



A Generalized Exponential Model based Analysis of Daily Low Temperature Data of Ahmedabad City using Markov Chain Approach

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ABSTRACT: The rise in atmospheric temperature over time is significantly affected by global warming. Many studies have been done on climate change. Analysis of meteorological phenomenon is a bit complex, so, this research paper is an attempt to model daily low temperature data with a Markov chain approach using generalized exponential model. We have used the data of daily low temperature of Ahmedabad, India, from 1st January 2001 to 31st December 2011. The analysis is carried out based on seasonal (winter, summer, and monsoon) and yearly data. The data is divided into 11 states for winter, summer, and yearly data and 6 states for monsoon with a smaller length of interval of each state. The transition probability matrices (TPMs) are prepared to transit among states. Generalized exponential model is also used to predict temperature for the next day. It is a bit challenging task to utilize statistical distribution and Markov chain model simultaneously to analyze daily temperature data. To verify our model, we have simulated the temperature for the same period as in the data and the results are compared. A simulation study is also carried out to generate temperature for future 5000 and 10,000 days and the results are shown. The simulated results are highly matched with the original data.

Keywords: Daily temperature, generalized exponential distribution, Indian season wise temperature, simulation, stochastic model, transition probability matrix.

I. INTRODUCTION

A lot of work has been done on analysing the hydrological and meteorological data since decades. Many studies have been done specifically, regarding the prediction of rainfall, temperature, evaporation, and precipitation. In meteorological data analysis temperature is one of the valued factors.

In this study we have considered Ahmedabad's daily low temperature data (in Celsius (°C)) from 01-01-2001 to 31-12-2011. Ahmedabad, the largest city in Gujarat state and lies at 23.03°N 72.58°E in western India at 53 meters (174 ft) above sea level on the banks of the Sabarmati river, in north-central Gujarat. The overall data is classified into three groups indicating the three seasons as monsoon (June to September), summer (February to May) and winter (October to January). In Ahmedabad daily low temperature lies almost between 5°C to 35°C. The historical data of 11 years, since 2001 to 2011 was collected from the website – <https://www.wunderground.com/history/daily/VAAH/date/2001-1-1> [1].

The data consists 4007 observations and they are bifurcated into three seasons as 1353 observations for winter, 1312 observation for summer and 1342 observations for monsoon season.

The data set is used for statistical analysis. In literature various types of time series models like ARIMA and exponential smoothing are utilized for analysis of such data.

We have considered the daily temperature as the stochastic random variable as time dependent event.

We have estimated average low temperature and average minimum-maximum of daily low temperature for

season wise as well as entire yearly. A prediction is being made for number of days and percentage for low temperatures, greater than 12, 14, 18, 20, 24 and 26 degree Celsius as well as less than 9, 11, 15, 17, 25 and 27 degree Celsius for winter, summer and yearly temperature. As there is some different temperature range in monsoon season, the number of days with low temperatures greater than 24 and 26 degree Celsius predicted. In a similar manner less than 25 and 27 degree Celsius for monsoon season is also considered for prediction.

In India, generally the trend of temperature had been analyzed using time series modelling [2]. The generalized exponential distribution is also used in design of rainfall estimation [3]. A specific stochastic model using three main variables as durations, magnitude and peak value have been developed for consecutive episodes of environmental observations [4]. The truncated exponential distribution has also been applied for the daily temperature data [5].

II. MATERIALS AND METHODS

We have used the generalized exponential distribution as a model for temperature data. It is also known as exponentiated exponential distribution. Generalized exponential distribution can be used to analyze the lifetime data in place of two parameter gamma or two parameter Weibull distribution.

Generalized exponential distribution was introduced around two decades ago. The three parameters generalized distribution fits well than the three parameter Weibull distribution or the three parameters gamma distribution [6]. Exponentiated exponential

distribution have somewhat similar properties to the properties of Weibull or a gamma distribution [7]. And further this distribution is expanded with six parameters generalized extended inverse Gaussian density function involving a confluent hypergeometric function of two variables [8]. A Bayesian model is also used for the prediction of the rainfall data [9]. Weibull-gamma composite is the bestfit model to describe the wind turbines actual power curves at the highest power.

