International Journal on Emerging Technologies **10**(2b): 163-171(2019)

ISSN No. (Print) : 0975-8364
ISSN No. (Online) : 2249-3255

Fixed Point Theorem on Metric Space using Modified Dominating Dualistic Kannan-Mapping

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(Received 02 May 2019, Revised 10 July 2019, Accepted 08 August 2019)
(Published by Research Trend, Website: www.researchtrend.net)

Abstract: In the present paper, a new mapping is introduced namely Modified Dominating Dualistic Kannan mapping. A new metric space namely ordered dualistic partial b-metric space is also introduced by adding additional condition in dualistic partial metric space. On the basis of the same, some fixed point theorems are proved. These results are more generalized than the result given by Nazam and Arsadin their paper in 2018 ("Some fixed point results in ordered dualistic partial metric spaces, Razmadze Mathematical Institute, 2018, https://doi.org/10.1016/j.trmi.2018.01.003")

Keywords: Fixed point, Dualistic partial b-metric space, Kannan Mapping, Modified Dualistic Kannan Mapping.

I. INTRODUCTION

In fixed point theory the most important result is Banach Contraction Principle (BCP), which was given by Stefan Banach in 1922. This principle state that "if (N, d) is a complete metric space and $A: N \to N$ is a contraction mapping that is $d(Ar, As) \leq k\, d(r,s)$ where $k \in (0,1)$ for all $r, s \in N$, then A has a unique fixed point". Since its inceptions in 1922, this contraction principle has seen many extension and generalization in different space [1-18]. In 1993, the concept of b-metric space was proposed by Bakhtin [21] by doing some changes in metric space. Steve G Matthew [19] extended the metric space to Partial Metric Space in 1994. In partial metric space distance of two same points is non zero as compare to metric space. Dualistic partial metric space was developed by Neill [20] as a more generalized form of partial metric Space. This space connects quasi metric and dualistic partial metric Space. One more new concept of partial b-metric space was given by S. Satish [22] in 2014 and he proved BCP on this space. Valero and Oltera [23] proved Banach contraction principleon a complete dualistic partial metric Space.Here, using new modified Dominating dualistic Kannan mapping, fixed point theorems are proved in ordered dualistic partial b-metric Space. This is an extension of work done by Nazam and Arshad [25]. We illustrate one example to support the main theorem.

II. PRELIMINARIES

Throughout this paper, collection of natural number is N, R^+ denotes all positive real numbers and R denotes real numbers. Some basic definitions are given below.

Definition 2.1 [19] Let $N \neq \emptyset$. Define a mapping $P: N * N \to [0,\infty)$ satisfies the following axioms: $\forall\, d^*, e^*, f^* \epsilon\, N$

$(P1) d^* = e^* \Leftrightarrow P(d^*,d^*) = P(d^*,e^*) = P(e^*,e^*)$;

$(P2) P(d^*,d^*) \leq P(d^*,e^*)$;

$(P3) P(d^*,e^*) = P(e^*,d^*)$;

$(P4) P(d^*,e^*) \leq P(d^*,f^*) + P(f^*,e^*) - P(f^*,f^*)$.

The pair (N, P) is known as partial metric space.

Neill [20] defined the dualistic partial metric space by extending the range $[0,\infty)$ to $(-\infty,\infty)$ in partial metric space.

Definition 2.2 [20] Let $N \neq \emptyset$, define a mapping $D: N * N \to R$ satisfies the following axioms: $\forall\, d^*, e^*, f^* \epsilon\, N$

$(D1) d^* = e^* \Leftrightarrow D(d^*,d^*) = D(e^*,e^*) = D(d^*,e^*)$;

$(D2) D(d^*,d^*) \leq D(d^*,e^*)$;

$(D3) D(d^*,e^*) = D(e^*,d^*)$;

$(D4) D(d^*,f^*) + D(e^*,e^*) \leq D(d^*,e^*) + D(e^*,f^*)$.

Then pair (N, D) is known as a dualistic partial metric space.

Definition 2.3 [25] Let $S: N \to N$ is a self-mapping and (N, D_b) be a dualistic partial b- metric space. S has a Convergence Comparison Property (CPP) if ∃ sequence $\{f_n\}$ of natural numbers such that $f_n \to f^*$, S satisfies $D_b(f^*,f^*) \leq D_b(S(f^*), S(f^*))$.

Definition 2.4 [21]. Let $N \neq \emptyset$ and $u \geq 1$ be a given positive number. A function

$d: N * N \to [0,\infty)$ is a *b*-metric on N if, $\forall\, d^*, e^*, f^* \epsilon\, N$, the satisfied following property:

$(b1) d(d^*,e^*) = 0 \; iff \; d^* = e^*$

$(b2) d(d^*,e^*) = d(e^*,d^*)$

$(b3) d(d^*,e^*) \leq u[d(d^*,f^*) + d(e^*,d^*)]$

Then pair (N, d) known as $b-$metric space.

