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Common Fixed Point Theorems with Generalize F-Contraction on Dualistic Partial B-Metric Space

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ABSTRACT: In the present paper a mapping is introduced namely generalize F-contraction mapping. A new metric space namely ordered dualistic partial b-metric space is also introduced by adding additional condition in dualistic partial metric space. On the basis of the same, some theorems are proved. This result is more generalized the result of Nazam and Arsad ("On common fixed point theorems in dualistic partial metric spaces"). An example is also given in the support of our result.

Keywords: Fixed point, Dualistic partial b-metric space, Quasi metric, Dualistic partial metric and f-contraction

I. INTRODUCTION

Jungck [9] introduced the idea of compatible mapping with the help of generalized weakly commuting mappings. There are many authors [1, 2, 8, 16, 17, 21] who proved many fixed point theorems for compatible mappings under the contractive type conditions. In 1922, the most important result of fixed point theory called Banach Contraction Principle (BCP), was established by Stefan Banach. This principle state that "if (N, d) is a complete metric space and if A: N→N is a contraction mapping that is d(Ar, As) ≤ k d(r, s), where k ϵ (0, 1) for all r, sϵ N than it has a unique fixed point ".One of such extension of metric space is Partial Metric Space introduced by Steve G Matthew [11] in 1922. In this metric space the distance between the two elements m and n is the distance of two elements is not necessarily zero. This research not only generalized the existing results of the metric space but also to establish new results with applications (see [4, 5, 7, 14, 20, 22]). Dualistic partial metric space was developed by Neill [15] after more generalized form of Partial Metric Space. This space connects quasi metric and dualistic partial metric space. Neill studied many topological properties of dualistic partial metric space. Fixed point theorem in dualistic partial metric space was proposed by Valero *et al.* [18] and proved the Banach fixed point theorem on complete dualistic partial metric space. Subsequently, Nazam *et al.* [3, 12, 13] introduced some fixed point results with applications in the dualistic partial metric space In 2014, one more idea of partial b-metric space is given by S. Satish [6] in 2014 and proved BCP on this space. Here, using generalize f-Contraction condition then prove fixed point theorem in ordered dualistic partial b metric space for weakly compatible mapping. This is an extension of work done by Muhammad Nazam [19]. One example is given in the support of this main result.

II. PRELIMINARIES

Throughout this paper, collection of natural number is N, R^+ denotes all positive real numbers and R denotes real numbers. Neill [15] defined the dualistic partial metric space by extending the range $[0,\infty)$ $to(-\infty,\infty)$ in partial metric space.

Definition 2.1 [15]: Let $N \neq \emptyset$, define a mapping $D: N * N \to R$ satisfies the following axioms: $\forall\, r_1'', r_2'', r_3'' \in N$

(i) $r_1''=r_2'' \Leftrightarrow D(r_1'',r_1'')=D(r_1'',r_2'')=D(r_2'',r_2'')$

(ii) $D(r_1'',r_1'') \le D(r_1'',r_2'')$

(iii) $D(r_1'',r_2'')=D(r_2'',r_1'')$

(iv) $D(r_1'',r_2'')+D(r_3'',r_3'') \le D(r_1'',r_3'')+D(r_3'',r_2'')$

Then pair (N, D) is known as a dualistic partial metric space.

Definition 2.2: [11]: Let $Y \neq \emptyset$. Define a mapping $d_p: Y \times Y \to [0,\infty)$ then d_p is called Partial Metric if satisfies the following property: $\forall\, r_1'', r_2'', r_3'' \in Y$

(i) $r_1''=r_2'' \Leftrightarrow d_p(r_1'',r_1'')=d_p(r_1'',r_2'')= \quad d_p(r_2'',r_2'')$

(ii) $d_p(r_1'',r_1'') \le d_p(r_1'',r_2'')$

(iii) $d_p(r_1'',r_2'')=d_p(r_2'',r_1'')$

(iv) $d_p(r_1'',r_2'') \le d_p(r_1'',r_3'')+d_p(r_3'',r_2'')-d_p(r_3'',r_3'')$

The pair (X, d_p) is known as partial metric space.

The pair (Y, d_b) is known as b-metric space.

Definition 2.3: [23, 24] Let $Y \neq \emptyset$. Define a mapping $d_b: Y \times Y \to [0,\infty)$ if satisfies the following axioms: $\forall\, r_1'', r_2'', r_3'' \in Y$

(i) $d_b(r_1'',r_2'') \Leftrightarrow r_1''=r_2''$

(ii) $d_p(r_1'',r_2'')=d_p(r_2'',r_1'')$

(iii) $d_p(r_1'',r_2'') \le u[d_p(r_1'',r_3'')+d_p(r_3'',r_2'')]$

Definition 2.3: [10] Let $T and\ F$ are self-mappings if $T(r_1'')=F(r_1'')$ for some$r_1'' \in M$, then $r_1'' is$ known as coincidence point and $T, F$are said to be weakly compatible if

$T(r_1'')=FT(r_1'')=TF(r_1'')=F(r_1'')$ for some $r_1'' \in M$

Example 1.2 Let $M = R \; and \; T, F: M \to M$ be given by

$$F(m) = 10m - 9 \; and T(m) = 9m - 8 \; for all m \in M$$

Then T, F are weakly compatible for m=1.

Proof: $FT(m) = 10T(m) - 9$

$= 10(9m - 8) - 9$

$= 90m - 80 - 9$

$= 90 - 89 = 1$

And $TF(m) = 9F(m) - 8$

$= 9(10m - 9) - 8$

$= 90m - 81 - 8$

$= 90 - 89 = 1$

⇒T, F are weakly compatible for m=1.

III. MAIN RESULTS

Here, we shall prove the theorem in dualistic partial b-metric space for the two weakly compatible mapping.

Definition 3.1: Let $N \neq \emptyset$, define a mapping $D_b: N * N \to R$ satisfies the following axioms: $\forall \, r_1'', r_2'', r_3'' \in N$

(i) $r_1''=r_2'' \Leftrightarrow D_b(r_1'',r_1'')=D_b(r_1'',r_2'')=D_b(r_2'',r_2'')$

(ii) $D_b(r_1'',r_1'') \leq D_b(r_1'',r_2'')$

(iii) $D_b(r_1'',r_2'')=D_b(r_2'',r_1'')$

(iv) $D_b(r_1'',r_2'')+D_b(r_3'',r_3'') \leq u[D_b(r_1'',r_3'')+D_b(r_3'',r_2'')]$

Pair (N, D_b) is known as dualistic partial b-metric space with coefficient $u \geq 1$.

