



A Linear Programming Approach to Optimizing Organization Transportation System

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ABSTRACT: Over the past few decades each and every organization, be it an industry for production, a unit of manufacturing, a government organization, a limited corporation, a banking sector or an educational institution is focusing on increasing profit margin. And in order to do that, one of the key factors is achieving maximum output with minimum resources (input). Therefore, it has become extremely important for organizations to plan the use of resources in the most optimum way. In this research, Linear Programming has been used to optimize the use of resources. Resources can be anything that act as an input. They can be men, material, money or other assets that can be used by an organization or an individual in order to function effectively. There are laws available in linear programming that are very effective in optimizing the resources. They have been used in this research and transportation system has been tested and tried to optimize. The results have been further compared with the existing system to provide a proper insight on whether they can be helpful in creating a proper plan for optimization for an organization.

I. INTRODUCTION

The transportation system being one of the major area of expenditure where each month quite a huge amount of finance is required in order to run the institution, and the institute being in a semi-urban area, which is around 14 kms from the nearest town, it becomes extremely necessary to bank on the transportation system in order to sustain in the market. The institute covers 4 major cities; Haldwani, Lal Kuan, Rudrapur and Ramnagar, and with the fact that the habitation is scarce near the locality of the college, the transportation system becomes a necessity. Also, in order to compete with the institutes of the area, and also keeping in mind the capacity of fee payers in this region, it becomes extremely essential to match the tuition and other fee with that of the board and university. So, in order to maximize the cost, or rather minimize the expenditure it becomes extremely essential to optimize the usage of transportation system. This has been tried in the transportation model.

II. LITERATURE REVIEW

In around last 65 years linear programming has been used extensively in military, financial, industrial, accounting, marketing and agricultural sectors. Although these are all diverse sectors, Linear Programming can be used to solve them. All linear

programming questions have four properties in common.

All the linear programming questions either maximize or minimize some quantity, they can be profit or cost. We usually call this property as the *Objective Function* of a linear programming question. The major concern for any manufacturing organization can be to maximize the profit. In case of any financial or military sector the concern can be to reduce the cost. Whatever the case may be, the Objective function has to be clearly written and stated [2].

The second property that any linear programming question has in the presence of *constraints* or restrictions [3]. That is to say it is the limit to which we can pursue the objective. Like, in any production company, the production can be limited by number of assembly lines or availability of manpower. Therefore, after the second property, we can state that maximization or minimization of a quantity subject to the limited resources.

There should be *alternative course of actions*. One or more alternative solutions to the problem in question must be available. So that we have options to choose from. If no alternative option is available, we will not need linear programming. Linear programming is all about choosing the best.

The objective and the constraints in any linear programming question should be *expressed in terms of a linear equation*. By saying linear equation what is

meant to be said that the equation should be a first order equation. Like, $2A + 4B = 8$ or $3A + 7B \geq 16$ are acceptable equations, however, $3A^2 + 4B^3 + 5AB = 24$ is not an acceptable equation because it has A^2 and also B^3 and then AB as terms in it. None of such can be included in a linear programming equation.

Apart from the above mentioned four properties, there are also some assumptions that need to be made while formulating a linear programming question. They are: certainty, proportionality, additivity, divisibility, non-negative variables. We should be certain that objectives and constraints are certain, that is they do not change with time. All the variables are in exact proportions. The total of all the objectives and constraints can be added and are divisible. Also, none of the objective or constraint is a negative number, as they all are real values and cannot be negative in any case.

Using all of these properties and assumptions a linear programming problem has to be formulated. Only then can a reliable solution be expected out of the solution. During this entire thesis, all these properties and assumptions have been kept in mind while formulating a problem.

All the three input variables considered to be essential resources have been formulated into objective functions and all the constraints have been duly considered. The alternatives have been studied and then linear equation has been formed to solve each and every case in the problem. An individual model has been formed for each case.

To solve a Linear Programming equation, we will be using a software called Linear Program Solver (LIPS) v1.11.1. It is an optimization package used for solving linear programming models. It has the capability to solve all kinds of problems such as Maximization problems, Minimization problems, integer problems, goal problems etc. It takes the optimization equation, variables and constraints as the input and generates the optimum solution as the output. It can also draw graphs for the output.

Some notable features of the program are:

- i. The program is based on efficient implementation of the modified simplex method that is specifically designed to solve large scale problems.
- ii. It provides not only an answer to the problem but also a detailed solution process as a sequence of simplex tables, so that it can also be used for teaching learning purposes of linear programming.
- iii. It provides the sequence of sensitivity analysis, which enable us to study the behavior of the model when the parameters are changed. They include changes in coefficient of the objective function, changes in variables, changes in

constraints, changes in rows and columns of matrices etc.