III. MAIN RESULTS

Here we introduce dualistic partial b-metric Space

Definition 3.1 Let $N \neq \emptyset$. The function $D_b: N * N \rightarrow R$satisfies the following axioms: if, $\forall\, d^*, e^*, f^* \epsilon\, N$

$(Db1) d^* = e^* \Leftrightarrow D_b(d^*, d^*) = D_b(e^*, e^*) = D_b(d^*, e^*);$

$(Db2) D_b(d^*, d^*) \leq D_b(d^*, e^*);$

$(Db3) D_b(d^*, e^*) = D_b(e^*, d^*);$

$(Db4) D_b(d^*, f^*) + D_b(e^*, e^*) \leq u[D_b(d^*, e^*) + (e^*, f^*)]$

Pair (N, D_b) is known as dualistic partial b-metric Space with coefficient $u \geq 1$.

Next we are giving one example related to this Definition.

Example 3.1 A function D_b: R × R → R defined by $D_b(d^*, e^*) = \sum_{i=1}^{n} |d_i^* - e_i^*|$.Clearly D_b satisfies (Db1)–(Db4) and hence D_b is a dualistic partial b-metric Space on R.

Proof: Let (Y, D_b) be a dualistic partial b-metric Space with coefficient u ≥ 1. Let $d^*, e^* f^* \in Y$ be an arbitrary point, then

(Db1) Let $d^* = e^*$ then,

$$D_b(d^*, e^*) = \sum_{i=1}^{n} |d_i^* - e_i^*|$$

$$\Leftrightarrow \sum_{i=1}^{n} |d_i^* - d_i^*|$$

$$\Leftrightarrow D_b(d^*, d^*)$$

$$\Leftrightarrow \sum_{i=1}^{n} |e_i^* - e_i^*|$$

$$\Leftrightarrow D_b(e^*, e^*)$$

(Db2) $D_b(d^*, e^*) = \sum_{i=1}^{n} |d_i^* - e_i^*|$

$$\geq \sum_{i=1}^{n} |d_i^* - d_i^*|$$

$$\geq D_b(d^*, d^*)$$

(Db3) $D_b(d^*, e^*) = \sum_{i=1}^{n} |d_i^* - e_i^*|$

$$= \sum_{i=1}^{n} |e_i^* - d_i^*|$$

$$= D_b(e^*, d^*)$$

(Db4) $D_b(d^*, e^*) = \sum_{i=1}^{n} |d_i^* - e_i^*|$

$$= \sum_{i=1}^{n} |d_i^* + f_i^* - f_i^* - e_i^* + f_i^* - f_i^*|$$

$$\leq \sum_{i=1}^{n} |d_i^* - f_i^*| + \sum_{i=1}^{n} |f_i^* - e_i^*| - \sum_{i=1}^{n} |f_i^* - f_i^*|$$

$$\leq u[\sum_{i=1}^{n} |d_i^* - f_i^*| + \sum_{i=1}^{n} |f_i^* - e_i^*| - \sum_{i=1}^{n} |f_i^* - f_i^*|]\ [\text{since } u \geq 1]$$

$$\leq u[D_b(d^*, f^*) + D_b(f^*, e^*) - D_b(f^*, f^*)]$$

$$\leq u[D_b(d^*, f^*) + D_b(f^*, e^*)] - D_b(f^*, f^*)$$

Therefore (Db1), (Db2), (Db3) and (Db4) are satisfied and So (Y, D_b) is a dualistic partial b-metric space.

Lemma 3.1 If (Y, D_b) is a dualistic partial b-metric space, then $d_{D_b}: Y \times Y \rightarrow R^+$ defined by $d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*)$ $\forall\, d^*, e^* \epsilon Y$ is called a Quasi Metric on Y such that $\tau(D_b) = \tau(D_{D_b})$.

Proof: Consider $d^*, e^* \in Y$.Then

$d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*)$ is always non negative because of $D_b(d^*, d^*) \leq D_b(d^*, e^*)$.

Now, we have to check that d_{D_b} is actually a quasimetric on Y. Let d^*, e^*, $f^* \in$ Y. It is obvious that $d^* = e^*$ provides that $d_{D_b}(d^*, e^*) = D_{D_b}(e^*, d^*) = 0$ Moreover, if $d_{D_b}(d^*, e^*) = D_{D_b}(e^*, d^*) = 0$ then

$D_b(d^*, e^*) - D_b(d^*, d^*) = D_b(e^*, d^*) - D_b(e^*, e^*) = 0$
Hence we obtain that $d^* = e^*$, since

$D_b(d^*, e^*) = D_b(d^*, d^*) = D_b(e^*, e^*)$. Furthermore

$$d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*)$$

$$\leq D_b(d^*, f^*) + D_b(f^*, e^*) - D_b(f^*, f^*) - D_b(d^*, d)$$

$$= d_{D_b}(d^*, f^*) + d_{D_b}(f^*, e^*).$$

Finally we show that $\tau(D_b) = \tau(D_{D_b})$. Indeed, let a y ∈ Y and ε > 0 and consider $y \in B_{d_{D_b}}(d^*, \epsilon)$. Then $d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*) < \epsilon$ and,

hence, $D_b(d^*, e^*) < \epsilon + D_b(d^*, d^*)$.

