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Effect of Magnetic Field, Two-temperature, Dual Phase Lag and Initial Stress on the Rayleigh Wave Propagation

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ABSTRACT: The governing equation of the Rayleigh wave of with two temperatures, initial stress and magnetic field and dual phase lag thermo-elasticity are solved. The governing equation are solved by the surface wave solution and particular solution satisfies the boundary condition to obtain the secular equation of Rayleigh wave for thermally insulated/isothermal on stress free surface in half-space. The secular equation is approximated for calculating the numerical value of the velocity, amplitude-attenuation factor of Rayleigh wave by using ortran programming for a given material. Hence, effect of magnetic field, initial stress, two-temperature, dual-phase-lag on amplitude- attenuation and velocity of propagation are shown graphically.

Keywords: Thermo-elasticity, Two Temperature, Rayleigh waves, generalized thermo-elasticity, Dual-phase-lag.

I. INTRODUCTION

Biot [1], was the first who investigate the classical phenomena of dynamic coupled thermo-elasticity and this theory was further extended to the generalized thermo-elasticity by Lord & Shulman [2] and Green & Lindsay [3]. In this theory they consider the propagation of heat as a wave phenomenon instead of diffusion phenomenon and hence it helps to predict the speed propagation of heat. J-ignaczak and Starzewski [4] ,introduced that heat in a medium predicts as a finite velocity of propagation. Hetnarski and Ignaczak [5], investigated different effects of propagation of wave in thermoelastic medium and this effect is used in many fields like as mineral and oil exploration. Geophysics, engineering. Deresiewicz [6] Singh [7-8], Sinha, Sinha [9], Othman , Song [10], in coupled thermoelasticity and also in generalized thermoelasticity they consider many different problems for plane-wave propagation. Tzou [11-13] developed that the dual phase lag model where the response of interaction between phonon and electron at macroscopic level is considered delayed as compare to microscopic level. In this model we use modified fourier law which have two different time-translation. Recently, Rayleigh wave propagation by using DPL model in isotropic thermoelastic solid half-space was investigated by Abouelregal [14]. Further the behavior of Rayleigh wave by using DPL model in initially stressed solid half space of anisotropic thermo-elastic surface under the effect of magnetic field by S. Kumari [15]. Gurtin and Williams [16,17] proposed the 2nd law of thermodynamics in which because of heat conduction in continuous body phenomena of entropy was governed. Chen *et al.,* [18, 19] was first who purposed a theory on two -temperature for heat conduction through a material. The main aim of that theory was to construct a material in such a way that the two different temperatures don't coincide within it. In this theory a parameter that is $a^* > 0$ for material is involve such that if $a^* \to 0$ then $\phi \to T$ where, ϕ is conductive temperature and T is thermodynamic temperature. This theory helps to make a two temperature model in which the distribution of temperature in electron and phonon was predicted under the process of ultra short laser in any metals. In certain conditions time dependent problem can be equal to two temperature theory was investigated by Warren and chen [20] , whereas Φ and T in particular wave propagation are different. In two temperature theory the motion of harmonic plane waves was studied by Puri and Jordan [21]. Theory for two-temperature generalized thermo-elasticity was developled by Youssef [22]. Further, the exact solution for two-temperature in dual phase lag model under the given two initial boundary conditions was derived Quintanilla and Jordan [23]. E. Karamany and Ezzat [24] and Ezzat and E.Karamany [25] and Ezzat *et al.*, [26] without energy dissipation in two-temperature thermoelasticity they proved uniqueness and reciprocity theorems. Ezzat *et al.*, [27] developed a theory with fractional order DPL heat transfer under two-temperature magneto-thermo-elasticity. Finite wave speed of thermo-elasticity under fractional order of two-temperature was proposed by Sur and Kanoria [28]. H.M. Youssef (2012) [29] gives a state-space approach on generalized themroelasticity of two-temperature having no energy dissipation with a medium subject to a moving source of heat. M.A. Ezzat and A.S. [30] Karamony purposed a model of magneto-thermoelasticity with two-temperature in which heat conduction law of problem of two-temperature theory for a thermo-elastic half-space that was subjected to the Ramp-theory heating.

