transmitter model is described. Section III presents the Rayleigh selective channel model considered in this work. Section IV discusses a coherent MIMO-OFDM detector. The channel estimation technique is provided in section V. In Section VI, we present simulation analyses for different scenarios and compare them. Finally, a conclusion is given in VII.

II. TRANSMITTER MODEL

Let us consider the baseband-equivalent ASTC-MIMOOFDM system, with n_t = 2 transmit antennas and n_r = 2 receive antennas, depicted in Fig. 1. The transmitted binary source sequence b_i is QPSK modulated. The QPSK symbols s_k are then ASTC encoded.

We note $v_k = [s_{4k-3}, s_{4k-2}, s_{4k-1}, s_{4k}]^T$ the ASTC encoder input and C_k its output at time k. As in [3], we express the ASTC output as

$$C_k = \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha(v_k[1] - \theta v_k[2]) & \alpha(v_k[3] + \theta v_k[4]) \\ \overline{\alpha}(v_k[3] \quad \overline{\theta} v_k[4]) & \overline{\alpha}(v_k[1] + \overline{\theta} v_k[2]) \end{pmatrix} \quad (1)$$

where

$$\theta = \frac{1+\sqrt{5}}{2} \qquad \overline{\theta} = \frac{1-\sqrt{5}}{2} \alpha = 1+i-i\theta \qquad \overline{\alpha} = 1+i-i\overline{\theta} \quad (2)$$

The ASTC output is converted to a serial stream and then fed to n_t OFDM modulators with n_c subcarriers and a cycle prefix (CP) of length n_g. The q^{th} (q = 1, 2) colon of C_k, given by equation 1, is transmitted by the qth antenna. We note $\{x_k^q\}$ the sequence at the qth OFDM modulator. After a serial to parallel conversion, the OFDM modulator uses an IFFT module and a CP is added. The overall vectors of length n_c+ n_g are transmitted over a frequency and time selective MIMO channels. The CP length n_g is assumed to be longer than the largest multipath delay spread in order to avoid OFDM inter symbol interference. Let us denote by $x_{n,k}$ the $n_t \times 1$ MIMO vector to be transmitted on the n^{th} subcarrier at time k. The k^{th} MIMO-OFDM symbol is then given by

$$u_k = \xi_1 \sqrt{n_c} (F^{-1} \otimes I_{n_t}) x_k$$

where F^{-1} is the n_c x n_c Fourier matrix, of which the $(n; k)^{th}$ element is exp $(-j2\pi nk/n_c)$, $\otimes$ denotes the

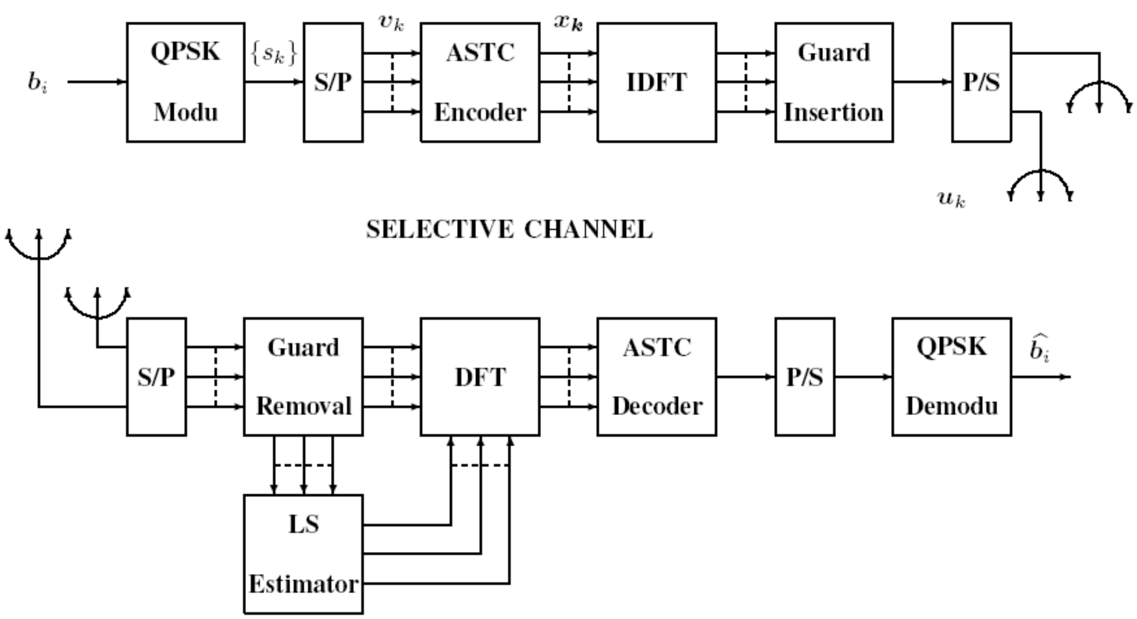

Fig. 1. ASTC-MIMO-OFDM Transceiver.