- iv. It also supports goal programming which supports weighted and Lexicography GP methods. Goal programming methods are intended for solving multi-objective optimization problems.

III. RESOURCES UNDER OBSERVATION

The transport system is the backbone of any organization, be it a manufacturing industry or a service industry [9]. As an educational institute is more of a service industry where output is educated students, it becomes extremely necessary for the institute to run an organized transportation system, as transport system becomes one of the key resources to any educational institute. If an institute has good connectivity to different areas or localities, it becomes a key motivation for students to pursue education in the institute.

As an example, the transportation system of Amrapali Group of Institutes has been studied and will be used for optimization in this report. A brief about the institute transportation system is being discussed here in order to understand the problem statement in a better way. Although, this optimization will be specifically for the use of this Institute, however, it can be implemented in any other educational institute will slight modifications.

The Amrapali Group of Institutes was established in the year 1999, and has four major campuses in the beautiful hill city of Haldwani, Distt. Nainital, Uttarakhand. It is situated 14-18 kilometers North-West to Haldwani. It comprises of four major institutes which are:

1. Amrapali Institute of Technology and Sciences (AITS)
2. Amrapali Institute of Computer Sciences and Application (AICA)
3. Amrapali Institute of Hotel Management (AIHM)
4. Faculty of Commerce and Business Management (FCBM)

Currently it runs a transportation system with 11 busses to pick and drop students and it connects to 4 nearby towns, Haldwani, Lalkuan, Ramnagar, Rudrapur. The purpose of this model is to minimize the transportation cost on a daily basis and provide an optimized solution for the benefit of the transport manager. Data collected has been the real time data observed over the period for the month of August, 2016. The details of the transportation model will be discussed further.

IV. DATA ACCUMULATION

The data accumulated below in this report is for the month of August, 2016. The data is not a dynamic data so it does not change over a period of time. It stays

constant for at least a period of 1 year. Hence it can be trusted and used for data analysis.

Now discussing in detail the resources observed in section 3.3, AITS has four branches. Each having a student capacity of 60 students, so that makes to a total of $60+60+60+60 = 240$ students. Out of which 72 students take the transport facility. AICA has two branches, each having a student capacity of 120 students, so that makes a total of $120+120 = 240$ students. Out of which 116 students take the transport facility. AIHM has two branches, each having a student capacity of 120 students, so that makes a total of $120+120 = 240$ students. Out of which 58 students take

the transport facility. FCBM has three branches, having a student capacity of $90+120+60 = 270$ students. Out of which 130 students take the transport facility. The data can be summarized into a table 1.

Students of the institute availing the transport facility are majorly from 4 local towns, situated not very far from one another. 48 students avail the facility from Ramnagar, 196 students avail the facility from Haldwani, 56 students avail the facility from Lalkuan, and 76 students avail the facility from Rudrapur. The data can be summarized into table 2.

Table 1: Students availing Transport Facility per Institute.

Institute	Students availing Transport Facility per Institute
AITS	72
AICA	116
AIHM	58
FCBM	130

Table 2: Students availing Transport Facility per Town.

Name of the Town	Students availing Transport Facility per Town
Ramnagar	48
Haldwani	196
Lalkuan	56
Rudrapur	76

Also, the distance of each Institute from each Town is summarized in the table below:

Table 3: Distance of Each Institute from Each Town.

Name of the Town	Distance in Kilometers			
	AITS	AICA	AIHM	FCBM
Ramnagar	29	28	25	27
Haldwani	14	15	18	16
Lalkuan	29	30	33	31
Rudrapur	45	46	49	47

The institute runs 11 buses with different seating capacities. It runs 2 buses with 48 seating capacity, 2 buses with 36 seating capacity, 3 buses with 32 seating capacity and 4 buses with 28 seating capacity. The town-bus data is given in the table 4.

Table 4: Town – Bus Data.

Name of the Town	Number of Buses	Seating Capacity
Ramnagar	1	48
Haldwani	2	36
	3	32
	1	28
Lalkuan	2	28
Rudrapur	1	48
	1	28

V. DATA ANALYSIS

Each 48 seater bus has an average of 4 km/liter, 36 seater bus has an average of 5 km/liter, 32 seater bus has an average of 5 km/liter and 28 seater bus has an average of 6 km/liter. Taking the diesel prices to be constant at ₹ 60/liter, we calculate the price per kilometer as:

1. For 48 seater bus: $60/4 = ₹ 15 / \text{km}$.
2. For 36 seater bus: $60/5 = ₹ 12 / \text{km}$.
3. For 32 seater bus: $60/5 = ₹ 12 / \text{km}$.
4. For 28 seater bus: $60/6 = ₹ 10 / \text{km}$.