Youssef and Harby (2007) [31] gave a theory on generalized thermoelasticity for two-temperature of an infinite solid having a spherical cavity which is subjected to different loading temperature in which equation obtained from above discussion is used to obtain the formulation of model. H.M. Youssef (2008) [32] solved a problem of 2-d for generalized thermoelasticity with two-temperature and half space which is subjected to Ramp-type heating. Youssef and Bassiouny (2008) [33] using above theories solve the problem of 1-d pizoelctric half-space by using its boundary condition in which material is objected to three types of heating effect- a) thermal shock-type b) Ramp-type c) Harmonic-type. H.M. Youssef (2009) [34] purposed a model in which cylindrical cavity is subjected to any moving source of heat which is solved by two theory of generalized thermoelasticty with two-temperature. In this analytical solution is obtained by laplace transforms and its numerical result, discussion and comparison is taken with the L.S. Model of 1-d generalized themoelasticity. Ezzat *et al.*, [35] Kumar and Mukhopadhyay [36] discussed effects on wave propagation of thermal relaxation time in context of two-temperature thermo-elasticity. The phenomena of plane waves reflected back on a two-temperature generalized thermoelastic solid half-space free surface was studied by Singh and Bala [37]. Kumar *et al.*, [38] investigated parameter on plane harmonic wave passes through an elastic medium that are 2nd relaxation time and two-temperature.

In the present work, behaviour of Rayleigh wave by using dual phase lag model in initially stressed solid half space of anisotropic thermo-elastic surface under the effect of magnetic field and two temperature are studied. The frequency equation of Rayleigh wave under the given surface with particular cases i.e. Thermally insulated space/Isothermal and some special case had been derived. In this the combined effect of magnetic field and two-temperature are shown graphically.

II. BASIC EQUATIONS

Following Tzou [11, 12] and Youssef, *et al.*, [22] the basic equations Dual Phase lag transversely isotropic thermo-elasticity.

1. Constitutive Equation

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad (1)$$

$$\sigma_{ij} = (c_{ijkl} + \delta_{ij} P_{ij}) e_{kl} - \beta_{ij}\Theta \quad (2)$$

2. The Equation of Motion

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + \rho F_i \quad (3)$$

3. The equation of energy

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + \rho F_i \quad (4)$$

4. The modified Fourier law

$$-K_{ij}\left(\Phi_{,j} + \tau_\theta \frac{\partial \Phi_{,j}}{\partial t}\right) = q_i + \tau_q \frac{\partial q_i}{\partial t} \quad (5)$$

5. The Relation between entropy, Strain and Temperature

$$\rho S = \frac{\rho c_E}{T_0}\Phi + \beta_{ij} e_{ij} \quad (6)$$

6. The basic equation of Maxwell's Electromagnetic field

$$\nabla.\boldsymbol{E} = -\mu_e \frac{\partial \boldsymbol{h}}{\partial t}, \quad \nabla.\boldsymbol{E} = 0, \quad (7)$$

$$\nabla.\boldsymbol{h} = j, \nabla.\mathbf{h} = 0$$

7. The equation for Maxwell stress

$$\bar{\sigma}_{ij} = \mu_e\left[H_i h_j + H_j h_i - (\mathbf{Hh})\delta_{ij}\right] \quad (8)$$

8. The Relation between two temperature:

$$\Phi - \Theta = a^* \Phi_{ii} \quad (9)$$

where i, j, k, l are from 1 to 3

Notations

$\Theta = \Phi - T_0$	=	small temperature increment,
T	=	absolute temperature,
T_0	=	uniform reference temperature s.t $\left\lvert\frac{\Theta}{T_0}\right\rvert \ll 1$
δ_{ij}	=	kronecker delta
Φ	=	the conductive temperature
a*	=	parameter of two temperature
ρ	=	density of the medium
q_i	=	vector of heat conduction
K_{ij}	=	thermal conductivity tensor components
C_E	=	specific heat at the constant strain
C_{ijkl}	=	tensor of the elastic constant
σ_{ij}	=	stress tensor components
P_{ij}	=	parameter of Initial stress
u_i	=	displacement vector
e_{ii}	=	strain tensor component
S	=	entropy per unit mass
β_{ii}	=	constitutive coefficients.
H	=	perturbed magnetic field over H_0
J	=	electric current density
$\mathbf{u_e}$	=	magnetic permeability,
h	=	$\nabla \times (\mathbf{u} \times \mathbf{H0})$ and **H , H0** + **h**.
τ_q	=	phase-lag of heat flux
τ_θ	=	temperature with $0 \le \tau_\theta \le \tau_q$