Kronecker product, I_{nt} represents the n_t identity matrix, x_k is an $n_c n_t$ x 1 vector given by

$$x_k = \begin{pmatrix} x_{1,k} \\ \vdots \\ x_{n_c,k} \end{pmatrix} \qquad (3)$$

and ξ_1 is the CP adding matrix given by

$$\xi_1 = \left[\begin{pmatrix} 0 & I_{n_g} \\ & I_{n_c} \end{pmatrix} \otimes I_{n_t} \right] \qquad (4)$$

The $n_t(n_c+n_g)$ length MIMO-OFDM symbol is transmitted over a time and frequency selective channel.

III. THE CHANNEL MODEL

As mentioned above, we assume that the MIMO-OFDM symbols are transmitted over a time and frequency selective Rayleigh channel and that the channel taps remain constant during a packet transmission. Consequently, the channel impulse response (CIR) between q^th^ transmitting antenna and p^th^ receiving antenna is modeled by a tapped delay line as

$$h_k^{p,q} = \sum_{l=0}^{L-1} h_k^{p,q}(l)\delta(k-l) \qquad (5)$$

where

$h_k^{p,q}(l)$ is the l^{th} path from the q^{th} transmitting antenna to p^{th} receiving antenna at time k and L is the largest order among all impulse responses. The channel taps sequence $\{h_k^{p,q}(l)\}$ is a correlated complex Gaussian process with zero mean, the same variance σ_k^2 h and the autocorrelation function

$$E\{h_k^{p,q}(l)\left[h_{k-k'}^{m,n}(l')\right]^*\} = \rho_{Rx}^{(m,p)}\rho_{Tx}^{(n,q)}J_0(2\pi f_m k')\delta(l-l') \qquad (6)$$

where J_0 is the Bessel function with zero order, f_m is the normalized Doppler shift $\rho_{Rx}^{(m,p)}$, $\rho_{Tx}^{(n,q)}$ refers respectively to the correlation coefficient between the received antennas (m, p) and the transmitted antennas (n, q).

The received signal at the p^{th} receiving antenna during the k^{th} MIMO-OFDM symbol is

$$y_k^p = \sum_{q=1}^{n_T}\sum_{l=0}^{L-1} h_k^{p,q}(l)u_k^q(k-l) + w_k^p \qquad (7)$$

where u_k^q is the symbol vector transmitted by the q^{th} antenna and w_k^p is a zero mean white Gaussian complex noise of variance $N0/2$.

Let us introduce the n_t x n_r matrices $h_k(l) = [h_k^{p,q}(l)]$ for $l = 0, \cdots, L-1$ and define the equivalent channel matrix

$$h_k = \begin{pmatrix} h_k(0) & & \\ \vdots & \ddots & \\ h_k(L-1) & \dots & h_k(0) \end{pmatrix} \qquad (8)$$

We can thus express the MIMO-OFDM received signal in a matrix notation as

$$y_k = h_k u_k + w_k \qquad (9)$$

represents w_k the AWGN at time k with n_r n_c i.i.d. elements.

IV. THE COHERENT DETECTION

At the receiver, after removing the CP, the signal is transformed back to the frequency domain by the mean of a DFT process. The signal at the DFT output is then given by

$$Z_k = \frac{1}{\sqrt{n_c}}\left[(F \otimes I_{n_t n_r})\xi_2\right] y_k \tag{10}$$

where the CP removing matrix ξ_2, which discards the $n_g n_r$ first elements of y_k, is defined as $[0_{n_c n_g} I_{n_g}]$.
By combining equations (9) and (10), we can re-express the DFT output as

$$\begin{aligned} Z_k &= \left[(F \otimes I_{n_t n_r})\xi_3(F^{-1} \otimes I_{n_t n_r})\right] x_k + W_k \\ &= H_k x_k + W_k \end{aligned} \tag{11}$$

where the circulant matrix ξ_3 is defined as $\xi_3\ \xi_2 h_k \xi_1$ and is the frequency domain noise with zero mean and variance σ^2_W and H_k is $n_t n_c$ frequency domain matrix defined as

$$H_k = \left[(F \otimes I_{n_t n_r})\xi_3(F^{-1} \otimes I_{n_t n_r})\right] \tag{12}$$