Consequently $y \in B_{d_{D_b}}(r^*, \epsilon)$ and $\tau(D_b) = \tau(D_{D_b})$.

Conversely if $y \in B_{D_b}(r^*, \epsilon)$ we have, $D_b(d^*, e^*) < \epsilon + D_b(d^*, d^*)$

Thus $d_{D_b}(d^*, e^*) = D_b(d^*, e^*) - D_b(d^*, d^*) < \epsilon$, $y \in B_{d_{D_b}}(d^*, \epsilon)$ and $\tau(D_b) = \tau(D_{D_b})$

Implies that $\tau(D_b) = \tau(D_{D_b})$.

Lemma 3.2

(1) If metric space $(W, d_s^{D_b})$ is complete then dualistic partial b-metric $(W, D_b)$is also complete and vice versa.

(2) A point $y \epsilon W$ and a sequence $\{y_n\}_{n \epsilon N}$ in W such that $\{y_n\}$ converge to y, with respect to $\tau(d_s^{D_b})$ iff $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = D_b(y, y) = \lim_{n\to\infty} D_b\,(y, y_n)$

Proof: We claim that a $\{y_n\}$ be a Cauchy sequence in (W, D_b)

Hence this is also Cauchy sequence in (W,$d_s^{D_b}$).

Let $\{y_n\}$ is a Cauchy sequence in (W, D_b)

Then ∃ α ∈ R s.t, given ε > 0, there is n_ε∈ N with $|D_b(y_n, y_m) - \alpha| < \frac{\epsilon}{2} \forall\, n, m \geq n_\epsilon$.

Hence, $d_{D_b}(y_n, y_m) = D_b(y_n, y_m) - D_b(y_n, y_n)$

$$= |D_b(y_n, y_m) - \alpha + \alpha - D_b(y_n, y_n)|$$

$$\leq |D_b(y_n, y_m) - \alpha| + |\alpha - D_b(y_n, y_n)|$$

$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

For all $n, m \geq n_\epsilon$. Similarly we show $d_{D_b}(y_n, y_m) < \epsilon$ for all $n, m \geq n_\epsilon$.

We conclude that $\{y_n\}$ is a Cauchy sequence in (W,$d_s^{D_b}$).

Implies we show that when (W,$d_s^{D_b}$) is complete than (W, D_b) is complete

If $\{y_n\}$ is a Cauchy sequence in $(\mathrm{W}, \mathrm{D_b})$,

Then also Cauchy sequence in (W,$d_s^{D_b}$)

Suppose that y ∈ N and the metric space (W,$d_s^{D_b}$) is complete such that $\lim_{n\to\infty} d_s^{D_b}\,(y, y_n) = 0$.

By lemma (3.1) we follow that $\{y_n\}$ is a convergent sequence in $(\mathrm{W}, \mathrm{D_b})$.

Further we show that $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = D_b(y, y)$.

Since $\{y_n\}$ is Cauchy sequence in $(\mathrm{W}, \mathrm{D_b})$ than

$$\lim_{n\to\infty} D_b\,(y_n, y_n) = D_b(y, y)$$

Consider ε > 0 then ∃ n_0 ∈ N such that $d_s^{D_b}(y, y_n) < \frac{\varepsilon}{2}$ whenever n ≥ n_0

Thus

$$|D_b(y,y) - D_b(y_n,y_n)| \leq |D_b(y,y) - D_b(y,y_n)| +$$

$$|D_b(y,y_n) - D_b(y_n,y_n)|$$

$$= d_s^{D_b}(y,y_n) + d_s^{D_b}(y_n,y)$$

$$< d_s^{D_b}(y,y_n) < \epsilon$$

Whenever n ≥ n_0

⇒(W, D_b) is complete.

Next we show that every Cauchy sequence $\{y_n\}$ in $(N, d_s^{D_b})$ be a Cauchy sequence in $(\mathrm{W}, \mathrm{D_b})$. Let $\epsilon = \frac{1}{2}$. Then ∃ n_0∈ N such that $D_b(y_n, y_m) < \frac{1}{2} \forall\, n, m \geq n_0$

Since

$$d_{D_b}(y_n, y_{n_0}) + D_b(y_n, y_n) = d_{D_b}(y_{n_0}, y_n) + d_{D_b}(y_{n_0}, y_{n_0}),$$

Then

$$|D_b(y_n, y_n)| = d_{D_b}(y_{n_0}, y_n) + d_{D_b}(y_{n_0}, y_{n_0}) - d_{D_b}(y_n, y_{n_0})$$

Consequently the sequence $(D_b(y_n, y_n))_n$ is restricted in R, and consequently so there is y in R s.t. a subsequence $(D_b(y_{n_k}, y_{n_k})_k$ is convergent to y, i.e.

$$\lim_{k\to\infty} D_b\,(y_{n_k}, y_{n_k}) = y.$$

It remains to show that $(D_b(y_n, y_n))_n$ be a Cauchy sequence in R.