III. FORMULATION OF PROBLEM

We consider a transversely isotropic homogeneous dual phase lag thermo-elastic half- space solid under the effect of magnetic field and two temperature with (x, y, z) in Cartesian coordinate system at uniform temperature previously. Origin of the co-ordinate system from the plane surface and the is normal to z-axis ($z \ge 0$) and was assumed at z = 0 at free from stress on a thermally insulated and isothermal surface. We considered the plane strain parallel to x-z plane, and **u** = $(u_1, 0, u_3)$ as displacement vector and $\mathbf{H}_0 = (0, H_0, 0)$ as constant magnetic field Using equation (1) –(9) we obtained the following:

$$(c_{11} + P_{11})\frac{\partial^2 u_1}{\partial x^2} + (c_{13} + c_{44} + P_{11})\frac{\partial^2 u_3}{\partial x \partial z} + \beta_1 \frac{\partial \Theta}{\partial x} = \rho \ddot{u}_i - \mu_e H_0^2\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_3}{\partial x \partial z}\right) \quad (10)$$

$$(c_{44}+P_{11})\frac{\partial^2 u_3}{\partial x^2}+(c_{13}+c_{44}+P_{11})\frac{\partial^2 u_1}{\partial x\partial z}$$
$$+(c_{44}+P_{33})\frac{\partial^2 u_3}{\partial z^2}-\beta_3\frac{\partial\Theta}{\partial z}$$
$$=\rho\ddot{u}_i-\mu_e H_0^2(\frac{\partial^2 u_1}{\partial x\partial z}+\frac{\partial^2 u_3}{\partial z^2}) \quad (11)$$

$$(1+\tau_\theta\frac{\partial}{\partial t})(K_1\frac{\partial^2\Phi}{\partial x^2}+K_3\frac{\partial^2\Phi}{\partial z^2})$$
$$=(1+\tau_q\frac{\partial}{\partial t})[\rho c_E\frac{\partial\Theta}{\partial t}+\beta_1 T_0\frac{\partial^2 u_1}{\partial x\partial t}$$
$$+\beta_3 T_0\frac{\partial^2 u_3}{\partial z\partial t} \quad (12)$$

$$\Phi-\Theta=a^*(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2}) \quad (13)$$

Where

$$K_1=K_{11},\ K_3=K_{33},\ \beta_1=\beta_{11},\ \beta_3=\beta_{33}$$

IV . SURFACE WAVE SOLUTION

For thermo-elastic surface wave the propagation in x-direction and the displacement and potential function are written as:

$$(u_1,u_3,\Phi)=[\phi_1(z),\phi_3(z),\varphi(z)]e^{ik(x-ct)} \quad (14)$$

Using the equation (13) into (10)-(12) and then using equation (14), we obtain the following homogenous system of the three equation:

$$[k^2(\frac{\rho c^2-c_{11}-P_{11}}{c_{44}+P_{11}})+D^2]\phi_1$$
$$+ik[\frac{c_{13}+\mu_e H_0^2}{c_4+P_{11}}+1]D\phi_3$$
$$-ik\frac{\beta_1}{c_{44}+P_{11}}[1-a^*(-k^2+D^2)]\varphi=0 \quad (15)$$

$$[ik(\frac{c_{13}}{c_{44}+P_{11}}+1)]D\phi_1$$
$$+[k^2(\frac{\rho c^2}{c_{44}+P_{44}}-1)+\frac{c_{33}+P_{33}}{c_{44}+P_{44}}D^2]\phi_3$$
$$-\frac{\beta_3}{c_{44}+P_{44}}[1-a^*(-k^2+D^2)]\varphi=0 \quad (16)$$

$$ik^3\varepsilon\frac{\rho c^2}{c_{44}+P_{11}}\beta_1\phi_1$$
$$+\beta_3\frac{\rho c^2}{c_{44}+P_{11}}\varepsilon k^2 D\phi_3$$
$$+[k^2\{\frac{\rho c^2}{c_{44}+P_{11}}\{1-a^*(-k^2+D^2)\}-K_1^*\}$$
$$+K_3^*D^2]\varphi=0 \quad (17)$$

where

$$\varepsilon=\frac{\beta_1^2 T_0}{\rho^2 c_E c^2},\ \tau^*=\frac{\tau_q+\frac{i}{\omega}}{1-i\omega\tau_\theta},\ K_1^*=\frac{K_1}{c_E(c_{44}+P_{11})\tau^*}$$
$$K_3^*=\frac{K_3}{c_E(c_{44}+P_{11})\tau^*},$$

