



## Design of a Double Acceptance Sampling Plan to Minimize a Consumer's Risk Considering an O.C. Curve; A Case Study

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**ABSTRACT :** This paper uses the Two-Point method of sampling plan design, whereby the person designing the plan specifies two points on the operating characteristics curve. The following important visualization tools of this method will be explained (Operating characteristics curve, Consumer's risk point, Decision-making regarding with the minimization of consumer's risk).

It will show how to minimize the consumer's risk by matching different types of sampling plans. Construction of different O.C curves for the sampling plans and selection of a beneficial sampling plan. This paper consists of the methods of minimization of consumer's risk in acceptance sampling. Means a step towards the quality improvement and quality control by eliminating the chances of acceptance of defective raw material and components.

**Keywords :** Consumer's Risk, O.C. Curve, Acceptable Quality Level (AQL), Lot Tolerance Percent defective (LTPD), Average outgoing quality level (AOQL)

### I. INTRODUCTION

Acceptance sampling involves a system of Principles and methods. Its Purpose is to develop decision rules to accept or reject the lot or sample based on a sample data factors which are :

1. The quality requirements of the Product in the marketplace.
2. Capability of the process.
3. Cost and logistic of sample taking.

The no inspection alternative is useful in situations where either the vendor's process is so good that defective units are almost never encountered or where there is no economic justification to look for defective units for example, if the vendor's process capability ratio 3 to 4 acceptance sampling is unlikely to discover any defective units. We generally use 100% inspection in situation where the component is extremely critical and passing any defectives would result in an unacceptably high failure lost at subsequent stages or where the vendor's process capability is inadequate to meet specifications.

It is pointed that acceptance sampling out that acceptance sampling is a middle ground between the extremes of 100% inspection and no inspection. It often provides a methodology for moving between these extremes as sufficient information is obtained on the control of the manufacturing process. While there is no direct control of quality in the application of an acceptance- sampling plan to an isolated lot, when that plan is applied to a stream of lots from a vendor, it becomes a means of providing

protection for both the producer of the lot and the consumer. It also provides for an accumulation of quality history regarding the process that is useful in process that produce the lot, and it may provide feedback that is useful in process control at the vendor's plant are not adequate. Finally it may place economic or psychological pressure on the vendor to improve the production process.

### II. THE DODGE-ROMIG SYSTEM

The Dodge romig acceptance sampling systems is made up of AQL schemes, although provisions exist in the standards for constructing AOQL and LQL schemes. The first acceptance sampling system to propose the idea of the average outgoing quality limit (AOQL) as an index was published by Dodge and in the Bell System Technical Journal (1924). This system was published subsequently as Sampling Inspection Tables (1959) by Dodge and is still widely used. An abbreviated compilation of the sampling plans along with a guide to their use has been published as ASTM E 259-98, entitled Standard Practice for the Use of Process Oriented AOQL and LTPD Sampling Plans (1999).

The AQL acceptance sampling systems are used to provide protection to the buyer or consumer from accepting very bad lots and to encourage the producer to provide good lots consistently. The direct effect on quality of AQL acceptance sampling plans is negligible in most situations. For example, if the quality of each lot is identical (say,  $p\%$  non-conforming), any AQL sampling system will accept some fraction of the lots inspected and reject the rest. Except for the minor effect of removing the nonconforming items found in the samples taken from accepted lots, the quality of

accepted lots will be the same as the quality of the submitted lots (in this example,  $p\%$  nonconforming). The use of AQL acceptance sampling systems has a direct effect on the accepted quality that is proportional to the variance of the quality of the submitted lots. To illustrate this, consider the following simple hypothetical example.

Dodge-Romig addressed the problem of controlling the average quality through sampling inspection more directly. By requiring that all rejected lots be 100% reinspected and the nonconforming items removed and replaced by good items, they found that they could calculate the average outgoing quality limit (AOQL). This limit was the worst average quality that would be passed regardless of the quality of the submitted lots or the distribution of the quality of submitted lots. Obviously, one way perfect quality could be received would be for all lots to be rejected, inspected 100%, nonconforming items replaced with good items, and resubmitted. (The reader has probably noticed that this and preceding discussions make a very strong assumption, that inspection is 100% efficient. That is, every nonconforming item inspected will be detected to be nonconforming. No inspection process is ever 100% efficient. Even automatic inspection with the best computer-driven test set has some positive probability of a nonconforming item being accepted and vice versa.)