Example 3.1: A function D_b: R × R → R by $D_b(f_1^*, f_2^*) = \max\{f_1^*, f_2^*\}$ Clearly D_b satisfies (Db1)–(Db4) and hence D_b is a dualistic partial b-metric space on R.

Proof: Let coefficient $u \geq 1 \; and \; (Y, D_b)$ be a dualistic partial b-metric space. Let $f_1^*, f_2^*, f_3^* \in$ Y be an arbitrary point, then

(1) $D_b(f_1^*, f_2^*) = \max\{f_1^*, f_2^*\}$

$D_b(f_1^*, f_1^*) = D_b(f_1^*, f_2^*) = D_b(f_2^*, f_2^*) \Leftrightarrow f_1^* = f_2^*$

(2) $D_b(f_1^*, f_2^*) = \max\{f_1^*, f_2^*\} \geq \max\{f_1^*, f_1^*\}$ |

$\geq D_b(f_1^*, f_1^*)$

(3) $D_b(f_1^*, f_2^*) = \max\{f_1^*, f_2^*\}$

$= \max\{f_2^*, f_1^*\}$

$= D_b(f_2^*, f_1^*)$

(4) $D_b(f_1^*, f_2^*) = \max\{f_1^*, f_2^*\}$

$= \max\{f_1^*, f_2^*\} + \max\{f^*, f^*\} - \max\{f^*, f^*\}$

$\leq \max\{f_1^*, f^*\} + \max\{f^*, f_2^*\} - \max\{f^*, f^*\}$

$\leq u[\max\{f_1^*, f^*\} + \max\{f^*, f_2^*\} - \max\{f^*, f^*\}][since \; u \geq 1]$

$\leq u[\max\{f_1^*, f^*\} + \max\{f^*, f_2^*\}] - \max\{f^*, f^*\}$

$\leq u[D_b(f_1^*, f^*) + D_b(f^*, f_2^*)] - D_b(f^*, f^*)$

So (Y, D_b) is a dualistic partial b-metric space.

Lemma 3.1 If (Y, D_b) is a dualistic partial b-metric space, then $d_{D_b}: Y \times Y \to R^+$ defined by $d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*) \quad \forall \, d^*, e^* \epsilon Y$

is called a Quasi Metric on Y such that $\tau(D_b) = \tau(D_{D_b})$.

Proof: Consider $d^*, e^* \in Y$.Then

$d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*)$ is always non negative because of $D_b(d^*, d^*) \leq D_b(d^*, e^*)$.

Now, we have to check that d_{D_b} is actually a quasi-metric on Y. Let$d^*, e^*, f^* \in$ Y. It is obvious that $d^* = e^*$ provides that $d_{D_b}(d^*, e^*) = D_{D_b}(e^*, d^*) = 0$ Moreover, if $d_{D_b}(d^*, e^*) = D_{D_b}(e^*, d^*) = 0$ then

$D_b(d^*, e^*) - D_b(d^*, d^*) = D_b(e^*, d^*) - D_b(e^*, e^*) = 0$
Hence we obtain that $d^* = e^*$, since

$D_b(d^*, e^*) = D_b(d^*, d^*) = D_b(e^*, e^*)$. Furthermore

$$d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*)$$

$\leq D_b(d^*, f^*) + D_b(f^*, e^*) - D_b(f^*, f^*) - D_b(d^*, d)$

$= d_{D_b}(d^*, f^*) + d_{D_b}(f^*, e^*)$.

Finally we show that $\tau(D_b) = \tau(D_{D_b})$. Indeed, let a y∈ Y and ε > 0 and consider $y \in B_{d_{D_b}}(d^*, \epsilon)$. Then $d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*) < \epsilon$ and,

hence, $D_b(d^*, e^*) < \epsilon + D_b(d^*, d^*)$.

Consequently $y \in B_{d_{D_b}}(r^*, \epsilon)$ and $\tau(D_b) = \tau(D_{D_b})$.

Conversely if $y \in B_{D_b}(r^*, \epsilon)$ we have, $D_b(d^*, e^*) < \epsilon + D_b(d^*, d^*)$

Thus $d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*) < \epsilon$, $y \in B_{d_{D_b}}(d^*, \epsilon)$ and $\tau(D_b) = \tau(D_{D_b})$

Implies that $\tau(D_b) = \tau(D_{D_b})$.

Remark: Let(Y, D_b) is a dualistic partial b-metric space. The function $d_{D_b}: Y \times Y \to R^+$ is known as quasi metric on Mdefined by

$$d_{D_b}(d^*, f^*) = D_b(d^*, f^*) - D_b(d^*, d^*) \forall d^*, f^* \epsilon Y$$

Moreover, if $d_{D_b}$is a dualistic quasi metric on Y, then $d^{s}_{D_b} = max\{d_{D_b}(d^*,f^*), d_{D_b}(f^*,d^*)\}$ defines a metric on Y.

Lemma 3.2

(1) If metric space$(W, d_s^{D_b})$ is complete then dualistic partial b-metric $(W, D_b)$is also complete and vice versa.

(2) A point $y \epsilon W$ and a sequence $\{y_n\}_{n\epsilon N}$in $W\ such\ that \{y_n\}$ converge to y, with respect to $\tau(d_s^{D_b})$ iff $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = D_b(y,y) = \lim_{n\to\infty} D_b\,(y, y_n)$

Proof: We claim that a $\{y_n\}$ be a Cauchy sequence in (W, D_b)

Hence this is also Cauchy sequence in (W,$d_s^{D_b}$).

Let $\{y_n\}$ is a Cauchy sequence in (W, D_b)

Then ∃ α ∈ R s.t, given ε > 0, there is n_ε∈ N with $|D_b(y_n, y_m) - \alpha| < \frac{\epsilon}{2} \forall\, n, m \geq n_\epsilon$.

Hence, $d_{D_b}(y_n, y_m) = D_b(y_n, y_m) - D_b(y_n, y_n)$

$$= |D_b(y_n, y_m) - \alpha + \alpha - D_b(y_n, y_n)|$$

$$\leq |D_b(y_n, y_m) - \alpha| + |\alpha - D_b(y_n, y_n)|$$

$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

For all $n, m \geq n_\epsilon$. Similarly we show $d_{D_b}(y_n, y_m) < \epsilon\ for\ all\ n, m \geq n_\epsilon$.

