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ARYABHATA-I: An early Indian Mathematician and his work

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ABSTRACT: The knowledge of History of early Indian Mathematics prior to 499 CE is not perfect and sufficient. In this paper we explore work done on mathematics by Arya Bhata-I and summarized in his book Arya Bhateeya.

Key Words: Arya Bhata-I, Arya Bhateeya, summarized.

I. INTRODUCTION

Kusuma Pura was the older name of Pataliputra situated in the Magadha state of ancient time, now a place near the modern Patna in Bihar. Aryabhata-I was one of the Mathematicians and Astronomers those belonged to the Kusuma Pura School of learning, but it is believed that he was a native of Kerala, a region of south India. It is due the facts that, the calendar based upon the systems given in the astronomical work of Aryabhata-I have been followed to some extent in Kerala even in the modern time. He mentioned in his work that he was twenty-three year old when the Kaliyuga's 3600^{th} year was progressing. This implies that he was born in the year 476 CE and his work had been completed in 499 CE. Not much about his life is known. He summarized his work in the book entitled Arya Bhateeya however, it is a very small book. It is believed that the important work Arya Bhateeya had been lost, fortunately, in 1864 an Indian scholar whose name is Bhau Daji recovered a copy of this work [1 p. 40-41 and references therein].The Arya Bhateeya is considered as the landmark work in history of mathematical sciences of early India. Various commentator like
Bhaskara-I, Somesvera, Parmeswera, Nilkantha Somayaji etc., wrote their valuable commentaries for the clear understanding of mathematical terms in this work. Aryabhata's one other work known as Aryabhata-siddhanta, which had been lost, but appeared through the references found in other works of mathematical or astronomical interest written after the time of Aryabhata-I [10 p. 1 and references therein].

The small work, Arya Bhateeya, was divided into two parts by Brahmagupta in his own work Brahma Sphuta Siddanta. The first part consists of ten verses from the introductory and concluding portions, while the second part, which is the larger one, consists of one hundred eight verses. These two parts are named as *Dasageetika* and *Aryāshtashatam* respectively. On the other hand Arya Bhata himself gave only a single name to his complete work, which was *Arya Bhateeya*. The second part i.e., *Aryāshtashatam* again ramified as *Ganita* or mathematics, *Kāla Kriya* or the calculation of time and the *Gōla* or sphere. The last two parts deal with the astronomical calculations and the first part, which consists of thirty three verses of the purely mathematical importance. Probably Aryabhata-I was the first Indian astronomer who gave the value of obliquity as 24^0, however, explicitly in his astronomical work [12 p. 107 and references therein]. The concept of the place value system of numbers was, possibly, first introduced by Aryabhata-I in his work Arya Bhateeya [13 p. 131 and references therein]. The mathematical contents dealt with mainly, the methods for extracting square roots and cube roots, some problems, the series, problems on geometry problems related to the quadratic equations and the solution of the indeterminate equations of the first degree.

Aryabhata-I was the first Indian mathematician who described the method of solution of the indeterminate equations. The problem of indeterminate equation was aroused in the following form. It is required to determine an integer 'N' which when divided 'm' leaves a remainder r_1, and when divided by 'n' leaves the remainder r_2.

Therefore, $N = mx + r_1 = ny + r_2$

Or, $ny - mx = p$

Where, $p = r_1 - r_2$

The equation $mx + p = ny$

Where m, n, p are integers, positive or negative, then the equation $mx + p = ny$ is known as indeterminate equation of the first degree. Clearly, the common factor of m and n should be also a factor common to p, otherwise the equation is inconsistent. However, the latter Indian mathematicians called the method of solution of indeterminate equation as *kuttaka*.

Aryabhata was undoubtedly, a real mathematician who discovered the method of solution of indeterminate equation of first degree. Probably, this is the most excellent achievement of Aryabhata-I in the area of pure mathematics [1 p. 95-96 and references therein].

Some important identities, which were appeared first time in the Aryabhata's work, are as the following [1 p. 45 and references therein]:

$\sum n^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots. + n^2 = [n(n+1)(2n+1)]/6$

$\sum n^3 = 1^3 + 2^3 + 3^3 + 4^3 + \ldots\ldots\ldots + n^3 = (1+2+3+\ldots..+n)^2 = [n^2(n+1)^2]/4$

$1+(1+2)+(1+2+3)+(1+2+3+4)+\ldots\ldots.$ Up to n terms $= [n(n+1)(n+2)]/6$

Arya Bhateeya described rules of two important operations, known as *Sa□krama□a*, i.e., the operation to find two unknown quantities when their sum and difference are given and these rules are as following:

"One should subtract the sum of the squares from the square of the sum". Implies that which is halved should be known as the product of two multipliers. In modern notations

$[(m+n)^2-(m^2+n^2)]/2 = mn.$

"The square root of the product multiplied by four added to the square of the difference is increased or decreased by the difference, and halved". In modern notations

$[\sqrt{\{4mn+(m-n)^2\}}+(m-n)]/2 = m;\ [\sqrt{\{4mn+(m-n)^2\}}-(m-n)]/2 = n.$

Bhaskara-I in his commentary on Arya Bhateeya of Aryabhata-I entitled as, Aryabhatiya Bhashya, described the first of these as an alternative method to find the product of two quantities [15 p. 58 and references therein].

Aryabhata-I constructed a table of sines, to find the approximate values by using a formula for $\{\sin (n+1)\theta - \sin n\theta\}$, which is in terms of $\sin n\theta$ and $\sin (n-1)\theta$ [4 p. 47 and references therein]. However, this sine table was included only the angles of multiples of $(15/4)^0$ and between 0^0 and 90^0 but, this is an important mathematical development in early India.