What Dodge and have done is to derive acceptance sampling plans that not only give the consumer quality at least as good as the desired AOQL, but do so with a minimum amount of inspection. Minimum amount of inspection refers to the overall inspection: that performed on the initial samples plus that involved with the complete reinspection of rejected lots. These acceptance-sampling plans were also designed with the understanding that lots are often “merely quantities whose size was determined by convenience in handling. In contrast with customer lots, which are specific in quantity and which commonly retain their identity, the inspection lots in manufacture were usually convenient subdivisions of a flow of product”.

The Dodge- plans were designed for use with rectifying inspection. In practice this may not be practical or possible (e.g., with destructive testing). In these cases the Dodge-plans may still be used, but the consumer is no longer guaranteed that the average outgoing quality will be better than. The AOQL value. In practice, however, the high rate of rejected lots with quality much worse (two times the AOQL value) and high rate of acceptance for lots with quality better than the AOQL value “will tend to compel the producer in his own interest, to maintain a process quality which at worst will be little, if any, poorer than the AOQL”.

**III. CASE STUDY AT BOSCH LTD**

**Company Profile :**

BOSCH limited is a joint venture company incorporated in 1982 for manufacture of hydraulic automotive brakes

system and their aggregates. Their promoters and collaborates are internationally known as Robert Bosch GMBH of Stuttgart, Germany and Kalyani group of India. They are originally equipment supplier to M/s Maruti Udyog Ltd., M/s Tata Engineering and locomotive co. Ltd, M/s Mahindra and Mahindra Ltd. M/s Bajaj Tempo Ltd. M/s Hindustan Motors Ltd. M/s Bajaj Auto Ltd. - M/s Piaggio Greaves Vehicle Ltd. And others.

Bosch manufactures and sales two broad categories of products.

1. Brake system aggregates of proprietary design of OEM.
2. Contract item manufactured as per customers drawing.

KBX is also supplying brakes aggregates components and service kits to the market in Australia, Europe and North America etc.

**IV. MATHEMATICAL MODEL**

For calculation of probabilities of acceptance, a Poisson’s Distribution method is used as given below.

$$e^{-x} \cdot e^x = e^{-x} + xe^{-x} + \frac{x^2 e^{-x}}{2!} + \frac{x^3 e^{-x}}{3!} + \dots$$

$\downarrow \quad \downarrow$   
 P(0) P(1) P(2) P(3)

**Existing Sampling plan adapted by a company:**

S No.	Lot size (N)	Sample size Measurement visual inspection			
		n	c	n	C
1	50-500	15	0	25	0
2	501-1000	20	0	30	0
3	1001-2000	25	0	35	0
4	2001-3000	30	0	40	0
5	3001-4000	35	0	35	0
6	4001-5000	40	0	50	0
7	5001-onwards	45	0	60	0

**Preparation of an O.C curve on existing sampling plan:**

**Lot size (N) = 1500**

∴ Sample size and acceptance number selected from sampling plan,

n = sample size = 25

c = acceptance number = 0

∴ Inspected the sample of 25 item and if number of defectives are less than equal to c (i.e. c = 0) then accept the lot or if it is more than c (c = 1, 2, 3 ...).

Then reject the lot.

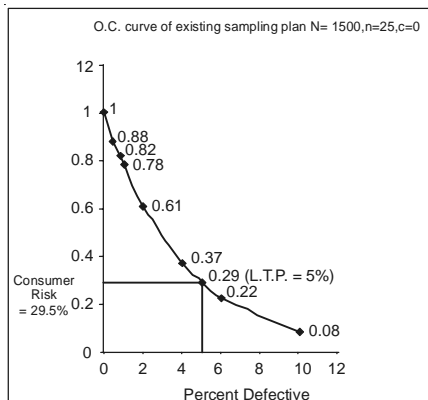
We have to draw an O.C curve for this assume the % defectives in a lots as 0.5%, 0.8%, 1%, 2%, 4%, 6% and 10%.

We have to calculate probability of acceptance (Pa) for above % defectives.

It can be calculate by Poisson's distribution given below.

S No.	Percent defectives in a lot (%) sample	Average number defectives in a = d - x	Probability of acceptance P(a) = P(0)
1	0.5%	0.125	0.88
2	0.8%	0.20	0.82
3	1%	0.25	0.78
4	2%	0.50	0.61
5	4%	0.10	0.37
6	6%	1.50	0.22
7	10%	2.50	0.08

An O.C. curve plotted on a graph is shown.