It is necessary for the non trivial solution of equation (15)-(17) is

$$D^6-AD^4+BD^2-C=0 \quad (18)$$

and A,B, and C can be written as

$$A=\frac{1}{K_3^*(r_3+s_1)-\zeta(r_3+s_1)k^2a^*-\overline{\beta}^2\xi\varepsilon a^*}$$
$$[k^2(r_2+s_1+1)^2(k^2a^*\xi-K_3^*)+$$
$$\overline{\beta}k^4\varepsilon\xi a^*(r_2+s_1+1)-k^4a^*\varepsilon\xi(r_3+s_1)$$
$$-k^2K_3^*(\xi-r_1-s_1)(r_3+s_1)$$
$$+k^4(\xi-r_1-s_1)\xi a^*(r_3+s_1)$$
$$+\overline{\beta}^2\xi\varepsilon a^*k^4(\xi-r_1-s_1)-k^2K_3^*(\xi-1)$$
$$+a^*k^4(\xi-1)\xi$$
$$-(r_3+s_1)k^2\xi(1+a^*k^2)+(r_3+s_1)k^2K_1^*$$
$$-\overline{\beta}^2\xi\varepsilon k^2(1+a^*k)] \quad (19)$$

$$B=\frac{1}{K_3^*(r_3+s_1)-\xi(r_3+s_1)k^2a^*-\overline{\beta}^2\xi\varepsilon a^*}$$
$$[k^4(\xi-1)\xi(1+a^*k^2)-K_1^*k^4(\xi-1)+$$
$$k^4K_3^*(\xi-r_1-s_1)(\xi-1)$$
$$+k^4\xi(\xi-r_1-s_1)(1+a^*k^2)(r_3+s_1)$$
$$-k^6a^*\xi(\xi-r_1-s_1)(\xi-1)$$
$$-k^4K_1^*(r_3+s_1)(\xi-r_1-s_1)$$
$$+k^4\overline{\beta}^2\xi\varepsilon(\xi-r_1-s_1)(1+a^*k^2)$$
$$+k^4\xi(1+a^*k^2)-k^4K_1^*(r_2+s_1+1)^2$$
$$+2k^4\overline{\beta}^2\varepsilon\xi(r_2+s_1+1)(1+a^*k^2)$$
$$-k^4\varepsilon\xi(1+a^*k^2)(r_3+s_1)$$
$$+k^6a^*\varepsilon\xi(\xi-1)] \quad (20)$$

$$C = \frac{1}{K_3^*(r_3+s_1)-\xi(r_3+s_1)k^2a^*-\overline{\beta}^2\xi\varepsilon a^*}$$
$$[(\xi-r_1-s_1)(\xi-1)k^6(\xi(1+a^*k^2)-K_1^*)$$
$$-k^6(1+a^*k^2)\varepsilon\xi(\xi-1)] \quad (21)$$

$$r_1 = \frac{c_{11}+P_{11}}{c_{44}+P_{11}},\ r_2 = \frac{c_{13}}{c_{44}+P_{11}},\ r_3 = \frac{c_{33}+P_{33}}{c_{44}+P_{11}},$$
$$s_1 = \frac{\mu_e H_0^2}{c_{44}+P_{11}}, \xi = \frac{\rho c^2}{c_{44}+P_{11}}$$

(i)

(ii)

The general solution of (18) can be written as

$$\phi_1(z) = [\sum^3 A_i e^{-m_i z} + \sum^3 A_i^* e^{m_i z}]e^{ik(x-ct)} \quad (22) \text{ (iii)}$$

$$\phi_3(z) = [\sum^3 B_i e^{-m_i z} + \sum^3 B_i^* e^{m_i z}]e^{ik(x-ct)} \quad (23)$$

$$\varphi(z) = [\sum^3 C_i e^{-m_i z} + \sum^3 C_i^* e^{m_i z}]e^{ik(x-ct)} \quad (24)$$