Since $\{y_n\}$ be a Cauchy sequence in (W,$d_s^{D_b}$), given ε > 0, ∃ n_ε∈ N s.t. $d_s^{D_b}(y_n, y_m) < \frac{\epsilon}{2} \forall$ n, m ≥ n_ε. Thus, for all n, m ≥ n_ε,

$$D_b(y_n, y_n) = d_{D_b}(y_m, y_n) + D_b(y_m, y_m) - d_{D_b}(y_n, y_m).$$

Therefore $\lim_{n\to\infty} D_b\,(y_n, y_n) = y$

whereas, $|D_b(y_n, y_m) - y| = |D_b(y_n, y_m) - D_b(y_n, y_n) + D_b(y_n, y_n) - y|$

$\leq D_b(y_n, y_m) + |D_b(y_n, y_n) - y| < \epsilon\ \forall$ n, m ≥ n_ε.

Hence $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = y$ and $\{y_n\}$ be a Cauchy sequence in $(\mathrm{W}, \mathrm{D_b})$.

Then $\{y_n\}$ be a Cauchy sequence in $(\mathrm{W}, \mathrm{D_b})$, and so it is convergent to a point y ∈ W with

$$\lim_{n\to\infty} D_b\,(y, y_n) = D_b(y, y) = \lim_{n,m\to\infty} D_b\,(y_n, y_m).$$

Given ε > 0, then ∃ n_ε∈ N

such that $D_b(y, y_n) - D_b(y, y) < \epsilon$

and $D_b(y, y) - D_b(y_n, y_n) < \epsilon$

whenever n ≥ n_ε as a consequence we have

$$d_{D_b}(y, y_n) = D_b(y, y_n) - D_b(y, y) < \epsilon,$$

and $d_{D_b}(y_n, y) = D_b(y, y_n) - D_b(y_n, y_n)$

$$\leq |D_b(y, y_n) - D_b(y, y) + D_b(y, y_n) - D_b(y_n, y_n)| < 2\epsilon$$

whenever n ≥ n_ε. Therefore (W,$d_s^{D_b}$) is complete.

$\lim_{n\to\infty} d_s^{D_b}\,(y, y_n) = 0$ iff $\lim_{n,m\to\infty} D_b\,(y_n, y_m) = D_b(y, y) = \lim_{n\to\infty} D_b\,(y, y_n)$.

Let a mapping $W: Y \to Y$ defined on Y and $(Y, \leq)$ be an ordered set to satisfy the property $y^* \leq W(y^*)\ \forall\ y^* \epsilon Y$

Then W is known as dominating mapping.

Definition 3.2 $W: Y \to Y$ is a self mapping and (Y, D_b) be a dualistic partial b-metric space. Then W has a Convergence Comparison Property due to Metric (CCPM) if for every sequence $\{d_n^*\}$ in Y such that

$d_n^* \to d^*$, W satisfies $D_b(d^*, e^*) \leq D_b(W(e^*), W(d^*)) - d(d^*, e^*)$

Definition 3.3 Let a mapping W:Y→Y is defined on Y and $(Y, \leq, D_b)$ is an ordered dualistic partial b-metric space, the mapping said to be MDDK-mapping if $\exists\, k\epsilon\left(0, \frac{1}{u}\right)$ $s.t.\, u \geq 1$

$$|D_b(W(d^*), W(e^*)| \leq \frac{k}{4}\left[\, |D_b(d^*, W(d^*)| + |D_b(e^*, W(e^*)| + \frac{|D_b(d^*, W(e^*)| + |D_b(W(d^*), e^*)|}{u}\right] \quad (1)$$

$\forall$ Comparable $d^*, e^* \epsilon Y$ and $d^* \leq W(d^*)$.

To avoid S be continuous, the subsequent property is required.

(S): for sequence $\{y_n\}$ in Y such that $\{y_n\} \leq y^* \epsilon Y \to y^* \in$Y, we have $\{y_n\} \leq y^* \forall\, n$.

Theorem 3.1 : Assume that $W: Y \to Y$ be a self-mapping and $(Y, \leq)$ is a Partially Ordered Set (POS). Let (Y, D_b) be a complete dualistic partial b-metric space s.t.

(a) W be a Modified Dominating Dualistic Kannan-mapping;

(b) W has condition (S);

(c) W possesses Convergence Comparison Property due to Metric

Then W be a fixed point.