If we consider lot tolerance percent defectives as 5% corresponding consumer's risk can be calculated from the graph.

From fig if we consider LTPD= 5%. Consumers risk is 29.5%

∴ Hence consumer's risk is 29.5% with the existing sampling plan. It must be minimized.

**Limitations and problems in existing sampling plan:**

1. Lot tolerance percent defectives are larger.
2. As this sampling plan is based upon single sampling, there may be possibility of acceptance of defective lots.
3. Sample size is very small, even though the cost of inspection is lower but probability of acceptance of such a lot may be more and hence we cannot judge by sampling of

small quantity of samples. And it is quiet harmful for an organization's quality objectives.

4. As acceptance number C=0, vendors may be supply by considering this acceptance number. However some tolerance limit should be given to the vendors for better co-ordination and commitment.

5. Consumer's risk in a single sampling plan is always larger and it is acceptance of such a lot, which would have been rejected. It can affect the consumer and his next production work and assembly work.

6. As the component for case, selected is wheel cylinder of a brake system. It is an important of that system and related with the human lives. Hence manufacturing organization should take care, that such a defective items should not be accepted.

7. Consumer's risk at a constant LTPD =5% is near about 29.5%. it is most harmful for an organization.

**Possible Remedies for Overcoming the Limitations:**

1. If increases the sample size and increases the acceptance number lot tolerance % defectives can be minimized. But still it is single sampling hence there may be possibility of acceptance of defective components.

2. By using double sampling inspection, LTPD can be minimized. Hence we can use the double sampling. But what will be the criteria about the decision of lot size and respective sampling size and acceptance numbers.

3. Can be use MIL-STD-105D Tables or Dodge -Romig sampling inspection. By using above plans we can minimize the risks in acceptance sampling.

4. But the MIL-STD-105D table provides plans that emphasize the protection of the producer against rejecting good lots. Also concept of minimum total inspection is not incorporated and there fore it is not assumed that rejected lots are 100 percent inspected.

5. The Dodge-Romig tables provide plans that emphasize the protection of the consumer against accepting bad lots. Beside this the plans incorporate the concept of minimum total inspection. The Dodge-Romig tables may be used for providing specified consumer protection at a minimum total inspection cost for a process for a previous run of parts; we can pick out of the table a plan that gives this assurance on a minimum amount of inspection.

The type of quality protection may be either for lot tolerance percent defectives (LTPD) or for average outgoing quality limit (AOQL). But we want a sampling plan based upon LTPD and having double sampling procedure.

From above discussion Dodge-Romig sampling plan can

be considered for the acceptance sampling.

**Design of double sampling plan for L.T.P.D = 5%:**

**Selection of sampling plan:**

**Lot size (N) = 1500**

**Sample size (n<sub>1</sub>) = 55                      Sample size (n<sub>2</sub>) =35.**

**Acceptance No (C<sub>1</sub>) = 0                      Acceptance No. (C<sub>2</sub>) = 1.**

Probability of acceptance and LTPD of the above plan can be calculated by taking 0.5, 0.8, 1, 2, 4, 6, and 10 percent of defectives per lot.

**1. Condition first:**

If there are zero defectives in first sample, *i.e.*  $[P(a)]_I = [P(0)]_I$

**2. Condition second:**

If there is one defective in first sample and zero defective in second sample,

$$i.e.. [P(a)]_{II} = [P(1)]_I \times [P(0)]_{II}$$

Then the lot will be accepted

∴ Probability equation is given by

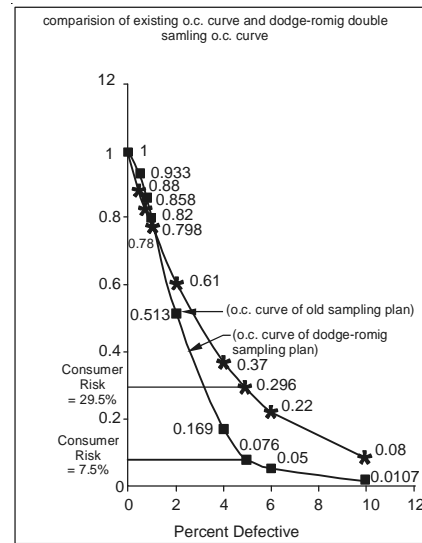
$$[P_{(a)}]_{combined} = [P(a)]_I + [P(a)]_{II}$$