Generalized exponential distribution has several properties similar to gamma distribution. The distribution is very similar to the Weibull distribution. Generalized exponential distribution reduces to exponential distribution when shape parameter is equal to 1. This distribution is right skewed unimodal distribution. The detail study regarding generalized exponential distribution and its properties are precisely explained [10]. The flood frequency of Polish rivers is also analyzed with generalized exponential distribution [11]. In health care industry diagnosis of any specific disease can also be predicted and compared with various data mining techniques like Bayesian network, decision tree and support vector machine [12].

The daily low temperature data is grouped into 11 states for winter, summer and the whole yearly data whereas 6 states are prepared for monsoon season. The states are shown in Table 1.

Let Y_t (where $t = 1, 2, \dots, N$) be the daily low temperature observation for the day t , and the states as $B_1, B_2, B_3 \dots B_{11}$.

Table 1: States for low temperature.

States	Temperature (°C) for winter, summer and entire year	Temperature (°C) for monsoon
1	5-8	21-22
2	9-10	23-24
3	11-12	25-26
4	13-14	27-28
5	15-16	29-30
6	17-18	31-33
7	19-20	-
8	21-22	-
9	23-24	-
10	25-26	-
11	27-35	-

We have measured the transition of the daily temperature from one state to another state. For example, on the date of 28th November 2003 the daily low temperature is 5°C so, it belongs to state 1, and the next day 29th November 2003 the temperature captured is 17°C which belongs to state 6. Hence, we say that there is a transition from state 1 to state 6. The transition probability for transition from state i to state j is denoted as p_{ij} , $i, j = 1, 2, \dots, 11$.

We have developed the transition probability matrix $P=[p_{ij}]_{11 \times 11}$ for winter, summer and yearly data and $P=[p_{ij}]_{6 \times 6}$ for monsoon data.

The transitional probability p_{ij} is calculated using the frequency of each state and frequency f_{ij} , which denotes the number of days transits from the state B_i (Current day) to state B_j (Next day). The transitional probability matrix (TPM) for the three seasons and yearly basis are given displayed in Table 2- 5 (where TD = Today and TM = Tomorrow).

Table 2: TPM for winter season.

TD	TM	Transition probability matrix for winter season										
	1	2	3	4	5	6	7	8	9	10	11	
1	0.1429	0.4286	0.1429	0.0000	0.1429	0.1429	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.3889	0.4444	0.1111	0.0000	0.0185	0.0185	0.0000	0.0000	0.0185	0.0000	0.0000
3	0.0224	0.1418	0.3955	0.3433	0.0746	0.0149	0.0000	0.0000	0.0000	0.0075	0.0000	0.0000
4	0.0046	0.0320	0.2055	0.4384	0.2603	0.0365	0.0091	0.0046	0.0000	0.0046	0.0046	0.0046
5	0.0045	0.0090	0.0362	0.2489	0.4389	0.1855	0.0452	0.0090	0.0136	0.0090	0.0000	0.0000
6	0.0000	0.0000	0.0142	0.0664	0.2417	0.4550	0.1943	0.0095	0.0142	0.0000	0.0047	0.0047
7	0.0050	0.0100	0.0000	0.0100	0.0300	0.2650	0.5200	0.1450	0.0100	0.0000	0.0050	0.0050
8	0.0000	0.0000	0.0000	0.0000	0.0088	0.0354	0.3097	0.4867	0.1062	0.0354	0.0177	0.0177
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0471	0.0588	0.2353	0.4824	0.1647	0.0118	0.0118
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0357	0.2738	0.5595	0.1310	0.1310
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0417	0.0417	0.0417	0.0417	0.5417	0.2917	0.2917

Table 3: TPM for summer season.

TD	TM	Transition probability matrix for summer season										
	1	2	3	4	5	6	7	8	9	10	11	
1	0.5714	0.4286	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.1176	0.3529	0.2353	0.1765	0.0588	0.0588	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.1739	0.3478	0.3478	0.0435	0.0870	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0385	0.0962	0.3654	0.3269	0.1154	0.0577	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0119	0.0238	0.1667	0.3452	0.2976	0.1190	0.0238	0.0000	0.0119	0.0000	0.0000
6	0.0000	0.0000	0.0159	0.0397	0.1667	0.4286	0.2460	0.1032	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.0827	0.1729	0.3534	0.2556	0.1053	0.0150	0.0150	0.0150
8	0.0000	0.0000	0.0000	0.0075	0.0150	0.0602	0.1729	0.3759	0.2932	0.0451	0.0301	0.0301
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0181	0.0904	0.1506	0.4277	0.2651	0.0482	0.0482
10	0.0000	0.0000	0.0051	0.0000	0.0000	0.0101	0.0101	0.0354	0.1515	0.4848	0.3030	0.3030
11	0.0027	0.0027	0.0027	0.0054	0.0054	0.0027	0.0054	0.0054	0.0323	0.1317	0.8038	0.8038

Table 4: TPM for monsoon season.

