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The Study of Anisotropic Model with Variable Cosmological Term

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ABSTRACT: This paper deals with the study of anisotropic model with variable cosmological term connect with the eradication of dense energy conditions and also with cosmological observations possess large negative pressure. The cosmological considerations provided the string indication that our universe has recently been in accelerated expansion phase during the last period. This paper describes the classical propositions about the nature of dark matter and dark energy which can be associated to the accelerating expansion of the universe to discover the origin of the extended universe. To consider this we deal with Bianchi type− I cosmological model with the ratio to appropriate function of an average scale factor.

Keywords: Acceleration, Cosmological Parameters, Dark energy, Dark matter, Hubble parameters

I. INTRODUCTION

Cosmology is the branch of science concerned with the study of the nature of the Universe and the entire at a big scale. In early 1990's, the growth of the universe fairly showed that it might have enough energy density to break its growth and re-collapse and also there may be that extent of little energy density, it would have no control over its expansion, with the passage of time, gravity might be certain to slow down the expansion. In observation, there is no slowing in the universe, but hypothetically the universe has long passed its peak and is slowly. All the matter of universe pulled together by the attractive force of Gravity. In accelerated expansion phase by [1-7], exposed that the expansion of the universe has not slowed down due to gravity as it was earlier. In recent years, there has been a prominent and an important evidence for the recognition of Einstein's cosmological constant (Λ). Λ is the function of temperature which is associated with the impulsive symmetry breaking procedure suggested by [8], therefore for the spatially homogenous expanding universe, Λ is measured as the function of time, as the result, the formation of the particles and the expansion of the universe leads to reduction of value of Λ. The universe is very old that is why the constant is small. The vacuum's energy density is considered by [9-11], depicts the models with dynamic decaying cosmological term. [12, 13], have discussed the inverse time square dependence of $\Lambda \propto t^{-2}$. [14, 15] have studied the literature of cosmological models with Λ proportional to the scale factor. In general relativity, [16-19] has done a lot of work on the anisotropic Bianchi type-I cosmological model for varying Λ. [20-23] have considered the perfect fluid Bianchi type–I model by assuming θ proportionate to σ with variable Λ. The possible confirmation that the Universe is fast-moving presented by the observational study of Type Ia supernovae. The Invention of cosmic acceleration was the most important finding in modern cosmology. The basis of cosmic acceleration is still unknown. As per General Relativity, the slowing rate of expansion should be led by gravity, if the Universe consists of ordinary matter or radiation, then this expansion of powerful explosion is known as "Big Bang". But, if we observe the chronology of the Universe, after the beginning of cosmic inflation, the Universe still expands in an accelerating rate. After that, Researchers originated three explanations, which were "cosmological constant", "Dark Energy" and "Dark Matter." A long-discarded version of Einstein's theory of gravity was "Cosmological constant." "Dark Energy" was tied to Einstein's cosmological constant, explaining the unrealistic expansion of the Universe, nevertheless the correct explanation was not known to theorists even then they gave the solution a name, the dark energy. Gravity, being the weakest fundamental force of the Universe, couldn't have alone played the role. Therefore, a hypothetical form of matter, known as "Dark Matter" was introduced. Hence, the origin of dark forces from the present epoch of the Big Bang was known. In recent years, an evidence suggests that there is something in our Universe which is invisible, as there is some new form of matter or energy. As per current measurements of NASA's WMAP (Wilkinson's Microwave Anisotropy Probe), the dark energy gives 68.3% (approx. 70%) of the total energy, where the mass-energy of dark matter contribute 26.8% (approx. 25%) and ordinary (baryonic) matter) contribute 4.9% (approx. 5%) , in today's universe, which were determined while making the accurate measurements of the cosmic microwave background fluctuations. The universe is spatially flat.
($\Omega_{total} = \Omega_{mass} + \Omega_{relativistic} + \Omega_{\Lambda} = 0.315 \pm 0.018 + 9.24 \times 10^{-5} + 0.6817 \pm 0.0018 = 1.00 \pm 0.02$)
accelerating, composed of photons and neutrinos, dark energy, normal mass (baryonic matter) in the current accepted model of modern cosmology. Therefore the role and nature of dark energy and dark matter is concerned with the fate and face of the Universe. They must show a substantial role in the chronology of the Universe in leading it to its present state. The expansion of the universe is accelerated by the small value of cosmological constant. The formation of anisotropic compact stars comes into existence by the help of competent candidates of cosmological constant. The analytical solution of Krori and Barua metric has

