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Some Problems on Holomorphic Sectional Curvature

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ABSTRACT: Purpose of the present paper is to study of Holomorphic Sectional Curvature. In section 2, we have defined and studied H-Projective Curvature tensor. Section 3 is devoted for Recurrent Sasakian manifolds and Ricci Recurrent Sasakian manifold.

I. INTRODUCTION

An n-dimensional Sasakian space M^n is an odd dimensional Riemannian space, which admits a Unit Killing vector field η^{λ} satisfying:

(1.1) $\eta_{k,\,i.j} = \eta_j\, g_{ik} - \eta_k\, g_{ij}$

Wherein a comma (,) followed by index denotes the operation of covariant differentiation with regard to the fundamental tensor g_{ij} of the Riemannian space.

(1.2) $R^h{}_{ijk} = \partial_i \left\{ {}_j{}^h{}_k \right\} - \partial_j \left\{ {}_i{}^h{}_j \right\} + \left\{ {}_i{}^h{}_j \right\}\left\{ {}_j{}^j{}_k \right\} - \left\{ {}_j{}^h{}_i \right\}\left\{ {}_i{}^l{}_k \right\}$

Whereas the Ricci tensor and the scalar curvature are respectively given by

(1.3) $R_{jk} = R^i{}_{ijk}$,

(1.4) $R = R_{jk}\, g^{jk}$

and

(1.5) $\partial_i = \left(\partial / \partial x^i \right)$

A tensor S_{ij} is defined as

(1.6) $S_{ij} = -F^a{}_i\, R_{aj}$

then we have

(1.7) $S_{ij} = -S_{ji}$

and

(1.8) $F^a{}_i\, S_{aj} = -S_{ia}\, F^a{}_j$

II. H-PROJECTIVE CURVATURE TENSOR

H-Projective Curvature tensor in the Sasakian space is defined as [5]:

(2.1) $P^h{}_{ijk} = R^h{}_{ijk} + \{1/(n+2)\}\left(R_{ij}\,\delta^h{}_j - R_{jk}\,\delta^h{}_i + S_{ik}\,F^h{}_j - S_{ik}F^h{}_j - S_{jk}F^h{}_i + 2S_{ij}\,F^h{}_k \right)$

Definition 2.1

A Sasakian manifold is called H-Projective Recurrent if it satisfies the following condition.

(2.2) $\nabla_l \, P^h{}_{ijk} = \lambda_l \, P^h{}_{ijk}$

Wherein λ_1 is H-Projective Recurrent vector.

Definition 2.2

A Sasakian manifold is said to be H-Projective Symmetric if it satisfied the following condition.

(2.3) $\nabla_l \, P^h{}_{ijk} = 0.$

Definition 2.3

A Sasakian manifold is termed as H-Projectively flat if

(2.4) $P^h{}_{ijk} = 0.$

H-Conformal (or Bockner) Curvature tensor in the Sasakian space is given by

(2.5) $$B^h{}_{ijk} = R^h{}_{ijk} + \{1/(n+4\}\big(R_{ik}\,\delta^h{}_j - R_{jk}\delta^h{}_i + g_{ik}\,R^h{}_j - g_{jk}R^h{}_i$$
$$+S_{ik}F^h{}_j - S_{jk}F^h{}_i + F_{ik}\,S^h{}_j - F_{jk}F^h{}_i + 2S_{ij}F^h{}_k + 2F_{ij}\,S^h{}_k\big)$$
$$-\{R/(n+2)(n+4)\}\big(g_{ik}\,\delta^h{}_j - g_{jk}\delta^h{}_i + F_{ik}\,F^h{}_j - F_{jk}F^h{}_i + 2F_{ij}\,F^h{}_k\big).$$

Definition 2.4

A Sasakian space satisfying the relation

(2.6) $\nabla_a B^h{}_{ijk} - \lambda_a \, B^h{}_{ijk} = 0$

is termed as Sasakian space with Recurrent H-Conformal Curvature tensor.

H- Conharmonic Curvature tensor is given by

(2.7) $$T^h{}_{ijk} = R^h{}_{ijk} + \{1/(n+4)\}\big(R_{ik}\delta^h{}_j - R_{ik}\delta^h{}_i + g_{ik}\,R^h{}_j - g_{jk}R^h{}_i$$
$$+S_{ik}F^h{}_j - S_{jk}F^h{}_i + F_{ik}\,S^h{}_j - F_{jk}\,S^h{}_i + 2S_{ij}F^h{}_k + 2F_{ij}S^h{}_k\big).$$

Definition 2.5

A Sasakian space satisfying the following condition

(2.8) $\nabla_a T^h{}_{ijk} - \lambda_a T^h{}_{ijk} = 0$

for some non-zero Recurrence vector λ_a will be called a Sasakian space with Recurrent H-Conharmonic Curvature tensor or Recurrent Bochner Curvature tensor.

H-Concircular tensor is given by

(2.9) $C^h{}_{ijk} = R^h{}_{ijk} + \{R/n(n+2)\}\big(g_{ik}\,\delta^h{}_j - g_{jk}\delta^h{}_i + F_{ik}\,F^h{}_j\big)$

Definition 2.6

A Sasakian space is called Sasakian space with Recurrent H- Concircular Curvature tensor, if it satisfies.