TD	TM	Transition probability matrix for monsoon season				
	1	2	3	4	5	6
1	0.0000	0.5000	0.2500	0.2500	0.0000	0.0000
2	0.0098	0.3431	0.5294	0.1078	0.0098	0.0000
3	0.0033	0.0879	0.7065	0.1874	0.0116	0.0033
4	0.0020	0.0225	0.2331	0.6708	0.0695	0.0020
5	0.0000	0.0085	0.0598	0.2821	0.5556	0.0940
6	0.0000	0.0000	0.0385	0.1154	0.3846	0.4615

Table 5: TPM for yearly data.

TD	TM	Transition probability matrix for yearly season									
	1	2	3	4	5	6	7	8	9	10	11
1	0.3571	0.4286	0.0714	0.0000	0.0714	0.0714	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0282	0.3803	0.4085	0.1268	0.0141	0.0282	0.0141	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.1321	0.5283	0.2830	0.0377	0.0189	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0116	0.0349	0.1512	0.4186	0.3023	0.0581	0.0233	0.0000	0.0000	0.0000	0.0000
5	0.0110	0.0220	0.0220	0.2418	0.2857	0.2967	0.1099	0.0000	0.0000	0.0110	0.0000
6	0.0000	0.0000	0.0090	0.0721	0.2523	0.4144	0.1892	0.0631	0.0000	0.0000	0.0000
7	0.0111	0.0111	0.0000	0.0111	0.0889	0.2333	0.4000	0.1667	0.0556	0.0111	0.0111
8	0.0000	0.0000	0.0000	0.0147	0.0000	0.0441	0.2059	0.3971	0.1912	0.0735	0.0735
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0306	0.0306	0.1327	0.3673	0.3367	0.1020
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0065	0.0032	0.0129	0.1032	0.6645	0.2097
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0053	0.0053	0.0321	0.1738	0.7834

A. Generalized exponential distribution for daily low temperature

The probability density function and the cumulative distribution function of generalized exponential distribution in case of j^{th} state can be represented as follows:

$$f_j(x) = \alpha_j \lambda_j (1 - e^{-\lambda_j x})^{\alpha_j - 1} e^{-\lambda_j x}, x > 0, \alpha_j > 0, \lambda_j > 0, \tag{1}$$

and,

$$F_j(x) = (1 - e^{-\lambda_j x})^{\alpha_j}, x > 0, \alpha_j > 0, \lambda_j > 0, j = 1, 2, \dots, 11. \tag{2}$$

The generalized exponential distribution fits well daily low temperature data.

Define $F_j(x) = P$ [Next day low temperature $\leq x$; when low temperature of today belongs to the state B_j], where $j = 1, 2, 3, \dots, 11$.

The mean and variance of the generalized exponential distribution can be given as:

$$\mu_1 = \frac{\psi(\alpha_j + 1) - \psi(1)}{\lambda_j}; \text{ where } \alpha_j > 0, \lambda_j > 0, j \text{ represents the state, } j=1, 2, \dots, 11. \tag{3}$$

$$\mu_2 = \frac{\psi'(1) - \psi'(\alpha_j + 1)}{\lambda_j^2}; \text{ where } \alpha_j > 0, \lambda_j > 0, j \text{ represents the state, } j=1, 2, \dots, 11. \tag{4}$$

Here ψ' is a derivative of ψ (di-gamma function) which is known as tri-gamma function.

With the help of method of moments, the parameters α_j and λ_j are estimated based on the observed data of j^{th} state for $j = 1, 2, \dots, 11$, season wise and yearly data. The results are shown in Table 6.

B. Simulation for comparison with original data

In this section first of all we have simulated the temperatures for the 4007 days starting from the first day of the observed data using the values of λ_j and α_j from Table 6. And the results are compared with the original data. The results obtained from simulation study exhibits almost close to the original data.