We conclude that $\{y_n\}$ is a Cauchy sequence in (W,$d_s^{D_b}$).

Implies we show that when (W,$d_s^{D_b}$) is complete than (W, D_b) is Complete

If $\{y_n\}$ is a Cauchy sequence in (W, D_b),

Then also Cauchy sequence in (W,$d_s^{D_b}$)

Suppose that y ∈ N and the metric space (W,$d_s^{D_b}$) is complete such that $\lim_{n\to\infty} d_s^{D_b}\,(y, y_n) = 0$.

By lemma (3.1) we follow that $\{y_n\}$ is a convergent sequence in (W, D_b).

Further we show that $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = D_b(y,y)$.

Since $\{y_n\}$ is Cauchy sequence in (W, D_b) than

$$\lim_{n\to\infty} D_b\,(y_n, y_n) = D_b(y,y)$$

Consider ε > 0 then ∃ n_0 ∈ N such that $d_s^{D_b}(y, y_n) < \frac{\varepsilon}{2}$ whenever n ≥ n_0

Thus

$$|D_b(y,y) - D_b(y_n,y_n)| \leq |D_b(y,y) - D_b(y,y_n)| + |D_b(y,y_n) - D_b(y_n,y_n)|$$

$$= d_s^{D_b}(y,y_n) + d_s^{D_b}(y_n,y)$$

$$< d_s^{D_b}(y,y_n) < \epsilon$$

Whenever n ≥ n_0

⇒(W, D_b) is complete.

Next we show that every Cauchy sequence $\{y_n\}$ in $(N, d_s^{D_b})$ be a Cauchy sequence in (W, D_b). Let$\epsilon = \frac{1}{2}$. Then ∃ n_0∈ N such that $D_b(y_n, y_m) < \frac{1}{2} \forall\, n, m \geq n_0$

Since

$$d_{D_b}(y_n, y_{n_0}) + D_b(y_n, y_n) = d_{D_b}(y_{n_0}, y_n) + d_{D_b}(y_{n_0}, y_{n_0}),$$

Then

$$|D_b(y_n, y_n)| = d_{D_b}(y_{n_0}, y_n) + d_{D_b}(y_{n_0}, y_{n_0}) - d_{D_b}(y_n, y_{n_0})$$

Consequently the sequence $(D_b(y_n, y_n))_n$ is restricted in R, and consequently so there is y in R s.t. a subsequence $(D_b(y_{n_k}, y_{n_k}))_k$ is convergent to y, i.e.

$$\lim_{k\to\infty} D_b\,(y_{n_k}, y_{n_k}) = y.$$

It remains to show that $(D_b(y_n, y_n))_n$ be a Cauchy sequence in R.

Since $\{y_n\}$ be a Cauchy sequence in (W,$d_s^{D_b}$), given ε > 0, ∃n_ε∈ N s.t.$d_s^{D_b}(y_n, y_m) < \frac{\epsilon}{2}$∀n, m ≥ n_ε. Thus, for all n, m ≥ n_ε,

$$D_b(y_n, y_n) = d_{D_b}(y_m, y_n) + D_b(y_m, y_m) - d_{D_b}(y_n, y_m).$$

Therefore $\lim_{n\to\infty} D_b\,(y_n, y_n) = y$

Whereas, $|D_b(y_n, y_m) - y| = |D_b(y_n, y_m) - D_b(y_n, y_n) + D_b(y_n, y_n) - y|$

$\leq D_b(y_n, y_m) + |D_b(y_n, y_n) - y| < \epsilon$ ∀ n, m ≥ n_ε.

Hence $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = y$ and $\{y_n\}$ be a Cauchy sequence in (W, D_b).

Then $\{y_n\}$ be a Cauchy sequence in (W, D_b), and so it is convergent to a point y ∈ W with

$$\lim_{n\to\infty} D_b\,(y, y_n) = D_b(y,y) = \lim_{n,m\to\infty} D_b\,(y_n, y_m).$$

Given ε > 0, then ∃n_ε∈ N

$such\ that\ D_b(y, y_n) - D_b(y,y) < \epsilon$

And $D_b(y,y) - D_b(y_n, y_n) < \epsilon$

Whenever n ≥ n_ε As a consequence we have

$d_{D_b}(y, y_n) = D_b(y, y_n) - D_b(y,y) < \epsilon,$

And

$$d_{D_b}(y_n, y) = D_b(y, y_n) - D_b(y_n, y_n)$$

$$\leq \left| D_b(y, y_n) - D_b(y, y) + D_b(y, y_n) - D_b(y_n, y_n) \right| < 2\epsilon$$

whenever n ≥ n_ϵ. Therefore (W,$d_s^{D_b}$) is complete.

Finally,
$\lim_{n\to\infty} d_s^{D_b}(y, y_n) = 0$iff $\lim_{n,m\to\infty} D_b(y_n, y_m) = D_b(y, y) = \lim_{n\to\infty} D_b(y, y_n)$.

Let a mapping $W: Y \to Y$ defined on Y and $(Y, \leq)$ be an ordered set to satisfy the property $y^* \leq W(y^*)\ \forall\ y^* \epsilon Y$

Then W is known as dominating mapping.

DEFINITION 3.2: Let T, F be two self mappings and (M,D_b)dualistic partial b-metric space. We assume the mapping T,F a generalize F- contraction if ∃ $\beta \in \left[0, \frac{1}{u}\right]$ with constant u ≥1 s.t.

$$|D_b(T(x^*), T(y^*))| \leq \beta \max \begin{bmatrix} |D_b(F(x^*), F(y^*))|, |D_b(T(x^*), T(y^*))|, \\ |D_b(F(y^*), T(y^*))|, \frac{|D_b(F(x^*), T(x^*))|}{u} \end{bmatrix} \quad (1)$$

∀ x, y $\epsilon\, M$.

Theorem 3.1

Let W, V: M→M are weakly compatible self mapping and (M, D_b) a dualistic partial b-metric space such that

1) W(M) ⊆ V (M)
2) W is a generalize F- contraction

Then W and V have a Unique Common Fixed point.