The approximation of π correct up to five significant figures was first given by Aryabhata-I for to calculate the circumference of a circle of diameter twenty thousand units in the following verse:

caturadhikam satamastagunam
dvasastistathasaharanam ayuta dvaya
viskambhasyasanno vrttav parinahah

That is, "add four to 100, multiply by eight, and then add 62,000. By this rule circumference of a circle with a diameter of 20,000 can be approached." This gives the ratio of the circumference to the diameter equal to $\{[(4+100)\times 8+62000]/20000\} = 62832/20000 = 3.1416$ which is very close to the modern value of $\pi = 3.14159$ correct up to five places. P. Jha also wrote in his paper [11] about this important contribution of Aryabhata, that:

"Aryabhata-I's value of π is a very close approximation to the modern value and the most accurate among those of the ancients. There are reasons to believe that Aryabhata devised a particular method for finding this value. It is shown with sufficient grounds that Aryabhata himself used it, and several later Indian mathematicians and even the Arabs adopted it. The conjecture that Aryabhat's value of π is of Greek origin is critically examined and is found to be without foundation. Aryabhata discovered this value independently and also realised that π is an irrational number. He had the Indian background, no doubt, but excelled all his predecessors in evaluating π. Thus the credit of discovering this exact value of π may be ascribed to the celebrated mathematician, Aryabhata-I."

Thus Aryabhata was not only approximated a more accurate value of π but he was also aware of its irrationality. However, a formal proof of the irrationality of π was given by a European mathematician only in 1761, whose name is Lambert [1 p. 153 and references therein].

The square root extracting method described in the Aryabhata's work was also found in the earlier Jainas canonical works in which, the square root of large numbers for their cosmology was calculated. However, the method of root extraction was not mentioned clearly but it was explained clearly by Aryabhata in his mathematical work.

The cube root extraction method found in Aryabhata's work have not been appeared in the earlier mathematical work so, most likely it was his original work. Algorithm for finding square root was also found in the works prior to Aryabhata's work but, algorithm for cube root was first clearly enunciated by him in his work *Arya Bhateeya* [14 p. 161 and references therein]. Aryabhata described his rule of cube extraction in the following verse:

aghanad bhajed dvitīīyat
trigu□ena ghansya mulavarge□a |
vargastripurvaguita□□
śodhya□ prathamad ghanaśca hganat ||5 (*Aryabhateeya, Ganitpada,* verse 5)

That is, "(Having subtracted the greatest possible cube from the last cube place and then having written down the cube root of the number subtracted in the line of the cube root), divide the second non cube place (standing on the right of the last cube place) by thrice the square of the cube root (already obtained); (then) subtract from the first non-cube place (standing on the right of the second non-cube place) the square of the quotient multiplied by thrice the previous (cube root); and (then subtract) the cube (of the quotient) from the cube place (standing on the right of the first non-cube place) (and write down the quotient on the right of the previous cube root in the line of the cube root, and treat this as the new cube root. Repeat the process, if there are still digits on the right) [5 p. 150 and references therein].

An example of cube root extraction method of Aryabhata, quoted from [5 p. 152- 154 and references therein] as follows:

Let the number be 34,965,783 and whose cube root is to be extracted by using Aryabhata's rule of cube extraction.

Step 1: Above each digit, mark out cubic places (with hat), and non-cubic places (with dash), starting from right. The units place is always considered as cubic place:

34965783

Step 2: Locate the last cubic place (i.e. the second digit from the left) and find the number that holds this position (=34):

↓
34965783

Step 3: Now, find the number's nearest lesser (or equal) cube (=27 i.e. 3^3) and subtract it from 34. Put the cube root of this lesser cube (3) in the root result area. The root result area now contains the most significant digit of the root result:

34965783 Root Result = 3
-2 7
7

Step 4: Locate the next place to the right and move its digit (9) down to the right of the result of the previous subtraction to form 79. Since we are now on a second non-cubic place, we divide 79 by three times the square of the current root and evaluate the quotient, i.e. $[79/(3\times3^2)] = 2$. Multiply this quotient with three times the square of current assembled root, i.e. $2\times3\times3^2= 54$ and subtract it from 79.

34965783
………. Root Result = 3
7 9
- 5 4
2 5

Step 5: Now, bring down the next digit in the number to form 256 and subtract three times the assembled root times the square of the last quotient, i.e. $3\times3\times2^2 = 36$. And then place this quotient as the next digit of the assembled root result, i.e. 10×3+2= 32.

34965783
………. Root Result = 3 2
-2 5 6
3 6
2 2 0

Step 6: Again, bring down the next digit to the right of the previous subtraction to form 2205 and subtract the cube of the last quotient, i.e. $2^3 = 8$ to get 2197.

34965783
………. Root Result = 3 2
-2 2 0 5
8
2 1 9 7

Step 7: Repeat the steps 4, 5 and 6 until the remainder is zero. The resulting calculation is as follows:

34965783
-2 7
7 9
-5 4
2 5 6
- 3 6
2 2 0 5 Root Result = 3 27
- 8
2 1 9 7 7
-2 1 5 0 4
4 7 3 8
-4 7 0 4
3 4 4
-3 4 3
0

Thus the cube root of the number 34965783 is 327.

II. CONCLUSION

Undoubtedly; Arya Bhata-I occupies unique place in the development of Mathematics and Astronomy in early centuries. He did his fundamental work on Number System, Trigonometry and Theory of Equations etc.

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