considered for this purpose. The Tolman–Oppenheimer–Volkoff (TOV) equations (which is Static, spherically symmetric perfect fluid models in general relativity), consists the stability and the surface redshift of the compact stars. It also has the radial dependence on cosmological constant. The Prime purpose of proposed work is to investigate Λ in the form of the perfect fluid which contains matter in the homogeneous anisotropic Bianchi type-I space time, we get the solution of the Einstein field equations for stiff matter by taking the cosmological term proportionate to the square of the Hubble parameter. The manuscript follows the sequence of sections as described: In section 2, the basic definitions of anisotropic models are given. In section 3, solutions of field equations are produced by proposing the cosmological term which is proportional to square of the Hubble parameter. In section 4, the conclusion drawn from the results and In section 5, the future research idea is given.

II. THE METRIC AND FIELD EQUATIONS

The line element describes the spatially homogenous and anisotropic Bianchi type-I space-time by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad (1)$$

The functions of t are $A^2(t)$, $B^2(t)$, $C^2(t)$. The spatial volume of this model is specified by

$$V = R^3 = ABC \quad (2)$$

We define Average scale factor R is $[ABC]^{1/3}$ of Bianchi type-I Universe. Hubble parameter is defined in anisotropic models is given by

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right]$$

The dots describes the ordinary time derivatives of the concerned quantity. Hubble parameter H is given by

$$H = \frac{1}{3}[H_1 + H_2 + H_3] \quad (3)$$

Here $H_1 = \frac{\dot{A}}{A}$ is the directional Hubble factors in the x direction, $H_2 = \frac{\dot{B}}{B}$ is the directional Hubble factors in the y direction and $H_3 = \frac{\dot{C}}{C}$ is the directional Hubble factors in the z direction respectively. The energy momentum tensor of a perfect fluid is characterized the Cosmic matter as

$$T_i^j = (\rho + \bar{p})\,\upsilon_i\upsilon_j + \bar{p}g_{i_j} \quad (4)$$

The energy density is denoted by ρ, the four-velocity vector of the element is denoted by υ_i, the scalar expansion is θ, ε is coefficient of bulk viscosity, $\bar{p}$ is dissipative pressure and equilibrium pressure is denoted by p satisfying the relation.

$$v_i\ v_j = -1,$$
$$p = \omega\rho,\ 0 \le \omega \le 1. \quad (5)$$

The Einstein's field equation with cosmological constant lambda and time dependent gravitational G is

$$R_{ij} - \frac{1}{2}R_1^1 g_{i_j} = \Lambda(t)g_{i_j} - 8\pi\, G(t)T_{i_j} \quad (6)$$

R is the Ricci scalar . The conservation principle, that is div $(T_i^j) = 0$ and commoving system of co-ordinate, the field equations for the metric (1) by substituting the value of energy momentum tensor (2) are given by

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G\bar{p} + \Lambda \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G\bar{p} + \Lambda \quad (8)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G\bar{p} + \Lambda \quad (9)$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G\rho + \Lambda \quad (10)$$

$$8\pi\,\dot{G}\rho + 8\pi G\left[\dot{\rho} + (\rho + p)\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right]\right] + \dot{\Lambda} = 0 \quad (11)$$

with respect to cosmic time t, dot (.) denotes ordinary differentiation. Energy conservation equation $(T_{i,j}^j = 0)$ gives,

$$\dot{\rho} + (\rho + p)\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = 0 \quad (12)$$

Equation (12) together with (13) puts G and Λ coupled field specified by

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0 \quad (13)$$

When G is a constant then Λ is constant. By taking (5) in (12) and integrating, $k > 0$, in particular we are supposing $w = 0$, take $\rho = \frac{k}{R^3}$ (14)