Proof: Let we start with $y_0^* \epsilon\, Y$ and sequence $\{y_n^*\}$ is obtained using the iterative process

if $y_0^* \neq W(y_0^*)$

$\Rightarrow$ y_0^* is not a fixed point of W.

Let $y_1^* = W(y_0^*)$

$y_2^* = W(y_1^*)$

Continuing in similar way,

$$y_n^* = W(y_{n-1}^*) \;\; \forall\, n\epsilon\, N.$$

If ∃ a +ve integer m such that $y_m^* = y_{m+1}^*$, we have $y_m^* = y_{m+1}^* = W(y_m^*)$, so y_m^* be a fixed point of W. Hence proof completed.

If $y_n^* \neq y_{n+1}^*$ $\forall\, n\epsilon N$, and W has a Dominating mapping,

$y_0^* \leq W(y_0^*) = (y_1^*)$, we have $y_0^* \leq (y_1^*)$ and $y_1^* \leq T(y_1^*)$ $\Rightarrow y_1^* \leq y_2^*$ further $y_2^* \leq W(y_2^*)$ implies $y_2^* \leq y_3^*$, progressing in the similar way we get;

$$y_0^* \leq y_1^* \leq y_2^* \leq y_3^* \leq \cdots \leq y_n^* \leq y_{n+1}^* \leq y_{n+2}^* \leq \cdots$$

Since $y_n^* \leq y_{n+1}^*$ for each $n\epsilon\, N$, therefore, by (1), we have

$$|D_b(y_1^*, y_2^*)\,| = \left|D_b\left(W(y_0^*), W(y_1^*)\right)\right|$$

$$\leq \frac{k}{4}\left[\left|D_b\left(y_0^*, W(y_0^*)\right)\right| + \left|D_b\left(y_1^*, W(y_1^*)\right)\right| + \left|\frac{D_b\left(y_0^*, W(y_1^*)\right) + D_b(W(y_0^*), y_1^*)}{s}\right|\right]$$

$$\leq \frac{k}{4}\left[|D_b(y_0^*, y_1^*)| + |D_b(y_1^*, y_2^*)| + \left|\frac{D_b(y_1^*, y_1^*)}{s}\right| + s\left[\frac{|D_b(y_0^*, y_1^*)| + |D_b(y_1^*, y_2^*)|}{s}\right] - \left|\frac{D_b(y_1^*, y_1^*)}{s}\right|\right]$$

[Using Db4]

$$\leq \frac{k}{4}\left[|D_b(y_0^*, y_1^*)| + |D_b(y_1^*, y_2^*)| + |D_b(y_0^*, y_1^*)| + |D_b(y_1^*, y_2^*)|\right]$$

$$\leq \frac{k}{4}\left[2\left[|D_b(y_0^*, y_1^*)| + |D_b(y_1^*, y_2^*)|\right]\right]$$

$$\leq \frac{k}{2}\left[|D_b(y_0^*, y_1^*)| + |D_b(y_1^*, y_2^*)|\right]$$

$$\left(1 - \frac{k}{2}\right) |D_b(y_1^*, y_2^*)| \leq \frac{k}{2}|D_b(y_0^*, y_1^*)|$$

$$|D_b(y_1^*, y_2^*)| \leq \left(\frac{k}{2-k}\right)|D_b(y_0^*, y_1^*)|$$

$$|D_b(y_1^*, y_2^*)| \leq \lambda\, |D_b(y_0^*, y_1^*)|$$

where $\lambda = \left(\frac{k}{2-k}\right)$ and $0 < \lambda < 1$

Similarly $|D_b(y_2^*, y_3^*)| \leq \lambda\, |\, D_b\left(y_1^*, W(y_1^*)\right)| \leq \lambda^2\, |\, D_b(y_0^*, y_1^*)\, |$

Continue

$$|\, D_b(y_n^*, y_{n+1}^*)\, | \leq \lambda^n\, |\, D_b(y_0^*, y_1^*)\, | \quad (2)$$

Since $|\, D_b(y_n^*, y_n^*)\, | = |\, D_b\left(W(y_{n-1}^*), W(y_{n-1}^*)\right)|$

$$\leq \frac{k}{4}\left[\left|D_b\left(y_{n-1}^*, W\left(y_{n-1}^*\right)\right)\right| + \left|D_b\left(y_{n-1}^*, W\left(y_{n-1}^*\right)\right)\right| + \left|\frac{D_b(W(y_{n-1}^*), y_{n-1}^*) + D_b\left(y_{n-1}^*, W(y_{n-1}^*)\right)}{s}\right|\right]$$

$$\leq \frac{k}{4}\left[\left|D_b\left(y_{n-1}^*, y_n^*\right)\right| + \left|D_b\left(y_{n-1}^*, y_n^*\right)\right| + \left|\frac{D_b(y_n^*, y_{n-1}^*) + D_b(y_{n-1}^*, y_n^*)}{s}\right|\right]$$

$$\leq \frac{k}{4}\left[2 + \frac{2}{s}\right]|D_b(y_{n-1}^*, y_n^*)|$$

$$\leq \frac{k}{4}\left[\frac{2s+2}{s}\right]|D_b(y_{n-1}^*, y_n^*)|$$

$$\leq \beta\, \lambda^{n-1}|D_b(y_0^*, y_1^*)| \quad (3)$$

where $\beta = \frac{k(2s+2)}{4s}$ and $0 < \beta < 1$

We prove that $\{y_n^*\}$ is a Cauchy sequence in (Y, $d_{D_b}^s$).