Proof: Let we start with $k_0^* \epsilon\, M$

Since $W(M) \subseteq V(M)$

choose $k_1^* \epsilon\, M$

s.t.$W(k_0^*) = V(k_1^*)$

$$W(k_1^*) = V(k_2^*)$$

Let $W(k_s^*) \neq W k_{s-1}^*)$

And $V(k_s^*) \neq V(k_{s+1}^*)\quad \forall\, n \in N$

By the equation (1), we have

$$\left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|$$

$$\leq \beta \max \begin{bmatrix} \left|D_b\left(V(k_s^*), V(k_{s+1}^*)\right)\right|, \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|, \\ \left|D_b\left(V(k_{s+1}^*), W(k_{s+1}^*)\right)\right|, \frac{\left|D_b\left(V(k_s^*), W(k_s^*)\right)\right|}{u} \end{bmatrix}$$

$$\leq \beta \max \begin{bmatrix} \left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right| \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|, \\ \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|, \frac{\left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|}{u} \end{bmatrix}$$

If max

$$\begin{bmatrix} \left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|, \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|, \\ \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|, \frac{\left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|}{u} \end{bmatrix}$$

$$= \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|$$

Then,

$|D_b(W(k_s^*), W(k_{s+1}^*))| \leq \beta\, |D_b(W(k_s^*), W(k_{s+1}^*))|$

which is contradiction.

If max

$$\begin{bmatrix} \left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right| \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|, \\ \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right| \frac{\left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|}{u} \end{bmatrix}$$

$$= \frac{|D_b(W(k_{s-1}^*), W(k_s^*))|}{u}$$

Then

$|D_b(W(k_s^*), W(k_{s+1}^*))| \leq \beta \left[\frac{|D_b(W(k_{s-1}^*), W(k_s^*))|}{u}\right]$[also u ≥ 1]
which is contradiction

$$\text{Max} \begin{bmatrix} \left|D_b\left(W(s_{n-1}^*), W(k_s^*)\right)\right|, \left|D_b\left(W(k_s^*), W(s_{n+1}^*)\right)\right|, \\ \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right| \frac{\left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|}{u} \end{bmatrix} =$$

$$\left|\left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|\right|$$

hus

$|D_b(W(k_s^*), W(k_{s+1}^*))| \leq \beta\, |D_b(W(k_{s-1}^*), W(k_s^*))|$

By continuing in this way

$$\left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right| \leq \beta^s \left|D_b\left(W(k_0^*), W(k_1^*)\right)\right| \quad \forall\ s \in N \qquad (2)$$

Now$\left|D_b\left(W(k_s^*), W(k_s^*)\right)\right|$

$$\leq \beta \max \begin{bmatrix} \left|D_b\left(V(k_s^*), V(k_s^*)\right)\right|, \left|D_b\left(W(k_s^*), W(k_s^*)\right)\right|, \\ \left|D_b\left(V(k_s^*), W(k_s^*)\right)\right| \frac{\left|D_b\left(V(k_s^*), W(k_s^*)\right)\right|}{u} \end{bmatrix}$$

If max $\begin{bmatrix} \left|D_b\left(V(k_s^*), V(k_s^*)\right)\right|, \left|D_b\left(W(k_s^*), W(k_s^*)\right)\right|, \\ \left|D_b\left(V(k_s^*), W(k_s^*)\right)\right|, \frac{\left|D_b\left(V(k_s^*), W(k_s^*)\right)\right|}{u} \end{bmatrix}$

$=|D_b(W(k_{s-1}^*), W(k_s^*))|$ [for u≥1]

$|D_b(W(k_s^*), W(k_s^*))| \leq \beta\, |D_b(W(k_{s-1}^*), W(k_s^*))|$

$\leq \ \beta^s |D_b(W(k_0^*), W(k_1^*))|$

Thus we have

$|D_b(W(k_s^*), W(k_s^*))| \ \leq \beta^s |D_b(W(k_0^*), W(k_1^*))|$ (3)

By equation (1), (2) and (3), we get

$$\left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right|$$

$$\leq \left|D_b\left(W(k_s^*), W(k_{s+1}^*)\right)\right| - \left|D_b\left(W(k_s^*), W(k_s^*)\right)\right|$$

$$\leq |D_b(W(k_s^*), W(k_{s+1}^*))| + |D_b(W(k_s^*), W(k_s^*))|$$

$$\leq \ \beta^s |D_b(W(k_0^*), W(k_1^*))| + \ \beta^n |D_b(W(k_0^*), W(k_1^*))|$$

$$\leq \ 2\,\beta^s |D_b(W(k_0^*), W(k_1^*))|$$

$$\leq \ 2\,\beta^s \gamma$$

And $d_{D_b}(W(k_{s+1}^*), W(k_{s+2}^*)) \leq \ 2\,\beta^{s+1}\gamma$

For a fixed natural number q_1, we have

$d_{D_b}\left(W(k_{s+q_1-1}^*), W(k_{s+q_1}^*)\right) \leq \ 2\,\beta^{s+q_1-1}\gamma$, $\forall s \in N$

Now using by triangular inequality for d_{D_b} Let s and t are two +veinteger such that $t \geq s$,than

$$d_{D_b}\left(W(k_s^*), W(k_{s+q_1}^*)\right) \leq d_{D_b}\left(W(k_s^*), W(k_{s+1}^*)\right) +$$

$$d_{D_b}\left(W(k_{s+1}^*), W(k_{s+2}^*)\right) + \text{-----} + d_{D_b}\left(W(k_{s+q_1-1}^*), W(k_{s+q_1}^*)\right)$$

$$\leq 2\,\gamma[\beta^s + \beta^{s+1} + \text{-----} + \beta^{s+q_1-1}]$$

So put $s + q_1 = t$

$d_{D_b}(W(k_s^*), W(k_t^*)) \ \leq 2\,\gamma\, \frac{\beta^s}{1-\beta}$ (4)

By interchanging s and t, we have

$$\left|d_{D_b}(W(k_{s+1}^*), W(k_s^*))\right| \leq \left|d_{D_b}(W(k_{s+1}^*), W(k_s^*))\right| - |D_b(W(k_{s+1}^*), W(k_{s+1}^*))|$$

$$\leq \left|d_{D_b}(W(k_{s+1}^*), W(k_s^*))\right| + |D_b(W(k_{s+1}^*), W(k_{s+1}^*))|$$

$$\leq \beta^s |D_b(W(k_1^*), W(k_0^*))| + \ \beta^{s+1} |D_b(W(k_1^*), W(k_0^*))|$$

$$\leq \ \gamma\,(1+\beta)\,\beta^s$$

$$\Rightarrow d_{D_b}\left(W(k_{n+2}^*), W(k_{n+1}^*)\right) \leq \gamma\,(1+\beta)\,\beta^{s+1}$$