For ease of calculation, we take the average of diesel consumption using the equation below:

$$\begin{aligned} & [\sum (\text{No. of buses of each seating capacity} \times \text{Average of price per kilometer})] / \text{Total number of buses.} \\ & = [(2 \times 15) + (2 \times 12) + (3 \times 12) + (4 \times 10)] / 11 \\ & = 130 / 11 = 11.81 \\ & = 12 \text{ (Approx.)} \end{aligned}$$

Therefore, it can be assumed that the diesel consumption of each bus costs us approx. ₹ 12 / km. Now moving ahead, let us calculate the diesel

consumption cost per student. It can be calculated using the formula below:

$$\begin{aligned} & [\text{Total number of buses} \times \text{Diesel consumption cost per bus}] / \text{Total number of students travelling by bus} \\ & = [11 \times 12] / 376 = 132 / 376 \\ & = ₹ 0.35 \text{ (Approx.)} \end{aligned}$$

So, the average diesel consumption cost per student is ₹ 0.35 / km.

VI. FORMULATION OF LP PROBLEM

Now we will set up the LP problem and solve it using LiPS software. Although the problem can also be solved using various other methods such as Simplex Method, Graphical Method, Hungarian Method etc., however, a software has been used to solve the problem here [26].

The purpose of this problem is to optimize the use of buses from various towns to different institutes by minimizing the cost. Thus minimizing the cost will be the objective function for the problem statement.

The complete problem has been stated in the table given below:

Table 5: Supply and Demand of Transportation system to AGI.

Town		Distances to Institutes in kilometers				Maximum Students Travelling / Day from each City
		Institutes				
Route	Route No.	AITS (1)	AICA (2)	AIHM (3)	FCBM (4)	
Ramnagar	1	29 kms	28 kms	25 kms	27 kms	48 Students
Haldwani	2	14 kms	15 kms	18 kms	16 kms	196 Students
Lalkuan	3	29 kms	30 kms	33 kms	31 kms	56 Students
Rudrapur	4	45 kms	46 kms	49 kms	47 kms	76 Students
Students travelling to each institute / Day using the college transport		72	116	58	130	

In the table above Distance from each Institute to each town has been shown. Any of the particular cell can be referred to as X_{ij} = Transportation cost from a particular town to particular institute. (Where i = route from a town and j = route to an institute) e.g. X_{11} means Bus on the route from Ramnagar to AITS = 29 kms, X_{23} means bus on the route from Haldwani to AIHM etc.

The table also shows the constraints clearly. They are as follows:

1. A minimum of 72 students must travel to AITS
2. A minimum of 116 students must travel to AICA

3. A minimum of 72 students must travel to AIHM
4. A minimum of 72 students must travel to FCBM
5. A maximum of 48 students can travel from Ramnagar
6. A maximum of 196 students can travel from Haldwani
7. A maximum of 56 students can travel from Lalkuan
8. A maximum of 76 students can travel from Rudrapur

The next step is to determine the cost to haul a student from each town to each institute:

Table 6: Round trip cost for each student.

Route	Route No.	AITS (1)	AICA (2)	AIHM (3)	FCBM (4)
Ramnagar	1	₹ 20.30	₹ 19.60	₹ 17.50	₹ 18.90
Haldwani	2	₹ 9.80	₹ 10.50	₹ 12.60	₹ 11.20
Lalkuan	3	₹ 20.30	₹ 21.00	₹ 23.10	₹ 21.70
Rudrapur	4	₹ 31.50	₹ 32.20	₹ 34.30	₹ 32.90

The calculations in the table above have been made as follows:

Distance from town i to institute j x 2 [Round Trip] x 0.35 [the average diesel consumption cost per student]

e.g. for X₁₁ we have 29 x 2 x 0.35 = 20.30

Now let us formulate the function of Linear Programming. The **Objective Function** will be:

$$\text{MIN } 20.30X_{11} + 9.80 X_{21} + 20.30 X_{31} + 31.50 X_{41} + 19.60 X_{12} + 10.50 X_{22} + 21.00 X_{32} + 32.20 X_{42} + 17.50 X_{13} + 12.60 X_{23} + 23.10 X_{33} + 34.30 X_{43} + 18.90 X_{14} + 11.20 X_{24} + 21.70 X_{34} + 32.90 X_{44}$$