Under the assumption that the sub carriers are perfectly orthogonal and that the CP length is larger that the expected delay spread, it can been shown that the transmitted symbols are affected by amplitude attenuation and phase rotation. The channel gain matrix H_k is then a block diagonal matrix given by

$$H_k = \begin{pmatrix} H_{1,k}(0) & & 0 \\ & \ddots & \\ 0 & & H_{n_c,k} \end{pmatrix} \tag{13}$$

where n^{th} block H_n, is a n_t x n_r matrix which represents the MIMO channel gain at the n^{th} subcarrier and can be written as

$$H_{n,k} = \sum_{l=0}^{L-1} h_k(l) \exp\left(-j2\pi \frac{nl}{n_c}\right) \tag{14}$$

As illustrated by Fig. 2, simulation of MIMOOFDM system, which will be discussed in section VI, shows that the frequency domain channel matrix is diagonal. The received frequencydomain signal $Z_{n,k}$ at the DFT output can be expressed as

$$Z_{n,k} = H_{n,k} x_{n,k} + W_{n,k} \tag{15}$$

Therefore, we can describe the MIMOOFDM transmission system as a set of parallel flat fading channels with correlated attenuation $H_{n,k}$.
The channel effect (attenuation) in equation (11) is then compensate as

$$\hat{x}_k = H_k^{\dagger} Z_k \tag{16}$$

where (.)† denotes the pseudo-inverse operator. Once the channel effect is compensate, the decision variable $\hat{x}_k$ is passed for decoding. The maximum likelihood Golden code decoding can be performed using the sphere decoder or the Schnorr-Euchner algorithms [3]. In this work, we propose to use the zero forcing suboptimum decoder which reduces the numerical complexity without significant performance loss.

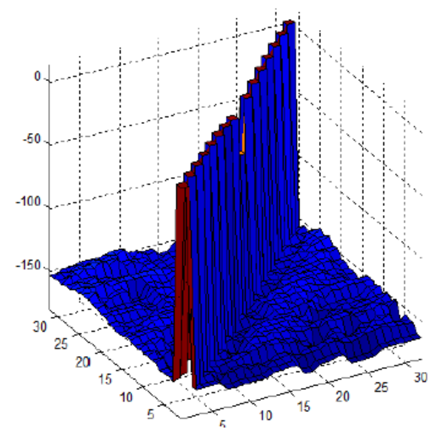

Fig. 2. Frequency domain channel matrix H_k for n_c 32.

A serial to parallel module, at each DFT output, is used reshape the signal $\hat{x}_k$ and to provide the sequences $\{\hat{c}_k^q\}$ for q = 1, ··· n_t. At time k, the decoder input is then and its output is given by

$$\hat{v}_k = \phi^{-1}\hat{c}_k \tag{17}$$

where

$$\phi = \frac{1}{\sqrt{5}}\begin{pmatrix} \alpha & \alpha\theta & 0 & 0 \\ 0 & 0 & i\overline{\alpha} & i\overline{\alpha\theta} \\ 0 & 0 & \alpha & \alpha\theta \\ \overline{\alpha} & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

V. CHANNEL ESTIMATION

As shown in equation (16), the signal correction needs the knowledge of the channel response which is generally unknown. In this section, we present a channel estimation method for OFDM systems using pilot symbols [8], [9]. For MIMOOFDM systems, pilots are inserted in both time and frequency domains as it is shown in Fig. 3. Let us denote O the number of pilots by an MIMOOFDM symbol and introduce the vector $x_k^{(o)}$ of length O defined as in equation (3) whose elements are the pilot symbols.

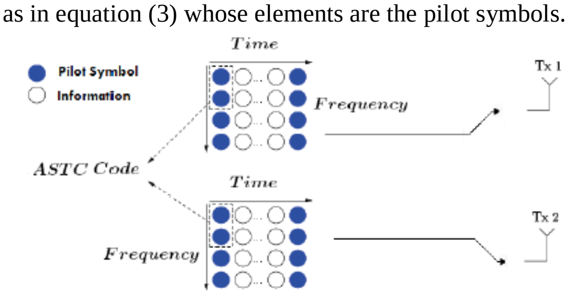

Fig. 3. A pilot symbol insertion in both time and frequency domains.

A straightforward channel estimation method at pilot location, which is based on the LS criterion, is given by

$$\hat{H}_k^{(o)} = \left[x_k^{(o)}\right]^{\dagger} Z_k^{(o)} \tag{19}$$

where subscript (o) indicates pilot symbol.