Shear tensor σ_{ij}, the non-vanishing component is defined as $\sigma_{ij} = u_{ij+}\ u_{ji} - \frac{2}{3}g_{i_j}\ u;_k^k$ are obtained as

$$\sigma_1^1 = \frac{4}{3}\frac{\dot{A}}{A} - \frac{2}{3}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] \quad (15)$$

$$\sigma_2^2 = \frac{4}{3}\frac{\dot{B}}{B} - \frac{2}{3}\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right] \quad (16)$$

$$\sigma_3^3 = \frac{4}{3}\frac{\dot{C}}{C} - \frac{2}{3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right] \quad (17)$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \left[\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}\right]\right] \quad (18)$$

By using the equations (18) and (3) , we get

$$\frac{\dot{\sigma}}{\sigma} = -\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = -3H \quad (19)$$

In the Einstein's field equations from (7) to (10) shear scalar σ, deceleration parameter q and Hubble parameter H can also be written as

$$H^2(2q - 1) - \sigma^2 + \frac{1}{R^2} = 8\pi\ G\ \bar{p} - \Lambda \quad (20)$$

$$3H^2 - \sigma^2 = 8\,\pi\,G\rho + \Lambda \quad (21)$$

H is the hubble parameter, q is the deceleration parameter, θ is the volume expansion , σ is shear scalar are given as

$$\theta = 3H = \frac{3\dot{R}}{R},\quad \sigma = \frac{k}{\sqrt{3}R^3},\ k > 0\ (\text{constant}) \quad (22)$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}$$

On integrating Eqs. (7) - (10) we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC} \quad (23)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{ABC} \quad (24)$$

Constants of integration are k_1, k_2. Using the equation (21) we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} \quad (25)$$

Implying that $\Lambda \ge 0$

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3} \text{ and } 0 < \frac{8\Pi\,\rho\,G}{\theta^2} < \frac{1}{3}$$

The upper limit of anisotropy lowers by the existence of positive Λ , the anisotropy is given by the negative Λ. The Eqn. (25) can be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\Pi\,\rho G}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c} \quad (26)$$

Critical density is $\rho_c = \frac{3H^2}{8\Pi G}$ and the vacuum density is $\rho_v = \frac{\Lambda}{8\Pi G}$

By the Eqns. (20) and (21)

$$\frac{d\theta}{dt} = -12\pi G\theta\rho - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2 \quad (27)$$

The rate of decrease slows down due to the presence of positive Λ and during time evolution although a negative Λ would encourage it, rate of volume expansion decreases.

From the equations (20) and (21) we obtained

$$\Lambda = (2 - \mathrm{q})H^2 - \frac{(1-w)\rho}{2} \quad (28)$$

If $\mathrm{q} \geq 2$ then $\Lambda \leq 0$

III. SOLUTION OF THE FIELD EQUATIONS

The five equations in six unknown (A, B, C, ρ, p and Λ) are provided by the system of Eqns. (5) and (7) to (10), one more equation is required to resolve the system entirely. Therefore we can propose that the cosmological term which is proportional to the square of the Hubble parameter.

$$\Lambda = \mathrm{a}H^2 = a\frac{\dot{R}}{R} \quad (29)$$

here a is a positive constant.

Current value of H, Ω and $\frac{\sigma}{\theta}$, can be acquired. Therefore stiff fluid ($w = 1$)

(20), (21), (29), we take $a = 1$,

$$H = \frac{1}{2t} \quad (30)$$

From (30) we get scale factor

$$R = e^{\frac{t_0}{2}} \times \sqrt{t} \quad (31)$$

where m is constant of integration, the metric (1) supposing the form, by taking (31), (23) and (24) as given by

$$ds^2 = -dt^2 + R^2\left[m_1^2(\exp\left\{\left[\frac{-2}{M\sqrt{t}}\right]\left[\frac{2k_1+k_2}{3}\right]\right\}\right]dx^2 + R^2\left[m_2^2(exp\left\{\left[\frac{-2}{M\sqrt{t}}\right]\left[\frac{k_2-k_1}{3}\right]\right\}\right]dy^2 + R^2\left[m_3^2(\exp\left\{\left[\frac{-2}{M\sqrt{t}}\right]\left[\frac{-2k_2-k_1}{3}\right]\right\}\right]dz^2 \quad (32)$$