Now $d_{D_b}(y_n^*, y_{n+1}^*) = D_b(y_n^*, y_{n+1}^*) - D_b(y_n^*, y_n^*)$

$$\leq |D_b(y_n^*, y_{n+1}^*)| + |D_b(y_n^*, y_n^*)|$$

$$\leq \lambda^n |D_b(y_0^*, y_1^*)| + \beta\, \lambda^{n-1} |D_b(y_0^*, y_1^*)|$$

$$\leq (\lambda^n + \beta\, \lambda^{n-1}) |D_b(y_0^*, y_1^*)|$$

$$\leq \lambda^n \left[1 + \frac{\beta}{\lambda}\right] |D_b(y_0^*, y_1^*)| \quad (4)$$

Continuing this way,

Thus $d_{D_b}(y_{n+k-1}^*, y_{n+k}^*) = D_b(y_{n+k-1}^*, y_{n+k}^*) - D_b(y_{n+k-1}^*, y_{n+k-1}^*)$

$$\leq |D_b(y_{n+k-1}^*, y_{n+k}^*)| - |D_b(y_{n+k-1}^*, y_{n+k-1}^*)|$$

$$\leq |D_b(y_{n+k-1}^*, y_{n+k}^*)| + |D_b(y_{n+k-1}^*, y_{n+k-1}^*)|$$

$$\leq \lambda^{n+k-1} \left[1 + \frac{\beta}{\lambda}\right] |D_b(y_0^*, y_1^*)|$$

Now using triangular inequality for d_{D_b}, Let n and m are two positive integer such that $m > n$.

Than

$$d_{D_b}(y_n^*, y_{n+k}^*) \leq d_{D_b}(y_n^*, y_{n+1}^*) + d_{D_b}(y_{n+1}^*, y_{n+2}^*) + \dots\dots\dots\dots + d_{D_b}(y_{n+k-1}^*, y_{n+k}^*)$$

$$\leq (\lambda^n + \lambda^{n+1} + \dots\dots\dots\dots + \lambda^{n+k-1}) \left(1 + \frac{\beta}{\lambda}\right) |D_b(y_0^*, y_1^*)|$$

$$\leq \frac{\lambda^n}{1-\lambda} \left(1 + \frac{\beta}{\lambda}\right) |D_b(y_0^*, y_1^*)|$$

$$\Rightarrow \quad d_{D_b}(y_n^*, y_m^*) \leq \frac{\lambda^n}{1-\lambda} \left(1 + \frac{\beta}{\lambda}\right) |D_b(y_0^*, y_1^*)|$$

Where n + k = m

Take limit $m, n \to \infty$, thus

$$\lim_{m,n\to\infty} d_{D_b}(y_n^*, y_m^*) = 0$$

(Since $\lim_{n\to\infty} \lambda^n = 0;\ 0 < \lambda < 1$)

$$\Rightarrow \quad \lim_{m,n\to\infty} d_{D_b}(y_n^*, y_m^*) = 0$$

Hence

$$\lim_{m,n\to\infty} d_{D_b}^s(y_n^*, y_m^*) = 0$$

Thus sequence $\{y_n^*\}$ in$(Y, d_{D_b}^s)$ be a Cauchy sequence and (Y, D_b) is a complete dualistic partial b-metric space Space,

From lemma (3.1), $(Y, d_{D_b}^s)$ is as well complete.

There for $\exists\, y^* \epsilon\, Y$ such that

$$\lim_{n\to\infty} d_{D_b}^s(y_n^*, y^*) = 0$$

Again from lemma (1), we have

$$\lim_{n\to\infty} D_b(y^*, y_n^*) = D_b(y^*, y^*) = \lim_{m,n\to\infty} D_b(y_n^*, y_m^*) \quad (5)$$