Where A_i, B_i, C_i and A_i^*, B_i^*, C_i^* are the constants and m_i be the roots of auxiliary equation of (18) which is cubic in m^2, and roots m_1^2 , m_2^2 , and m_3^2 are related as

$$m_1^2 + m_2^2 + m_3^2 = A \quad (25)$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = B \quad (26)$$

$$m_1^2 m_2^2 m_3^2 = C \quad (27)$$

For the surface wave roots are complex in general (W.L.O.G) we may assume that . $\mathrm{Re}(m_i) > 0$. We assume only m_i that satisfy the following condition:

$$\phi_1(z), \phi_3(z), \varphi(z) \rightarrow \infty \text{ as } z \rightarrow \infty \quad (28)$$

Using the above radiation condition the relation (21) – (24) reduces the following solutions in the half space $z > 0$ as

$$\phi_1(z) = [\sum^3 A_i e^{-m_i z}]e^{ik(x-ct)} \quad (29)$$

$$\phi_3(z) = [\sum^3 F_i e^{-m_i z}]e^{ik(x-ct)} \quad (30)$$

$$\varphi(z) = [\sum^3 F_i^* e^{-m_i z}]e^{ik(x-ct)} \quad (31)$$

where $B_i = F_i A_i$ and $C_i = F_i^* A_i$

$$F_i = -i\frac{m_i}{k}\left[\frac{\overline{\beta}(\xi - r_1 - s_1 + \frac{m_i^2}{k^2}) + (r_2 + s_1 + 1)}{\overline{\beta}\frac{m_i^2}{k^2}(r_2 + s_1 + 1) - \{\xi - 1 + (r_3 + s_1)\frac{m_i^2}{k^2}\}}\right]$$

$$F_i^* = -\frac{i\varepsilon k}{\frac{\beta_1}{c_{44}+P_{11}}[1 - a^*(-k^2 + \frac{m_i^2}{k^2})]}$$
$$\left[\frac{\overline{\beta}(\xi - r_1 - s_1 + \frac{m_i^2}{k^2}) + (r_2 + s_1 + 1)}{\overline{\beta}\varepsilon + (r_2 + s_1 + 1)(1 - \frac{K_1^*}{\xi} + \frac{K_3^*}{\xi}\frac{m_i^2}{k^2})}\right]$$

V. DERIVATION OF THE FREQUENCY EQUATION

The boundary conditions of stress free surface of the body at z=0:

$$\sigma_{zz} + \overline{\sigma}_{zz} = 0 \quad (32)$$

$$\sigma_{zx} + \overline{\sigma}_{zx} = 0 \quad (33)$$

$$\frac{\partial \Theta}{\partial z} + h\Theta = 0 \quad (34)$$

The solution (29)-(31) satisfy the boundary condition (32)-(34) and written in homogenous system of equation in terms of A_1, A_2 and A_3

$$\sum_{i=1}^{3}[(c_{33}+P_{33}-\mu_e H_0^2)m_i F_i - ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_i^2)\}F_i^*]A = 0 \quad (35)$$

$$\sum^3 (m_i - ikF_i)A_i = 0 \quad (36)$$

$$\sum_{i=1}^{3} F_i^* A_i[\{1-a^*(-k^2+m_i^2)\}m_i - h] = 0 \quad (37)$$

For non-trivial solution of Eqns. (35)-(37) the determinant should be vanish

Equation (38) is the frequency equation of the Rayleigh wave in dual Phase lag with initially stressed transversely isotropic , thermo-elastic half- space with two temperature and magnetic field .

$$F_1^*[\{1-a^*(-k^2+m_1^2)\}m_1 - h]]$$
$$\{[(c_{33}+P_{33}-\mu_e H_0^2)m_2 F_2 - ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_2^2)\}F_2^*][m_3 - ikF_3]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_3 F_3 - ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_3^2)\}F_3^*][m_2 - ikF_2]\}$$
$$-F_2^*[\{1-a^*(-k^2+m_2^2)\}m_2 - h\}]$$
$$\{[(c_{33}+P_{33}-\mu_e H_0^2)m_1 F_1 - ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_1^2)\}F_1^*][m_3 - ikF_3]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_3 F_3 - ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_3^2)\}F_3^*][m_1 - ikF_1]\}$$

$$+F_3^*[\{1-a^*(-k^2+m_3^2)\}m_3-h]]$$
$$\{[(c_{33}+P_{33}-\mu_e H_0^2)m_1F_1-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_1^2)\}F_1^*][m_2-ikF_2]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_2F_2-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_2^2)\}F_3^*][m_1-ikF_1]\}=0 \tag{38}$$