Calculations are as follows

**From the above conditions we can calculate probability of an acceptance:**

S No.	% Defectives in a lot	$[P_{(o)}]_I$	$+ [P_{(a)}]_{II}$	$= [P_{(a)}]_{combined}$
1	0.5	0.759	0.174	0.933
2	0.8	0.644	0.214	0.858
3	1	0.576	0.222	0.798
4	2	0.0332	0.181	0.513
5	4	0.110	0.059	0.159
6	6	0.036	0.014	0.050
7	10	0.0040	0.00672	0.0107

**We can plot an O.C curve as shown in Graph.**



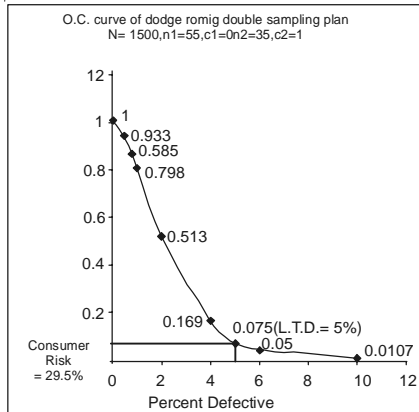
From O.C curve the consumer's risk (b) is minimized up to 7.5% for a constant LTPD = 5%.

Hence consumers risk is minimized up to 7.5 %.

**V. RESULT AND CONCLUSION**

In acceptance sampling procedure the decision-making about the acceptance and rejection of the incoming material takes place. In previous discussion it is clear that various sampling methods and sampling plans for decision-making. Each sampling plan can be launched after forming of a O.C curve for the particular LTPD. Means it can understand the probability of acceptance (Pa) for the respective percent defectives and then the sampling plan is selected for actual practice. As the sampling is at random, there may be both risks as stated in literature, consumer's risk and producer's risk. Now consumer's risk is very harmful for an organization, as there may be possibility of acceptance of defective lot with maximum LTPD. Various methods are there for minimization of this risk but it was time consuming and also cost increasing. In this paper it is considered an existing sampling plan of BOSCH limited.

**Comparison of existing and Designed new Double sampling plan:**



**Sampling plan designed for implementation with LTPD = 5% :**

An O.C Curve is plotted by Poisson’s distribution method. LTPD selected in

Whole case study was as 5 percent. Now the existing plan shows the 29.5 percent of consumer’s risk. This is very harmful for an organization. That is there was a 29.5% chance of acceptance of the lots, which contains 5 percent defectives in each. It is taken a decision to minimize this risk by applying various techniques for these three trials are carried. in first, by increasing the sample size and acceptance number and two trials were on the basis of trial and error basis by making an o.c.curve.

All the considerations are already stated in the case and literature.

From the selected sampling plans it is prepared an O.C curve which shows the minimization of the consumer’s risk. In the case study it is succeeded to minimize the consumer’s risk by selecting a sampling plan from the trial and error double sampling and at 5% lot tolerance percent defective. The new selected sampling plan is given to an organization for the implementation. Last but not least, the overall conclusion we can state that the consumer’s risk should be always minimum for the manufacturing organization. In current days quality is most important and methods are stepped towards the zero defects and hence if we prevent the acceptance of defectives incoming material our further cost of failure can be saved effectively. From overall paper and the case study and the result and discussion it is tried of best to minimize the consumer’s risk in acceptance sampling and tried to give more relief to the BOSCH limited

in their acceptance sampling procedure. It is believed that new suggested sampling plan would be beneficial for an organization.

Lot size (N)	Trial -1		Trial - 2		
	$n_1$	$c_1$	$n_2$	$n_1+n_2$	$c_2$
50-75	38	0	–	–	–
76-100	44	0	21	65	1
101-200	49	0	26	75	1
201-300	50	0	30	80	1
301-400	55	0	30	85	1
401-500	55	0	30	85	1
501-600	55	0	30	85	1
601-800	55	0	35	90	1
801-1000	55	0	35	90	1
1001-2000	55	0	35	90	1
2001-3000	55	0	65	120	2
3001-4000	55	0	65	120	2
4001-5000	55	0	65	120	2

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