Subject to:

Constraints:

- X₁₁ + X₂₁ + X₃₁ + X₄₁ ≥ 72 [Students travelling to AITS]
- X₁₂ + X₂₂ + X₃₂ + X₄₂ ≥ 116 [Students travelling to AICA]
- X₁₃ + X₂₃ + X₃₃ + X₄₃ ≥ 58 [Students travelling to AIHM]
- X₁₄ + X₂₄ + X₃₄ + X₄₄ ≥ 130 [Students travelling to FCBM]

$$X_{11} + X_{12} + X_{13} + X_{14} \leq 48 \quad \text{[Students travelling from Ramnagar]}$$

$$X_{21} + X_{22} + X_{23} + X_{24} \leq 196 \quad \text{[Students travelling from Haldwani]}$$

$$X_{31} + X_{32} + X_{33} + X_{34} \leq 56 \quad \text{[Students travelling from Lalkuan]}$$

$$X_{41} + X_{42} + X_{43} + X_{44} \leq 76 \quad \text{[Students travelling from Rudrapur]}$$

VIII. OPTIMIZED SOLUTION

In order to find the optimized solution to the Linear Programming problem function stated above, we use the software “LiPS” – Linear Program Solver v1.11.1.

For the computer solution, we create a new model by clicking on the ‘New’ and then select ‘Table Model’. In the ‘Model Parameters’, we enter ‘Number of Variables’ = 16, ‘Number of Constraints’ = 8, ‘Number of Objectives’ = 1. Then we set ‘Optimization Direction’ to ‘Minimization’. Following is the screenshot of the entries in the table

	X11	X21	X31	X41	X12	X22	X32	X42	X13	X23	X33	X43	X14	X24	X34	X44		RHS
Objective	20.30	9.80	20.3	31.5	19.6	10.5	21.0	32.2	17.5	12.6	23.1	34.3	18.9	11.2	21.7	32.9	->	MIN
Row1	1	1	1	1													>=	72
Row2					1	1	1	1									>=	116
Row3									1	1	1	1					>=	58
Row4													1	1	1	1	>=	130
Row5	1				1				1				1				<=	48
Row6		1				1				1				1			<=	196
Row7			1				1				1				1		<=	56
Row8				1				1				1				1	<=	76
Lower Bound	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
Upper Bound	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF		
Type	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT	CONT		

Fig. 1. Screenshot of the entries in the table of TTM.

Clicking on the 'Solve' button, solves the function to minimization and finds the optimal solution after 13 iterations. **The minimum cost incurred per day optimized in the best possible way comes out to be**

₹ 6582.80 rounded to the nearest digit gives us ₹ 6583.00.

Following is the table of Results found at individual Variables:

```
>> Optimal solution FOUND
>> Minimum = 6582.8
```

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cost	Reduced Cost
x11	0	20.3	-5.6
x21	0	9.8	0
x31	0	20.3	0
x41	72	31.5	0
x12	0	19.6	-4.2
x22	56	10.5	0
x32	56	21	0
x42	4	32.2	0
x13	48	17.5	0
x23	10	12.6	0
x33	0	23.1	0
x43	0	34.3	0
x14	0	18.9	-2.8
x24	130	11.2	0
x34	0	21.7	0
x44	0	32.9	0

Fig. 2. Screenshot of the result generated in TTM.

Further Discussions

Looking at the Figure 2 above, it can be observed that in order to minimize the fuel cost, the transportation system should use the following:

- 72 students from Rudrapur need to be dropped at AITS
- 56 students from Haldwani need to be dropped at AICA
- 56 students from Lal Kaun need to be dropped at AICA
- 4 students from Rudrapur need to be dropped at AICA
- 48 students from Ramnagar need to be dropped at AIHM
- 10 students from Haldwani need to be dropped at AIHM
- 130 students from Haldwani need to be dropped at FCBM

This model has sufficient scope for additional data. For increased accuracy in implementation of this model, more information may be added, as a result the objective function would be more complex. Also, if the data changes or additional buses be added to

the model, they can be accommodated by changing the functional equation, the value of constraints and finding a new optimized solution to the problem.

IX. SUMMARY AND CONCLUSION

In the transportation model, 4 cities were connected to 4 different institutes with 11 busses. Optimization has been done to minimize the running cost of the busses on a daily basis. The model has also shown how the city-institute drop system can be planned in order to get the optimized result.

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