Particular cases:

Thermally insulated surface: When $h \to 0$ then the frequency Eqn. (38) reduces to

$$F_1^*[\{1-a^*(-k^2+m_1^2)\}m_1\}]$$
$$\{[(c_{33}+P_{33}-\mu_e H_0^2)m_2F_2-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_2^2)\}F_2^*][m_3-ikF_3]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_3F_3-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_3^2)\}F_3^*][m_2-ikF_2]\}$$
$$-F_2^*[\{1-a^*(-k^2+m_2^2)\}m_2\}]$$
$$\{[(c_{33}+P_{33}-\mu_e H_0^2)m_1F_1-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_1^2)\}F_1^*][m_3-ikF_3]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_3F_3-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_3^2)\}F_3^*][m_1-ikF_1]\}$$
$$+F_3^*[\{1-a^*(-k^2+m_3^2)\}m_3\}]$$
$$\{[(c_{33}+P_{33}-\mu_e H_0^2)m_1F_1-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_1^2)\}F_1^*][m_2-ikF_2]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_2F_2-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_2^2)\}F_3^*][m_1-ikF_1]\}=0 \tag{39}$$

Isothermal surface : When put $h \to \infty$ then the frequency equation (38)

$$[(c_{33}+P_{33}-\mu_e H_0^2)m_2F_2-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_2^2)\}F_2^*][m_3-ikF_3]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_3F_3-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_3^2)\}F_3^*][m_2-ikF_2]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_1F_1-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_1^2)\}F_1^*[m_3-ikF_3]$$
$$+[(c_{33}+P_{33}-\mu_e H_0^2)m_3F_3-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_3^2)\}F_3^*][m_1-ikF_1]$$
$$[(c_{33}+P_{33}-\mu_e H_0^2)m_1F_1-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_1^2)\}F_1^*][m_2-ikF_2]$$
$$-[(c_{33}+P_{33}-\mu_e H_0^2)m_2F_2-ik(c_{13}-\mu_e H_0^2)$$
$$+\beta_3\{1-a^*(-k^2+m_2^2)\}F_3^*][m_1-ikF_1]=0 \tag{40}$$

Special Cases

(i) In absence of two temperature , $a^*=0$, initial stress $P_{11}=P_{33}=0$ and magnetic field $H_0=0$ then the frequency equation (38) can be written as

$$F_1^*[m_1-h]\{[c_{33}m_2F_2-ikc_{13}+\beta_3F_2^*][m_3-ikF_3]$$
$$-[c_{33}m_3F_3-ikc_{13}+\beta_3F_3^*][m_2-ikF_2]\}$$
$$-F_2^*[m_2-h\}]\{[c_{33}m_1F_1-ikc_{13}+\beta_3F_1^*][m_3-ikF_3]$$
$$-[c_{33}m_3F_3-ikc_{13}+\beta_3F_3^*][m_1-ikF_1]\}$$
$$+F_3^*[m_3-h\}]\{[c_{33}m_1F_1-ikc_{13}+\beta_3F_1^*][m_2-ikF_2]$$
$$-[c_{33}m_2F_2-ikc_{13}+\beta_3F_3^*][m_1-ikF_1]\}=0 \tag{41}$$

Where F_i, F_i^* calculated according and Eqn. (41) is same as the frequency equation given by singh *et al.*, (2013)

(ii) If we put $\tau_\theta=0$ and the considered only τ_q then the DPL thermo-elasticity reduces to Lord-Shulman generalized thermo-elasticity.

(iii) If we put $\tau_\theta=\tau_q \to 0$ then the DPL thermo-elasticity reduces to coupled thermo-elasticity.

(iv) If we put $\tau_\theta=\tau_q \to 0$ and $\varepsilon=0$ then the DPL thermo-elasticity reduces to un coupled thermo-elasticity.