C. Simulation for predicting the future

The simulation is also carried out for future days, starting from the end of the observed data. The simulation is done for 5000 days as well as for 10,000 days in case of the season wise and yearly data using R software.

Table 6: Values of α_j and λ_j for each state.

State	Winter season (October to January)		Summer season (February to May)		Monsoon season (June to September)		All seasons combine (January to December)	
	α_j	λ_j	α_j	λ_j	α_j	λ_j	α_j	λ_j
1	0.990486	326.5767	1.199516	2487.911	0.738121	6334020	1.489796	648.0234
2	1.654744	5123189	1.541808	1349585	0.696336	8515149	2.279639	2000149794
3	1.290355	1623976	1.308405	1869153	0.647778	8675623	1.908816	2000608388
4	1.157554	3444917	1.138925	3114523	0.637846	21345068	1.625894	2001887721
5	1.012174	3421753	0.990725	2914402	0.576911	12740704	1.421589	2001181401
6	0.898028	3729918	0.925789	5711287	0.564141	24633906	1.258754	2001307715
7	0.845852	8091023	0.836179	6793088	-	-	1.128311	2003272924
8	0.762772	6930886	0.766407	8161500	-	-	1.024165	2002661594
9	0.69822	7351549	0.69932	7382843	-	-	0.934584	2004361872
10	0.649586	7995810	0.705125	36289265	-	-	0.861767	200473638
11	0.585997	6804045	0.588215	8733744	-	-	0.786253	2003094568

D. Simulation algorithm

For simulation purpose the primary state is being considered based on the state of first day from the observed data. And the last day is being considered for the future prediction.

The algorithm steps are mentioned below:

1. Let the primary state is j ($j = 1, 2, 3, \dots, 11$). The uniform random number between 0 to 1 is generated, say q .
2. The random value (q) is compared one by one state wise with cumulative transition probabilities of the state j , till it exceeds the cumulative transition probability of the state, say k . Hence, the future state will be k th state.
3. Select the appropriate values of parameters for the k th state from Table 6.
4. Substitute the values of the parameters in the cumulative distribution function of doubly truncated generalized exponential distribution.

$$F(x | a_j < x < b_j) = \frac{(1 - e^{-\lambda_j x})^{\alpha_j} - (1 - e^{-\lambda_j a_j})^{\alpha_j}}{(1 - e^{-\lambda_j b_j})^{\alpha_j} - (1 - e^{-\lambda_j a_j})^{\alpha_j}} \quad (5)$$

Where (a_j, b_j) is the limit of the j^{th} state, $j = 1, 2, \dots, 11$ and then x will be replaced by a random number between 0 to 1 in the left side of the above equation.

5. Equating the value of random number q with the right side of the cumulative distribution function and using inverse transformation the temperature x is generated for the next day.
6. Taking the state k as primary state repeats the above steps M times to generate temperature for M days.

From the simulated results the descriptive statistics (Min, Max, Average and Standard Deviation) for season wise and entire year are presented in Table 7-10. For the data used and the predicted data the average temperature for winter, summer and monsoon remains closely to 17°C, 23°C and 27°C respectively. From the simulated data we have generated the number of days and the percentages having temperature above 12°C, 14°C, 18°C, 20°C, 24°C or 26°C in winter, summer and the whole year (all seasons combined) for the next year. Similarly, the results are also generated for temperature below 9°C, 11°C, 15°C, 17°C, 25°C and 27°C for winter, summer and the whole year for the next year. For the monsoon season the numbers of days have an average temperature above 24°C and 26 °C as well as numbers of days have an average temperature below 25°C and 27°C are estimated.

E. Simulation based on the square of TPM

In this section we have predicted temperature for future days from the end of the observed data. The one step transition probability matrix is constructed based on transition from today's to tomorrow's data. Therefore, the squared of TPM (T^2) is used for prediction of temperature of the day after tomorrow. The predicted results for different choice of temperatures are presented in Table 15-18. The results are compared with the predictions made on the basis of generalized exponential distribution.

Table 7: Descriptive statistics for observed and predicted temperature (°C) of winter season.

Statistics	Winter			
	Observed N=1353	Predicted		
		N=1353	N=5000	N=10000
Minimum temp.	5	6	4	4
Maximum temp.	35	35	35	35
Average temp.	17	17	17	17
Standard deviation of temp.	4.47	4.89	4.78	4.67

Table 8: Descriptive statistics for observed and predicted temperature (°C) of summer season.