(2.10) $\nabla_a \, C^h{}_{ijk} - \lambda_a \, C^h{}_{ijk} = 0$

for some non-zero Recurrence vector λ_a.

III. RICCI RECURRENT SASAKIAN MANIFOLDS

A Sasakian space is said to be recurrent if, we have

(3.1) $\nabla_a R^h{}_{ijk} - \lambda_a R^h{}_{ijk} = 0$

for some non-zero recurrence vector λ_a.

Definition 3.2

A Sasakian space is termed as Ricci Recurrent if it satisfies the relation

(3.2) $\nabla_a R_{ij} - \lambda_a R_{ij} = 0,$

Remark 3.1

It is noteworthy that a Ricci Recurrent Sasakian space is also known as Semi Recurrent Sasakian space.

Multiplying equation (3.2) by g^{ij}, we obtain

(3.3) $\nabla_a R - \lambda_a R = 0$

Remark 3.2

From (3.1), if follows that every Sasakian Recurrent space is Sasakian Ricci-Recurrent, but the converse is not necessarily true.

IV. HOLOMORPHIC SECTIONAL CURVATURE

The Holomorphic Sectional Curvature of a Sasakian space with regard to a vector v^h is given by

(4.1) $K_{mjlh} F^m{}_k v^k v^j F^l{}_i v^i v^h + K\left(g_{kj} v^k v^j g_{ih} v^i v^h\right) = 0.$

Remark 4.1

If the Holomorphic Sectional Curvature is constant with regard to any vector at all points then the space is said to be a space of Constant Holomorphic Sectional Curvature.

Definition 4.1

A vector v^h in the Sasakian manifold is called H-Projective vector if it satisfies the relation

(4.2) $L_v P^h{}_{ijk} = 0.$

Transvecting equation (4.2) by g_{hm}, we get

(4.3) $L_v P_{ijkm} = 0$

Wherein L_v, denotes the operator of Lie derivative.

In a Sasakian space of Constant Holomorphic Sectional Curvature, the Curvature tensor is given by

(4.4) $K_{kjih} = (K/4)\left\{\left(g_{hk} g_{ij} - g_{jh} g_{ik}\right) + \left(F_{hk} F_{jh} - F_{jh} F_{ik} - 2F_{jk} F_{ih}\right)\right\}$

Therefore, if the Sasakian space is of Constant Holomorphic Sectional Curvature, then

(4.5) $\nabla_l P_{ijkh} = 0$

Transvecting equation (4.5) with g^{kh} yields

(4.6) $\nabla_l P_{ij} = 0$

In this regard, we have the following heorem:

Theorem 4.1

A Sasakian space of Constant Holomorphic Sectional Curvature is H-Projective Symmetric.

Contracting equation (4.4) by g^{ih}, we obtain

(4.7) $K_{jk} = (K/2)\left(F^m{}_j F_{mk} - F^m{}_k F_{mi} + 2F F_{jk}\right)$

Transvecting equation (2.1) by g_{hm}, we get

(4.8) $$P_{ijkm} = R_{ijkm} + \{1/(n+2)\}\left(g_{jm} R_{ik} - g_{im} R_{jk} + S_{ik} F_{jm} - S_{jk} F_{im} + 2S_{ij} F_{km}\right)$$

Differentiating equation (4.8) convariantely, we obtain

(4.9) $$\nabla_l P_{ijkm} = \nabla_l R_{ijkm} + \{1/(n+2)\}\{g_{jm}(\nabla_l R_{ik}) - g_{im}(\nabla_l R_{jk})$$
$$+\nabla_l(S_{ik} F_{jm}) - \nabla_l(S_{jk} F_{im}) + 2\nabla_l(S_{ij} F_{km})\}$$

Case I

If a Sasakian manifold is H-Projectively Flat then equation (2.1) becomes reduced in the form

(4.10) $$R^h{}_{ijk} = -\{1/(n+2)\}\left(R_{ik}\,\delta^h - R_{jk}\,\delta^h{}_i + S_{ik} F^h{}_j - S_{jk} F^h{}_i + 2S_{ij} F^h{}_k\right)$$

Transvecting equation (4.10) by g_{hm}, we obtain

(4.11) $$R_{ijkm} = -\{1/(n+2)\}\left(R_{ik}\delta^h{}_j - R_{jk}\delta^h{}_i + S_{ik} F^h{}_j - S_{jk} F^h{}_i + 2S_{ij} F^h{}_k\right)$$

Transvecting equation (4.8) with g^{km} and using equation (1.8), we get

(4.12) $$P_{ij} = R_{ij} + \{2/(n+2)\}\left(S_{in} F^n{}_j + FS_{ij}\right)$$

Differentiating equation (4.12) covariantely, we obtain

(4.13) $$\nabla_l P_{ij} = \nabla_l R_{ij} + \{2/(n+2)\}\{\nabla_l(S_{in} F^n{}_j) + \nabla_l(F S_{ij})\}$$

Case II

If the Sasakian space is of Constant Holomorphic Sectional Curvature, then equation (4.13) becomes reduced in the form

(4.14) $$\nabla_l R_{ij} = -\{2/(n+2)\}\{\nabla_l(S_{ik} F^k{}_j) + \nabla_l(F S_{ij})\}.$$

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