Since $\lim_{m,n\to\infty} d_{D_b}(y_n^*, y_m^*) = 0$

There for $\lim_{m,n\to\infty} D_b(y_n^*, y_m^*) = 0$

By (5), we have

$$D_b(y^*, y^*) = \lim_{n\to\infty} D_b(y_n^*, y^*) = 0$$

Following (S), (1) and (Db4), we have

$$D_b(y^*, W(y^*)) \leq s\left[D_b(y^*, y_n^*) + D_b(y_n^*, W(y^*))\right] - D_b(y_n^*, y_n^*)$$

$$\leq s\left[D_b(y^*, y_n^*) + \left|D_b(y_n^*, W(y^*))\right|\right] + |D_b(y_n^*, y_n^*)|$$

$$\leq s\left[D_b(y^*, y_n^*) + \frac{k}{4}\left[\left(|D_b(y_{n-1}^*, y_n^*)| + |D_b(y^*, W(y^*))| + \frac{|D_b(y_n^*, y^*) + D_b(y_{n-1}^*, W(y^*))|}{s}\right)\right]\right] + |D_b(y_n^*, y_n^*)|$$

$$\left(1 - \frac{ks}{4}\right) D_b(y^*, W(y^*)) \leq s\left[D_b(y^*, y_n^*) + \frac{k}{4}\left[|D_b(y_{n-1}^*, y_n^*)| + \frac{|D_b(y_n^*, y^*) + D_b(y_{n-1}^*, W(y^*))|}{s}\right]\right] + |D_b(y_n^*, y_n^*)|$$

Letting $n \to \infty$

$$\left(1 - \frac{ks}{4}\right) D_b(y^*, W(y^*)) \leq 0$$

[Using $\{y_n^*\} \to h^*$]

$$D_b(y^*, W(y^*)) \leq 0$$

But also $0 = D_b(y^*, y^*) \leq D_b(y^*, W(y^*))$

Thus $D_b(y^*, W(y^*)) = 0$

Since W has CCPM. So,

$$D_b(y^*, y) \leq D_b(W(y^*), W(y)) - d(y^*, y)$$

Taking $y = y^*$

$$D_b(y^*, y^*) \leq D_b(W(y^*), W(y^*)) - d(y^*, y^*)$$

$$\Rightarrow \quad 0 = D_b(y^*, y^*) \leq D_b(W(y^*), W(y^*)) \quad (6)$$

By axiom (Db2), we have

$$D_b(W(y^*), W(y^*)) \leq D_b(y^*, W(y^*)) = 0 \quad (7)$$

This inequality (6) and (7)

$$D_b(W(y^*), W(y^*)) = 0$$

Thus

$$D_b(y^*, W(y^*)) = D_b(W(y^*), W(y^*)) = D_b(y^*, y^*)$$

By using (Db1), we have

$$y^* = W(y^*)$$

y^* is a fixed point of W.

Example 3.2 We assume $Y = (-\infty, 0)$

$(Y, \leq, D_b)$ *is* a complete ordered dualistic partial b − metric space .

Define $W: Y \to Y$

$$W(u) = \begin{cases} -\frac{1}{2} & if\ u \in (-1,0) \\ -1 & if\ u \in (-\infty,-1) \end{cases}$$

and define $D_b(u,v) = \max\{u,v\}$

Proof: (1) Dominating property:

$$|D_b(W(u),W(v))| \leq \frac{k}{4}\left[|D_b(u,W(u))|+|D_b(v,W(v))|+ \frac{|D_b(u,W(v)|+|D_b(W(v),u)|}{s}\right]$$

Let $u = -\frac{1}{2}$ and $v = -1$

$$\left|D_b\left(W\left(-\frac{1}{2}\right),W(-1)\right)\right| \leq \frac{k}{4}\left[\left|D_b\left(-\frac{1}{2},W\left(-\frac{1}{2}\right)\right)\right|+|D_b(-1,W(-1))| + \frac{|D_b(-\frac{1}{2},W(-1)|+|D_b(W(-1),-\frac{1}{2})|}{s}\right]$$

$$\left|D_b\left(\left(-\frac{1}{2}\right),(-1)\right)\right| \leq \frac{k}{4}\left[\left|D_b\left(-\frac{1}{2},\left(-\frac{1}{2}\right)\right)\right|+|D_b(-1,(-1))| + \frac{|D_b(-\frac{1}{2},-1)|+|D_b(-1,-\frac{1}{2})|}{s}\right]$$

$$-\frac{1}{2} \leq \frac{k}{4}\left[-\frac{1}{2} - 1 - \frac{1}{s}\right]$$

$$-\frac{1}{2} \leq -\frac{k}{4}\left[\frac{3}{4}+\frac{1}{s}\right] \qquad k \in \left(0,\frac{1}{s}\right)$$

Satisfy dominating property and $s \geq 1$

(2) CPMM property:

$$D_b(u,v) \leq D_b(W(u),W(v)) - d(u,v)$$

$$D_b\left(-\frac{1}{2},-1\right) \leq D_b\left(W\left(-\frac{1}{2}\right),W(-1)\right) - d\left(-\frac{1}{2},-1\right)$$

$$-\frac{1}{2} \leq -\frac{1}{2} - \left|-\frac{1}{2}+1\right|$$

$$-\frac{1}{2} \leq -\frac{1}{2} - \frac{1}{2}$$

$$-\frac{1}{2} \leq -\frac{1}{4}$$

Satisfying the CPMM property

Theorem 3.2 Assume that $W: Y \to Y$ be a self − mapping and $(Y, \leq)$ is a partial ordered set. Let (Y, D_b) be a complete dualistic partial b-metric space s.t.