Statistics	Summer			
	Observed N=1312	Predicted		
		N=1312	N=5000	N=10000
Minimum temp.	5	7	5	5
Maximum temp.	35	35	39	39
Average temp.	23	23	23	23
Standard deviation of temp.	5.13	5.41	5.48	5.41

Table 9: Descriptive statistics for observed and predicted temperature (°C) of monsoon season.

Statistics	Monsoon			
	Observed N=1342	Predicted		
		N=1342	N=5000	N=10000
Minimum temp.	22	20	20	19
Maximum temp.	33	37	38	38
Average temp.	27	27	27	27
Standard deviation of temp.	1.64	2.63	2.54	2.55

Table 10: Descriptive statistics for observed and predicted temperature (°C) of yearly data.

Statistics	Yearly data			
	Observed N=4007	Predicted		
		N=4007	N=5000	N=10000
Minimum temp.	5	4	4	4
Maximum temp.	35	37	37	37
Average temp.	22	22	22	22
Standard deviation of temp.	5.55	5.69	5.62	5.49

Table 11: Frequency of simulated low temperature with different ranges for winter season.

Temperature	Observed frequency in days (N=1353)	Observed percentage (%) (N=1353)	Predicted frequency in days (N=1353)	Predicted percentage (%) (N=1353)	Predicted frequency in days (N=5000)	Predicted percentage (%) (N=5000)	Predicted frequency in days (N=10000)	Predicted percentage (%) (N=10000)
<9°C	7	0.52%	2	0.15%	19	0.38%	34	0.34%
<11°C	61	4.51%	50	3.70%	258	5.16%	494	4.94%
<15°C	414	30.60%	475	35.11%	1658	33.16%	3246	32.46%
<17°C	637	47.08%	665	49.15%	2435	48.70%	4872	48.72%
<25°C	1245	92.02%	1226	90.61%	4564	91.28%	9212	92.12%
<27°C	1329	98.23%	1289	95.27%	4794	95.88%	9641	96.41%
>12°C	1158	85.59%	1124	83.07%	4199	83.98%	8448	84.48%
>14°C	939	69.40%	878	64.89%	3342	66.84%	6754	67.54%
>18°C	505	37.32%	514	37.99%	1860	37.20%	3683	36.83%
>20°C	306	22.62%	340	25.13%	1173	23.46%	2341	23.41%
>24°C	108	7.98%	127	9.39%	436	8.72%	788	7.88%
>26°C	24	1.77%	64	4.73%	206	4.12%	359	3.59%

Table 12: Frequency of simulated low temperature with different ranges for summer season.

Temperature	Observed frequency in days (N=1312)	Observed percentage (%) (N=1312)	Predicted frequency in days (N=1312)	Predicted percentage (%) (N=1312)	Predicted frequency in days (N=5000)	Predicted percentage (%) (N=5000)	Predicted frequency in days (N=10000)	Predicted percentage (%) (N=10000)
<9°C	7	0.53%	5	0.38%	31	0.62%	56	0.56%
<11°C	24	1.83%	28	2.13%	110	2.20%	195	1.95%
<15°C	99	7.55%	88	6.71%	379	7.58%	760	7.60%
<17°C	183	13.95%	189	14.41%	735	14.70%	1451	14.51%
<25°C	741	56.48%	772	58.84%	2800	56.00%	5654	56.54%
<27°C	939	71.57%	949	72.33%	3583	71.66%	7211	72.11%
>12°C	1265	96.42%	1255	95.66%	4783	95.66%	9608	96.08%
>14°C	1213	92.45%	1224	93.29%	4621	92.42%	9240	92.40%
>18°C	1003	76.45%	1001	76.30%	3776	75.52%	7586	75.86%
>20°C	870	66.31%	863	65.78%	3335	66.70%	6604	66.04%
>24°C	571	43.52%	540	41.16%	2200	44.00%	4346	43.46%
>26°C	373	28.43%	363	27.67%	1417	28.34%	2789	27.89%

Table 13: Frequency of simulated low temperature with different ranges for monsoon season.