Now using by triangular inequality for d_{D_b} Let s and t are two +ve integer such that $t \geq s$, than

$$d_{D_b}\left(W(k_{s+q_1}^*), W(k_s^*)\right) \leq d_{D_b}\left(W(k_{s+q_1}^*), W(k_{s+q_1-1}^*)\right) + \text{--} + d_{D_b}\left(W(k_{s-1}^*), W(k_s^*)\right)$$

$$\leq \gamma\,(1+\beta)[\beta^{s+q_1-1} + \text{-----} + \beta^s]$$

$$\leq \gamma(1+\beta)\frac{\beta^s}{1-\beta}$$

$d_{D_b}(W(k_t^*), W(k_s^*)) \ \leq \gamma\,(1+\beta)\ \frac{\beta^s}{1-\beta}$ (5)

Using (3.4) and (3.5),

$$d_{D_b}^s(W(k_s^*), W(k_t^*)) \leq max\left\{2\gamma\ \frac{\beta^s}{1-\beta}, \gamma\ (1+\beta)\ \frac{\beta^s}{1-\beta}\right\}$$

$\Rightarrow \{W(k_s^*)\}_{s\epsilon N}$ in $\left(M, d_{D_b}^s\right)$ is a Cauchy sequence.

Since (M, D_b) is a complete dualistic partial b − metric space,

By lemma 3.2,$\left(M, d_{D_b}^s\right)$ is as well complete.

Then ∃ an element $k\epsilon\ W(M) \subset M$

Such that $\lim_{n\to\infty} d_{D_b}^s(W(k_s^*), k) = 0$

Where $\gamma = |D_b(W(k_0^*), W(k_1^*))|$

By lemma 3.2, we get

$D_b(k,k) = \lim_{n\to\infty} D_b(W(k_s^*), k) = 0 = \lim_{n,m\to\infty} D_b(W(k_t^*), W(k_s^*)) = 0$ (6)

By equation (3),

$$\lim_{n\to\infty} D_b(W(k_t^*), W(k_s^*)) = 0$$

And equation (5),

$$\lim_{n,m\to\infty} D_b(W(k_s^*), W(k_t^*)) = 0$$

By equation (6), we get

$D_b(k,k) = \lim_{n\to\infty} D_b(W(k_s^*), k) = 0$

As $k\epsilon\ W(M) \subset V(M)$, ∃ $k_1 \epsilon\ M$

Such that $k = \left(V(k_1)\right)$ and equation (3.7)

$$D_b\left(V(k_1), V(k_1)\right) = 0$$

Also $0 \leq d_{D_b}\left(V(k_1), W(k_1)\right) = D_b\left(V(k_1), W(k_1)\right) - D_b\left(V(k_1), V(k_1)\right)$

$$D_b\left(V(k_1), W(k_1)\right)$$

By the equation (1), we have

$$\left|D_b\left(V(k_{n+1}^*), W(k_1)\right)\right| = \left|D_b\left(W(k_n^*), W(k_1)\right)\right|$$

$$\leq \beta max.\left\{\begin{matrix}\left|D_b\left(V(k_n^*), W(k_1)\right)\right|, \left|D_b\left(W(k_n^*), W(k_1)\right)\right|, \\ \left|D_b\left(V(k_1), W(k_1)\right)\right|, \frac{\left|D_b(V(k_s^*), W(k_s^*))\right|}{u}\end{matrix}\right\}$$

$$\leq \beta max.\left\{\begin{matrix}\left|D_b\left(W(k_{s-1}^*), V(k_1)\right)\right|, \left|D_b\left(W(k_s^*), W(k_1)\right)\right|, \\ \left|D_b\left(V(k_1), W(k_1)\right)\right|, \frac{\left|D_b\left(W(k_{s-1}^*), W(k_s^*)\right)\right|}{u}\end{matrix}\right\}$$

Taking limit $n \to \infty$

$$\left|D_b\left(V(k_1), W(k_1)\right)\right| \leq \beta\left|D_b\left(V(k_1), W(k_1)\right)\right|$$

Implies that $D_b(V(k_1),W(k)) = 0 = D_b(V(k_1),V(k)) = D_b(W(k_1),W(k_1))$.

By axioms$(Db1)$,

$\Rightarrow V(k_1) = W(k_1)$

Thus
$k = V(k_1) = W(k_1)$ is a coincidence point of W and V.

Since W and V are weakly compatible mapping $k = V(k_1) = W(k_1)$

$\Rightarrow W(k) = WV(k_1) = VW(k_1) = V(k)$

By equation (1),

$$|D_b(W(k_1),W(k))|$$

$$\leq \beta max.\left\{\begin{matrix} |D_b(V(k_1),V(k))|, |D_b(W(k_1),W(k))|, \\ |D_b(V(k),W(k))|, \frac{|D_b(V(k_1),W(k_1))|}{u} \end{matrix}\right\}$$

$$\leq \beta |D_b(W(k),W(k))|$$

$$\leq \beta |D_b(W(k_1),W(k))|$$

Thus $D_b(W(k_1),W(k)) = 0 = D_b(W(k_1),W(k_1)) = D_b(W(k),W(k))$

By axioms $(Db1)$,

$\Rightarrow W(k_1) = W(k)$

Hence $k = W(k_1) = W(k)$

$\Rightarrow$ k is common fixed point of W and V.

Uniqueness:

$|D_b(k,g)| = |D_b(W(k),W(g))|$

$$\leq \beta \max\left\{\begin{matrix} |D_b(V(k),V(g))|, |D_b(W(k),W(g))|, \\ |D_b(V(g),Wg)|, \\ \frac{|D_b(V(k),W(k))|}{u} \end{matrix}\right\}$$

$$\leq \beta |D_b(k,g)|$$

$\Rightarrow |D_b(k,g)| = 0$

$\Rightarrow k = g$

Hence k has a common fixed point of W and V.

Next we are presenting the Example of complete dualistic partial-metric space.