(v) If we put

$$c_{11}=\lambda+2\mu, c_{13}=\lambda, c_{44}=\mu,$$
$$\beta_1=\beta_3=\beta, K_1=K_3=K,$$
$$P_{11}=P_{33}=0, a^*, H_0=0$$

(vi) then the equation (35) reduces to the isotropic elasticity. And then if put $K=0, \beta=0, \varepsilon=0$ and after a long calculation the frequency equation (38) reduces to

$$(2-\frac{c^2}{c_2^2})^2=4(2-\frac{c^2}{c_1^2})^{\frac{1}{2}}(1-\frac{c^2}{c_2^2})^{\frac{1}{2}}$$

Which is the frequency equation of Rayeigh wave.

VI. NUMERICAL RESULT AND DISCUSSION

When ε is small at normal temperature and $\varepsilon \ll 1$ using in equation (19)-(21), we obtain the following approximated roots using the relation (19)-(21)

$$\frac{m_1^2}{k^2}=(\frac{c_{11}+P_{11}+\mu_e H_0^2}{c_{44}+P_{11}}-\frac{\rho c^2}{c_{44}+P_{11}}) \tag{42}$$

$$\frac{m_2^2}{k^2}=(\frac{c_{44}+P_{11}-\rho c^2}{c_{33}+P_{33}+\mu_e H_0^2}) \tag{43}$$

$$\frac{m_2^2}{k^2}=(\frac{(c_{44}+P_{11})^2K_1^*-\rho c^2(1+a^*k^2)}{(c_{44}+P_{11})K_3^*-a^*k^2\rho c^2}) \tag{44}$$

We restricted only the case of thermally insulated surface only for the numerical computation of the dimensionless speed of Rayleigh wave. Therefore the equation (38) is approximated and with the help of equation (42)-(44) solved numerically to obtain in the speeds of Rayleigh wave for certain range of dimensionless constants. We numerically calculated the non dimensional propagation of velocity of Rayleigh wave for thermally insulated half space and approximated frequency equation (39) using (42)-(44)

Following the physical constants of crystal of Zinc (Chadwick and Seet [from paper 5] are considered. For the given frequency range $2Hz \le \omega \le 8Hz$ at H=0.2Oe P = 0.5 Pa, $a^* = 0, a^* = 0.02$ and $a^* = 0.002$ by using DPL theory, non-dimensional Rayleigh wave velocity was calculated by $\frac{\rho c^2}{d_{44}}$

$$c_{11} = 1.628 \times 10^{11} Nm^{-2},\ c_{33} = 1.562 \times 10^{11} Nm^{-2}$$

$$c_{13} = 0.508 \times 10^{11} Nm^{-2},\ c_{44} = 0.385 \times 10^{11} Nm^{-2}$$

$$\beta_1 = 5.75 \times 10^6 Nm^{-2} \deg^{-1},\ \ \beta_3 = 5.17 \times 10^6 Nm^{-2} \deg^{-1}$$

$$K_1 = 1.24 \times 10^2 Wm^{-1} \deg^{-1},\ \ K_3 = 1.34 \times 10^2 Wm^{-1} \deg^{-1}$$

$$C_e = 3.9 \times 10^2 JKg^{-1} \deg^{-1},\ \ \rho = 7.14 \times 10^3 Kgm^{-3}$$

$$T_0 = 296K,\ \tau_q = 0.005s,\ \ \tau_\theta = 0.0005$$

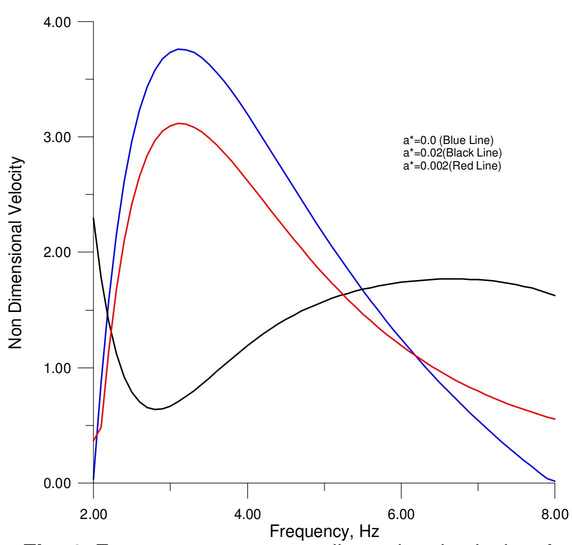

Fig. 1: Frequency versus non-dimensional velocity of Rayleigh wave in DPL theory when at H = 0.2 oe.