Temperature	Observed frequency in days (N=1342)	Observed percentage (%) (N=1342)	Predicted frequency in days (N=1342)	Predicted percentage (%) (N=1342)	Predicted frequency in days (N=5000)	Predicted percentage (%) (N=5000)	Predicted frequency in days (N=10000)	Predicted percentage (%) (N=10000)
<25°C	106	7.90%	232	17.29%	488	9.76%	1116	11.16%
<27°C	709	52.83%	660	49.18%	2699	53.98%	5424	54.24%
>24°C	1236	92.10%	1120	83.46%	4512	90.24%	8884	88.84%
>26°C	633	47.17%	682	50.82%	2301	46.02%	4576	45.76%

Table 14: Frequency of simulated low temperature with different ranges yearly data (combined all seasons).

Temperature	Observed frequency in days (N=4007)	Observed percentage (%) (N=4007)	Predicted frequency in days (N=4007)	Predicted percentage (%) (N=4007)	Predicted frequency in days (N=5000)	Predicted percentage (%) (N=5000)	Predicted frequency in days (N=10000)	Predicted percentage (%) (N=10000)
<9°C	14	0.35%	26	0.65%	26	0.52%	41	0.41%
<11°C	85	2.12%	109	2.72%	123	2.46%	212	2.12%
<15°C	513	12.80%	515	12.85%	609	12.18%	1201	12.01%
<17°C	812	20.26%	801	19.99%	966	19.32%	1886	18.86%
<25°C	2081	51.93%	2355	58.77%	2989	59.78%	6036	60.36%
<27°C	2966	74.02%	2934	73.22%	3647	72.94%	7428	74.28%
>12°C	3751	93.61%	3737	93.26%	4689	93.78%	9406	94.06%
>14°C	3480	86.85%	3492	87.15%	4391	87.82%	8799	87.99%
>18°C	2847	71.05%	2890	72.12%	3655	73.10%	7363	73.63%
>20°C	2515	62.77%	2708	67.58%	3397	67.94%	6736	67.36%
>24°C	1912	47.72%	1638	40.88%	2011	40.22%	3964	39.64%
>26°C	1027	25.63%	1073	26.78%	1353	27.06%	2572	25.72%

Table 15: Predicted days based on square of TPM of winter season.

Temperature	<9°C	<11°C	<15°C	<17°C	<25°C	<27°C	>12°C	>14°C	>18°C	>20°C	>24°C	>26°C
Predicted Days	7	68	415	640	1247	1329	1158	938	502	304	106	24

Table 16: Predicted days based on square of TPM of summer season.

Temperature	<9°C	<11°C	<15°C	<17°C	<25°C	<27°C	>12°C	>14°C	>18°C	>20°C	>24°C	>26°C
Predicted Days	7	24	99	183	739	938	1265	1213	1005	872	573	375

Table 17: Predicted days based on square of TPM of monsoon season.

Temperature	<25°C	<27°C	>12°C	>14°C
Predicted Days	106	709	1236	633

Table 18: Predicted days based on square of TPM of yearly data (combined all seasons).

Temperature	<9°C	<11°C	<15°C	<17°C	<25°C	<27°C	>12°C	>14°C	>18°C	>20°C	>24°C	>26°C
Predicted Days	18	94	533	823	2008	2927	3731	3460	2818	2522	1985	1066

III. RESULTS AND DISCUSSION

From the columns 2 to 5 of the Table 11-14, we observed that the simulated results for the same period used in the data are almost similar. The results forecast for next 5000 and 10,000 days from the last day of the observed data are given in the columns 6 to 9 of the Table 11-14. Not much difference is observed between the results obtained under 5000 and 10,000 simulations. Also, we found that the percentage of days for selected categories of temperature remains almost similar to the results obtained through the observed data. Which shows that the generalized exponential model works good for predicting daily low temperature.

The prediction for next 4007 days is also made using square of TPM. The classification of days for the selected categories of temperature are shown in Table 15-18. The results are almost closed to the classified frequencies of the observed data.

IV. CONCLUSION

In this paper we have generated Markov chain model with the help of TPM. The daily low temperature can be projected using two different approaches – applying generalized exponential model and using square of TPM. The temperature prediction results obtained through the above mentioned methods are almost identical.

V. FUTURE SCOPE

The similar model can be used for predicting the low temperature for other places to confirm the model's accuracy. For the statistical analysis of daily low temperature some further statistical models can also be developed, and a comparison can be made with the model used within this paper.

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Conflict of Interest. The authors of this manuscript confirm that there is no any conflicts of interest associated with the research article.

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