Example 3.2: Let $N = and D_b: N \times N \rightarrow N$ by $D_b(k,g) = max\{k,g\} \; \forall \; k,g$

and define the mapping $T,F: N \rightarrow N \; by$

$$T(k) = \frac{k^5}{7} \; and \; F(k) = \frac{k^5}{3} \; \forall k \epsilon N$$

Then(N, D_b) is a complete dualistic partial b − metric space

Proof:$\left[\frac{-1}{7},0\right] = T(k) \subset F(k) = \left[\frac{-1}{3},0\right]$

F,T are weakly compatible : $T(k) = T(F(k)) = F(T(k)) = F(k)$

$$T(F(k)) = T\left(\frac{k^5}{3}\right) = \frac{\left(\frac{k^5}{3}\right)}{7} = \frac{k^5}{3^5 \times 7} = 0 \; for \; k = 0$$

$$and \; F(T(k)) = F\left(\frac{k^5}{7}\right) = \frac{\left(\frac{k^5}{7}\right)}{3} = \frac{k^5}{7^5 \times 3} = 0 \; for \; k = 0$$

Implies that $T(F(k)) = F(T(k))$

F,T are weakly compatible mapping for coincidence point k= 0

Without loss of generality let $k > g$ and

Thus

$$|D_b(T(k),T(g))| = \left|max.\left\{\frac{k^5}{7},\frac{g^5}{7}\right\}\right| = \left|\frac{k^5}{7}\right|$$

$$|D_b(F(k),F(g))| = \left|max.\left\{\frac{k^5}{3},\frac{g^5}{3}\right\}\right| = \left|\frac{k^5}{3}\right|$$

$$|D_b(F(g),T(g))| = \left|max.\left\{\frac{g^5}{3},\frac{g^5}{7}\right\}\right| = \left|\frac{g^5}{7}\right|$$

$$|D_b(T(k),F(k))| = \left|max.\left\{\frac{k^5}{7},\frac{k^5}{3}\right\}\right| = \left|\frac{k^5}{7}\right|$$

For generalize f-contraction mapping:

$|D_b(T(k),T(g))|$

$$\leq \beta \max.\left[\begin{matrix} |D_b(F(k),F(g))|, |D_b(T(k),T(g))|, \\ |D_b(F(g),T(g))|, \\ \frac{|D_b(T(k),F(k))|}{u} \end{matrix}\right]$$

$$\left|\frac{k^5}{7}\right| \leq \beta \; max.\left[\left|\frac{k^5}{3}\right|, \left|\frac{k^5}{7}\right|, \left|\frac{g^5}{7}\right|, \frac{1}{u}\left|\frac{k^5}{7}\right|\right] Since \; u \geq 1$$

$$\left|\frac{k^5}{7}\right| \leq \beta \; max.\left[\left|\frac{k^5}{7}\right|\right] \forall \; k \epsilon N$$

Therefore F-contraction condition is hold and β = 1.

Definition 3.3: Let W and V be two self-mapping and dualistic partial b-metric space (M, D_b). We say the mapping W, V a generalize F- contraction with constant u≥1 such that

$$|D_b(W(x),W(y))| \leq a|D_b(V(x),W(x))| + b|D_b(V(y),W(y))|$$

$$+c\frac{[|D_b(V(x),W(y))|+|D_b(W(x),W(x))|]}{u} \quad (8)$$

$\forall\ x, y\epsilon\ M, where\ a, b, c \geq 0\ and\ a + b + 2c < 1$

Theorem 3.2: Let W, V: M→M weakly compatible self-mappings and (M, D_b) a dualistic partial b-metric space such that

1) $W(M) \subset V(M)$

2) W is a generalize F- contraction

Then W and V have a Unique Common Fixed point.

Proof: Let we start with $k_0^* \epsilon\ M$

Since$W(M)\ \subseteq V(M)$

choose $k_1^* \epsilon\ M$

such that $W(k_0^*) = V(k_1^*)$

$W(k_1^*) = V(k_2^*)$

Continue in similar way

$$W(k_s^*) = V(k_{s+1}^*)$$

If $W(k_s^*)$ =$V(k_{n-1}^*)$ =$V(k_s^*)$ for same $s\epsilon N$

Then $u = W(k_s^*)$ =$V(k_s^*)$ is a coincidence point of W and V.

Let $\quad W(k_s^*) \neq Wk_{s-1}^*)$

And $V(k_s^*) \neq V(k_{s+1}^*) \quad \forall n\ \in N$

By the equation (8), we have

$$\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right|$$

$$\leq a\left|D_b\left(V(k_s^*),W(k_s^*)\right)\right|+b\left|D_b\left(V(k_{s+1}^*),W(k_{s+1}^*)\right)\right|$$

$$+ c\frac{\left[\left|D_b\left(V(k_s^*),W(k_{s+1}^*)\right)\right|+|D_b(W(k_s^*),W(k_s^*))|\right]}{u}$$

$$\leq a|D_b(W(k_{s-1}^*),W(k_s^*))| + b\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right| +$$

$$c\frac{\left[\left|D_b\left(W(k_{s-1}^*),W(k_{s+1}^*)\right)\right| + \left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|\right]}{u}$$

$$\leq a|D_b(W(k_{s-1}^*),W(k_s^*))| + b\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right| +$$

$$c\left[\frac{u\left[\left|D_b\left(W(k_{s-1}^*),W(k_s^*)\right)\right| + \left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right|\right]}{s} - \frac{\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|}{u} + \frac{\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|}{u}\right]$$

$$\leq a|D_b(W(k_{s-1}^*),W(k_s^*))| + b\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right| +$$

$$c\left[|D_b(W(k_{s-1}^*),W(k_s^*))| + \left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right|\right]$$

$$(1\text{-}b\text{-}c)\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right| \leq a\left|D_b\left(W(k_{s-1}^*),W(k_s^*)\right)\right|$$

$$+c\left|D_b\left(W(k_{s-1}^*),W(k_s^*)\right)\right|$$

$$\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right| \leq \frac{a+c}{1-b-c}|D_b(W(k_{s-1}^*),W(k_s^*))|$$

$$\leq \alpha|D_b(W(k_{s-1}^*),W(k_s^*))|$$

and so on,

$$\leq \alpha^n|D_b(W(k_0^*),W(k_1^*))| \quad (9)$$

ere $\alpha = \frac{a+c}{1-b-c}$ and $a + b + 2c < 1$

Let $k_s \epsilon M$ then the, we have

$$\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right| \leq\ a\left|D_b\left(V(k_s^*),W(k_s^*)\right)\right| + b\left|D_b\left(V(k_s^*),W(k_s^*)\right)\right|$$