In Fig. 1 we observe that frequency of Rayleigh wave depends on the different values of a*. in graph we see that at a*=0 as frequency of Rayleigh wave increases than its non-dimensional velocity is also increases sharply up to certain point and decreases sharply, further with the increase in a* increase in the velocity became slow and further at higher value of a* i.e. at 0.02 velocity starts decreasing with increase in frequency and increase after a certain point.

Fig. 2 indicates the effect of initial stress on non-dimensional velocity at different value of a*. By using DPL theory we determine the non-dimensional velocity of Rayleigh wave in the different range of initial-stress i.e. $0 \le P \le 1.6\,Pa$ at $\omega = 5Hz$ and H=0.2Oe, $a^* = 0, a^* = 0.02$ and $a^* = 0.002$ by using formula $\frac{\rho c^2}{c_{44} + P_{11}}$. From the graph we observe that non-dimensional velocity decreases with increase in initial-stress and then slightly increased and with increase in the value of a* corresponding curve slightly shifts downward.

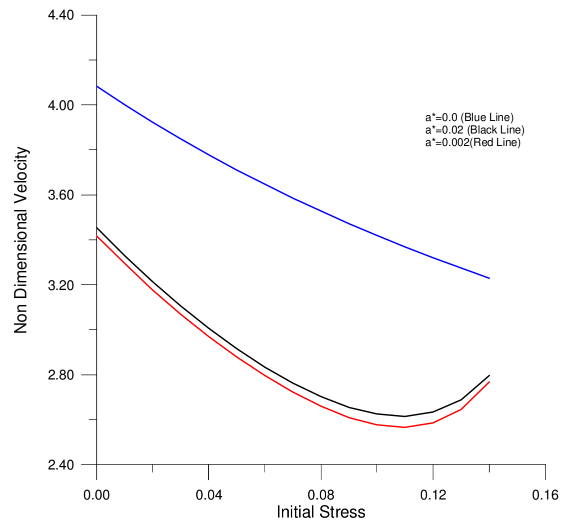

Fig. 2. Dependence of non-dimensional velocity of Rayleigh wave on the initial stress in DPL theory at *w* = 5 Hz and H=0.2 oe.

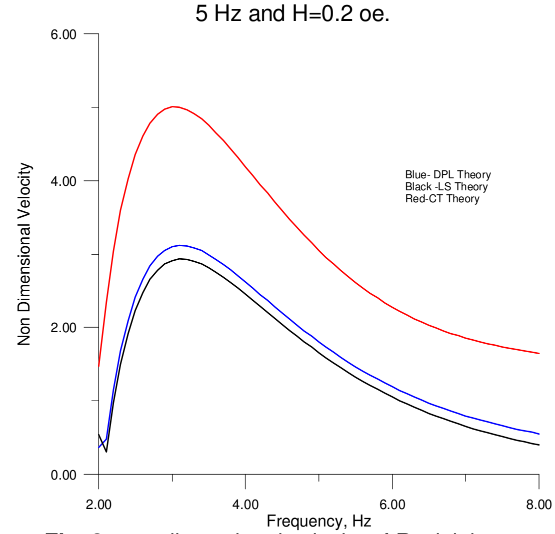

Fig. 3: non-dimensional velocity of Rayleigh wave versus Frequency for DPL, L-S, C.T. theory at *P* = 0.5. Pa and *H* = 0.4 O e and two temperature parameter a*=0.02

For DPL theory, L-S Theory, and C.T. Theory we determine the non-dimensional velocity of Rayleigh wave in the given range of frequency i.e.

$2Hz \le \omega \le 8Hz$ at P = 0.5 Pa, H=0.2Oe and a^* = 0.002. From Fig.3 we observe that with increase frequency of Rayleigh wave its non-dimensional velocity increases and then up to certain point it decreases sharply a^* = 0.002.

VII. Conclusion

The general solution of governing equation of transversely isotropic dual phase lag with two temperature and magnetic field were obtained using the surface wave solution. We reduce general solution to particular solution by using suitable radiation condition in the given half-space. Using this particular solution we find the frequency equation for Rayleigh wave by using the suitable boundary conditions on the surface which is stress free and insulated thermally in half-space. For numerical calculation, approximate the frequency equation for a particular model of material. The dimensionless velocity against the frequency two-temperature, initial stress was plotted. Effect of two temperature, initial stress and dual phase were shown graphically.

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