The channel frequency response at nonpilot positions is then estimated by interpolating the channel estimates at neighboring pilot symbol positions. Several efficient interpolation techniques for OFDM channel estimation have been investigated in [8]. In this work, we use the linear interpolation for its simplicity.

VI. SIMULATION ANALYSIS

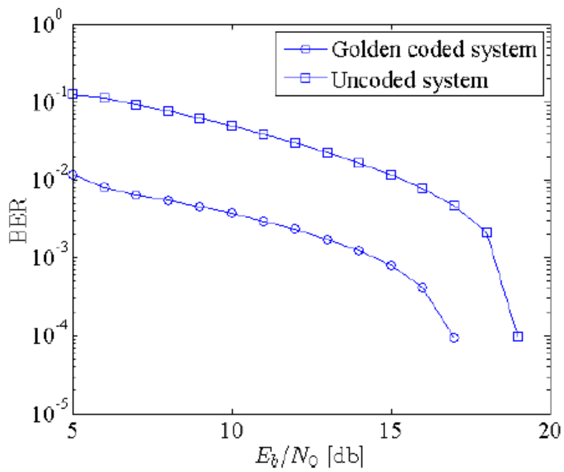

Fig. 4. Normalized Doppler frequency effect.

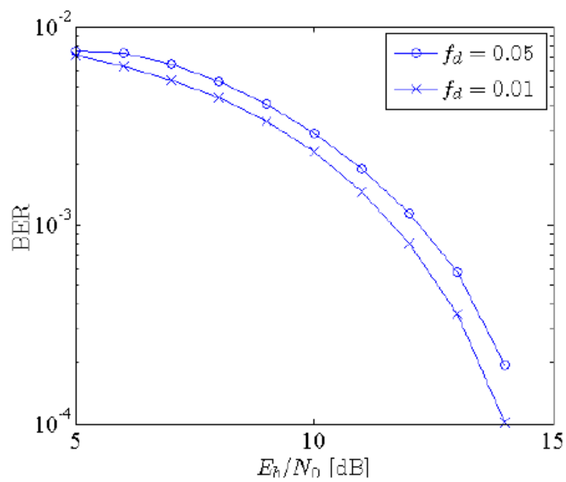

Fig. 5. Effect of the Doppler frequency shift.

To investigate the performance of the studied ASTC MIMOOFDM system, a series of Monte Carlo simulations were carried out. All simulations were performed for at least 100 transmission bit errors, and BER curves were averaged over 50 Monte Carlo trials.

Fig. 4 depicts the BER versus E_b/N_0 performance for Golden coded and uncoded MIMOOFDM systems. The gain obtained by the Golden code is 2.2 dB for a BER of 10^{-3}. Fig. 5 shows the BER versus E_b/N_0 performance for tow normalized Doppler frequencies, when the

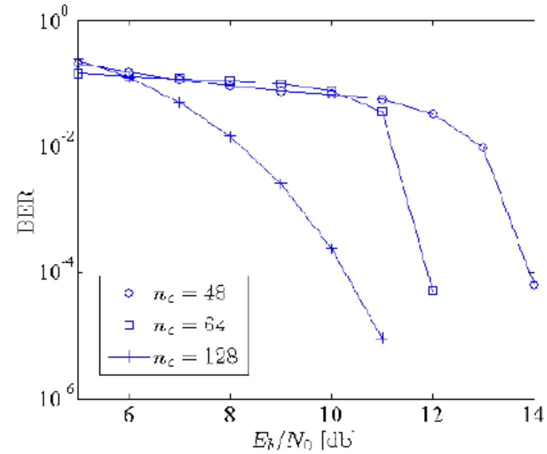

Fig. 6. Performances for different subcarrier number n_c.

vehicle speed *is* $v_h = 30km/h$ and the system is operating in the 9 GHz band. We note that the performances are better when the normalized fading rate f_d decreases. The discrepancy between curves corresponding to f_d 0.01 and f_d 0.05 is about 0.5 dB at a BER of 10^{-3}.

The BER performances for different subcarrier number (n_c = 48, 64 and 128) are given in Fig. 6. As expected, the BER performances increases as as the number of subcarrier increases. At a BER of 10^{-4}, the gain obtained by increasing nc from 64 to 128 is about 2 dB.

VII. CONCLUSION

The Golden code is an optimal ASTC code with a full rate and a full diversity that has a maximal coding gain and good performance in flat fading channels. For a high bit rate transmission over a multi path channel, the received signal is affected by the channel ISI. In this paper, we have considered the MIMOOFDM system with GC coding. The OFDM technique allows to overcome the channel selectivity. Thus, the GC codes maintain their properties and achieve good BER performances.

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