In above equation m_1, m_2, m_3 are constants and $M = e^{\frac{3t_0}{2}}$

From the model (32), density ρ, cosmological constant Λ and the spatial V are

$$V = R^3 = \left[e^{\frac{t_0}{2}} \times \sqrt{t}\right]^3 \quad (33)$$

$$\rho = \frac{k}{\left[e^{\frac{t_0}{2}} \times \sqrt{t}\right]^3} \quad (34)$$

$$\Lambda = \frac{1}{[2t]^2} \quad (35)$$

Expansion scalar $\theta = \frac{3}{2t}$ (36)

Shear $\sigma = \frac{k}{\sqrt{3}} \times \frac{1}{\left[e^{\frac{t_0}{2}} \times \sqrt{t}\right]^3}$ (37)

For this model the deceleration parameter

$$q = 1 \quad (38)$$

$$\frac{\sigma}{\theta} = \frac{\frac{k}{\sqrt{3}} \times \frac{1}{\left[e^{\frac{t_0}{2}} \times \sqrt{t}\right]^3}}{\frac{3}{2t}} = \frac{2}{3\sqrt{3}}\frac{k}{e^{\frac{3t_0}{2}}t^{1/2}} \quad (39)$$

Ω is defined as the ratio between the vacuum density and matter density is given by

$$\Omega = \frac{\Lambda}{\rho} = \frac{\frac{1}{[2t]^2}}{\frac{k}{\left[e^{\frac{t_0}{2}} \times \sqrt{t}\right]^3}} = \frac{e^{\frac{3t_0}{2}}t^{-1/2}}{4k} \quad (40)$$

IV. CONCLUSION

We explored variable Λ like the form of the perfect fluid with containing matter in the homogeneous anisotropic Bianchi type-I cosmological model by assuming the cosmological term proportionate to the square of the Hubble parameter. From the model (32), we established that the expansion scalar θ is infinite and the spatial volume V is zero at $t = 0$. The universe begins evolving with zero volume and an infinite rate of expansion at $t = 0$. The model has a point-type" singularity at the early era. At the initial singularity, the pressure, cosmological term, Hubble factor, shear scalar and energy density diverges. As t increases, shear scalar, density and pressure decrease and approaches to zero asymptotically. At t = 0, the cosmic scenario begins from a big bang and remains till $t = \infty$. The model tends to isotropy for a big value of t as $\frac{\sigma}{\theta} \rightarrow 0$ then $\mathrm{t} \rightarrow \infty$. Since $R \rightarrow 0$ concluding that initially the declaration parameter q for the model is 1. Therefore from the model (32) depicts an accelerating universe.

In this paper we investigated the nature of dark forces and extended their effects to analyse the various cosmological phase transitions that occurred whose evidence can be found in the chronology of the Universe, along with the observational evidence and studied the Big Bang Cosmological Models which signified the existence of Dark Energy and Dark Matter, moreover various scalar field models such as cosmological inflation, quintessence, and other elements. The presence of dark matter and dark energy noticed in the big bang universe which can be sketched back to inflationary universe. It was perceived that inferred inertial mass of a portion of the virtual matter convert into inertial mass of real particles when the inflation ends. The late-time accelerating expansion of the universe was due to the proposition that required the inertial of the visible matter be in space time accelerating frame of reference. It was observed that mass is free fall in the inflation any universe and when inflation end it transformed into the dark matter in the universe. Therefore, it was observed that nothing in the universe that derived the accelerating expansion of the universe.

V. FUTURE SCOPE

With the advancement in astrophysical studies and latest technologies, there is lot more to be observed, analysed and learned in our never-ending universe. According to multiverse theory, there can be possibilities of many alternative universes with different timelines, different laws of physics, never-ending possibilities, and new concepts.

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CONFLICT OF INTEREST

Author has no any conflict of interest.

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