$$+\ c\frac{\left[\left|D_b\left(V(k_s^*),W(k_s^*)\right)\right| + \left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|\right]}{u}$$

$$\leq a|D_b(W(k_{s-1}^*),W(k_s^*))| + b|D_b(W(k_{s-1}^*),W(k_s^*))| +$$

$$c\frac{\left[|D_b(W(k_{s-1}^*),W(k_s^*))| + \left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|\right]}{u}$$

$$\leq a|D_b(W(k_{s-1}^*),W(k_s^*))| + b|D_b(W(y_{n-1}^*),W(y_n^*))| +$$

$$c\left[\frac{u\left[|D_b(W(k_{s-1}^*),W(k_s^*))| + |D_b(W(k_{s-1}^*),W(k_s^*))|\right]}{u} - \frac{|D_b(W(k_{s-1}^*),W(k_s^*))|}{u} + \frac{\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|}{u}\right]$$

$$\leq a|D_b(W(k_{s-1}^*),W(k_s^*))| + b|D_b(W(k_{s-1}^*),W(k_s^*))|$$

$$+\ c\left[|D_b(W(k_{s-1}^*),W(k_s^*))| + \left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|\right]$$

$$(1\text{-}c)\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right| \leq a\left|D_b\left(W(k_{s-1}^*),W(k_s^*)\right)\right| +$$

$$b\left|D_b\left(W(k_{s-1}^*),W(k_s^*)\right)\right| + c\left|D_b\left(W(k_{s-1}^*),W(k_s^*)\right)\right|$$

$$\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right| \leq \frac{a+b+c}{1-c}|D_b(W(k_{s-1}^*),W(k_s^*))|$$

$$\leq \alpha|D_b(W(k_{s-1}^*),W(k_s^*))|$$

and so on, $\leq \alpha^n|D_b(W(k_0^*),W(k_1^*))| \quad (10)$

Where $\alpha = \frac{a+b+c}{1-c}$ and $a + b + 2c < 1$

Now,

$$=\left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right|$$

$$-\left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|$$

$$\leq \left|D_b\left(W(k_s^*),W(k_{s+1}^*)\right)\right| + \left|D_b\left(W(k_s^*),W(k_s^*)\right)\right|$$

$$\leq \alpha^n\left|D_b\left(W(k_0^*),W(k_1^*)\right)\right| + \alpha^n\left|D_b\left(W(k_0^*),W(k_1^*)\right)\right|$$

$$\leq 2\alpha^n\left|D_b\left(W(k_0^*),W(k_1^*)\right)\right|$$

For a fixed natural number q_1, we have

$d_{D_b}\big(W(k^*_{s+q_1-1}),W(k^*_{s+q_1})\big) \le 2\alpha^{s+q_1-1}\gamma, \forall\, s\epsilon N$

where $\gamma = |D_b(W(y^*_0),W(y^*_1))|$

Now using by triangular inequality for d_{D_b} Let s and t are two +ve integer such that $t \geq s$, than

$$d_{D_b}\big(W(k^*_s),W(k^*_{s+q_1})\big) \le d_{D_b}\big(W(k^*_s),W(k^*_{s+1})\big) +$$

$$d_{D_b}\big(W(k^*_{s+1}),W(k^*_{s+2})\big) + \text{-----} + d_{D_b}\big(W(k^*_{s+q_1-1}),W(k^*_{s+q_1})\big)$$
$$\le 2\gamma[\alpha^s + \alpha^{s+1} + \text{-----} + \alpha^{s+q_1-1}]$$

So put $s+q_1 = t$

$$d_{D_b}\big(W(k^*_s),W(k^*_t)\big) \le 2\gamma \frac{\alpha^s}{1-\alpha} \tag{11}$$

By interchanging s and t, we have

$$\big|d_{D_b}\big(W(k^*_{s+1}),W(k^*_s)\big)\big| \le \big|d_{D_b}\big(W(k^*_{s+1}),W(k^*_s)\big)\big| - |D_b(W(k^*_{s+1}),W(k^*_{s+1}))|$$

$$\le |d_{D_b}(W(k^*_{s+1}),W(k^*_s))| + |D_b(W(k^*_{s+1}),W(k^*_{s+1}))|$$

$$\le \alpha^s|D_b(W(k^*_1),W(k^*_0))| + \alpha^{s+1}|D_b(W(k^*_1),W(k^*_0))|$$

$$\le \gamma\,(1+\alpha)\,\alpha^s$$

$\Rightarrow d_{D_b}\big(W(k^*_{n+2}),W(k^*_{n+1})\big) \le \gamma\,(1+\alpha)\,\alpha^{s+1}$Now using by triangular inequality for d_{D_b} Let s and t are two +ve integer such that $t \geq s$, than

$$d_{D_b}\big(W(k^*_{s+q_1}),W(k^*_s)\big) \le d_{D_b}\big(W(k^*_{s+q_1}),W(k^*_{s+q_1-1})\big) +$$

$$\text{--} + d_{D_b}\big(W(k^*_{s-1}),W(k^*_s)\big)$$

$$\le \gamma\,(1+\alpha)[\alpha^{s+q_1-1} + \text{------} + \alpha^s]$$

$$\le \gamma(1+\alpha)\frac{\alpha^s}{1-\alpha}$$

$$d_{D_b}(W(k^*_t),W(k^*_s)) \le \gamma\,(1+\alpha)\,\frac{\alpha^s}{1-\alpha} \tag{12}$$

Using (11) and (12),

$$d^s_{D_b}(W(k^*_s),W(k^*_t)) \le max\left\{2\gamma\,\frac{\alpha^s}{1-\alpha}, \gamma\,(1+\alpha)\,\frac{\alpha^s}{1-\alpha}\right\}$$

$\Rightarrow \{W(k^*_s)\}_{s\epsilon N}$ in $(M, d^s_{D_b})$ is a Cauchy sequence.

Since (M, D_b) is a complete dualistic partial b − metric space,

By lemma 3.2, $(M, d^s_{D_b})$ is as well complete.

Then ∃ an element $k\epsilon\, W(M) \subset M$

Such that $\lim_{n\to\infty} d^s_{D_b}(W(k^*_s),k) = 0$

By lemma 3.2, we get

$$D_b(k,k) = \lim_{n\to\infty} D_b(W(k^*_s),k) = 0 = \lim_{n,m\to\infty} D_b(W(k^*_t),W(k^*_s)) = 0 \tag{13}$$

By equation (10),

$$\lim_{n\to\infty} D_b(W(k^*_t),W(k^*_s)) = 0$$

And equation (3.12),

$$\lim_{n,m\to\infty} D_b(W(k^*_s),W(k^*_t)) = 0$$

By equation (13),we get

$$D_b(k,k) = \lim_{n\to\infty} D_b(W(k^*_s),k) = 0 \tag{14}$$

as $k\epsilon\, W(M) \subset V(M), \exists\, k_1 \epsilon\, M$

such that $k = (V(k_1))$ and equation (14)

$$D_b(V(k_1),V(k_1)) = 0$$

Also $0 \le d_{D_b}(V(k_1),W(k_1))$

$$= D_b(V(k_1),W(k_1)) - D_b(V(k_1),V(k_1))$$

$$= D_b(V(k_1),W(k_1))$$

By the equation (8), we have

$$|D_b(V(k^*_{s+1}),W(k_1))| = |D_b(W(k^*_s),W(k_1))|$$

$$\le a|D_bV((k^*_s),W(k^*_s))| + b|D_b(V(k_1),W(k_1))| +$$

$$c\,\frac{[|D_b(V(k^*_s),W(k_1))| + |D_b(W(k^*_s),W(k^*_s))|]}{s}$$

$$\le a|D_b(W(k^*_{s-1}),W(k^*_s))| + b|D_b(V(k_1),W(v_1))|$$

$$+c\,\frac{[|D_b(W(k^*_{s-1}),W(k_1))| + |D_b(W(k^*_s),W(k^*_s))|]}{s}$$

Taking limit $n \to \infty$

$$|D_b(V(k_1),W(k_1))| \le b\,|D_b(V(k_1),W(k_1))|$$

Implies that $D_b(V(k_1),W(k_1)) = 0 = D_b(V(k_1),V(k_1)) = D_b(W(k_1),W(k_1))$.

By axioms $(Db1)$,

$\Rightarrow V(k_1) = W(k_1)$

Thus $k = V(k_1) = W(k_1)$ is a coincidence point of W and V.

SinceW and V are weakly compatible mapping $k = V(k_1) = W(k_1)$

$\Rightarrow W(k) = WV(k_1) = VW(k_1) = V(k)$

By equation (8),

$$|D_b(W(k_1),W(k))| \le a|D_b(V(k_1),W(k_1))| + b|D_b(V(k),W(k))|$$

$$+c\,\frac{[|D_b(V(k_1),W(k))| + |D_b(W(k_1),W(k_1))|]}{s}$$

$\leq b\,|\,D_b(W(k),W(k))\,|$

$\leq b\,|\,D_b(W(k_1),W(k))\,|$

Thus
$D_b(W(k_1),W(k))=0=D_b(W(k_1),W(k_1))=D_b(W(k),W(k))$

By axioms$(Db1)$,

$\Rightarrow W(k_1)=W(k)$

Hence $k=W(k_1)=W(k)$

Similarly $V(k)=k$

That is k is a common fixed point of W and V.

Example 3.3: Assume that $Y=[-1,0]$

and $D_b: Y\times Y\to Y$ by $D_b(g,h)=max.\{g,h\}\ \forall g,h\epsilon Y$

Then(Y,D_b) is a complete dualistic partial b − metric space and define the mapping

$T,F: Y\to Y$ by

$$T(g)=\frac{g^5}{7}\ and\ F(g)=\frac{g^5}{3}\ \forall g\epsilon Y$$

Proof:$\left[\frac{-1}{7},0\right]=T(g)\subset F(g)=:\left[\frac{-1}{3},0\right]$

F,T are weakly compatible $:T(g)=T(F(g))=F(T(g))=F(g)$

Than $T(F(g))=T\left(\frac{g^5}{3}\right)=\frac{\left(\frac{g^5}{3}\right)}{7}=\frac{g^5}{3^5\times 7}=0\ for\ g=0$

And $F(T(g))=F\left(\frac{g^5}{7}\right)=\frac{\left(\frac{g^5}{7}\right)}{3}=\frac{g}{7^5\times 3}=0\ for\ g=0$

Implies that $T(F(g))=F(T(g))$

F,T are weakly compatible mapping for coincidence point $g=0$

Without loss of generality we can assume that $g>h$ and

Thus

$$|D_b(T(g),T(h))|=\left|max\left\{\frac{g^5}{7},\frac{h^5}{7}\right\}\right|=\left|\frac{g^5}{7}\right|$$

$$|D_b(F(g),T(g))|=\left|max\left\{\frac{g^5}{3},\frac{g^5}{7}\right\}\right|=\left|\frac{g^5}{7}\right|$$

$$|D_b(F(h),T(h))|=\left|max\left\{\frac{h^5}{3},\frac{h^5}{7}\right\}\right|=\left|\frac{h^5}{7}\right|$$

$$|D_b(F(g),T(h))|=\left|max\left\{\frac{g^5}{3},\frac{h^5}{7}\right\}\right|=\left|\frac{h^5}{7}\right|$$

$$|D_b(T(g),T(g))|=\left|max\left\{\frac{g^5}{7},\frac{g^5}{7}\right\}\right|=\left|\frac{g^5}{7}\right|$$

For generalize f-contraction mapping:

$$|D_b(T(g),T(h))|\leq a|D_b(F(g),T(g))|+b|D_b(F(h),T(h))|+c\frac{[|D_b(F(g),T(h))|+|D_b(T(g),T(g))|]}{s}$$

$$\left|\frac{g^5}{7}\right|\leq a\left|\frac{g^5}{7}\right|+b\left|\frac{h^5}{7}\right|+c\left[\frac{\left|\frac{h^5}{7}\right|+\left|\frac{g^5}{7}\right|}{s}\right]$$

$$s\geq 1\ and\ a,b,c\geq 0$$

Therefore generalize F-contraction condition is hold.

IV.CONCLUSION

In the above proved fixed point theorems on ordered dualistic partial b-metric space, generalize f-contraction mapping is used. This result is generalized the result of Nazam and Arshad [24] and is an extension of their result.

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