(a) W be a Modified Dominating Dualistic Kannan-mapping;

(b) W has condition (S);

(c) W possesses Convergence Comparison Property

Then W has fixed point.

Proof: Let we start with $y_0^* \epsilon\, Y$ and sequence $\{y_n^*\}$ is obtained using the iterative process

if $y_0^* \neq W(y_0^*)$

$\Rightarrow$ y_0^* is not a fixed point of W.

Let $y_1^* = W(y_0^*)$

$y_2^* = W(y_1^*)$

Continuing in similar way,

$$y_n^* = W(y_{n-1}^*)\ \forall\, n \epsilon\, N.$$

If ∃ a +ve integer m such that $y_m^* = y_{m+1}^*$, we have $y_m^* = y_{m+1}^* = W(y_m^*)$, so y_m^* be a fixed point of W,

hence proof completed.

If $y_n^* \neq y_{n+1}^*$ ∀ $n \epsilon N$, and W has a Dominating mapping,

$y_0^* \leq W(y_0^*) = (y_1^*)$, we have $y_0^* \leq (y_1^*)$ and $y_1^* \leq T(y_1^*)$

$\Rightarrow y_1^* \leq y_2^*$ further $y_2^* \leq W(y_2^*)$ implies $y_2^* \leq y_3^*$, progressing in the similar way we get;

$$y_0^* \leq y_1^* \leq y_2^* \leq y_3^* \leq \cdots \leq y_n^* \leq y_{n+1}^* \leq y_{n+2}^* \leq \cdots$$

Since $y_n^* \leq y_{n+1}^*$ for each $n \epsilon\, N$, therefore, by (3.1), we have

$$|D_b(y_1^*, y_2^*)| = |D_b(W(y_0^*), W(y_1^*))|$$

$$\leq \frac{k}{4}\left[\left|D_b\left(y_0^*,W(y_0^*)\right)\right| + \left|D_b\left(y_1^*,W(y_1^*)\right)\right| + \left|\frac{D_b\left(y_0^*,W(y_1^*)\right)+D_b\left(W(y_0^*),y_1^*\right)}{s}\right|\right]$$

$$\leq \frac{k}{4}\left[|D_b(y_0^*,y_1^*)|+|D_b(y_1^*,y_2^*)|+\left|\frac{D_b(y_1^*,y_1^*)}{s}\right| + s\left[\frac{|D_b(y_0^*,y_1^*)|+|D_b(y_1^*,y_2^*)|}{s}\right] - \left|\frac{D_b(y_1^*,y_1^*)}{s}\right|\right]$$

[Using Db4]

$$\leq \frac{k}{4}\left[|D_b(y_0^*,y_1^*)|+|D_b(y_1^*,y_2^*)|+|D_b(y_0^*,y_1^*)|+|D_b(y_1^*,y_2^*)|\right]$$

$$\leq \frac{k}{4}\left[2[|D_b(y_0^*,y_1^*)|+|D_b(y_1^*,y_2^*)|]\right]$$

$$\leq \frac{k}{2}\left[|D_b(y_0^*,y_1^*)|+|D_b(y_1^*,y_2^*)|\right]$$

$$\left(1-\frac{k}{2}\right)|D_b(y_1^*,y_2^*)| \leq \frac{k}{2}|D_b(y_0^*,y_1^*)|$$

$$|D_b(y_1^*,y_2^*)| \leq \left(\frac{k}{2-k}\right)|D_b(y_0^*,y_1^*)|$$

$$|D_b(y_1^*,y_2^*)| \leq \lambda\, |D_b(y_0^*,y_1^*)|$$

where $\lambda = \left(\frac{k}{2-k}\right)$ and $0 < \lambda < 1$

Similarly

$$|D_b(y_2^*,y_3^*)| \leq \lambda |D_b\left(y_1^*,W(y_1^*)\right)| \leq \lambda^2 |D_b(y_0^*,y_1^*)|$$

Continue

$|D_b(y^*_n, y^*_{n+1})| \leq \lambda^n |D_b(y^*_0, y^*_1)|$ (8)

Since $|D_b(y^*_n, y^*_n)| = |D_b(W(y^*_{n-1}), W(y^*_{n-1}))|$

$$\leq \frac{k}{4}\left[\left|D_b\left(y^*_{n-1}, W(y^*_{n-1})\right)\right| + \left|D_b\left(y^*_{n-1}, W(y^*_{n-1})\right)\right| + \left|\frac{D_b(W(y^*_{n-1}), y^*_{n-1}) + D_b\left(y^*_{n-1}, W(y^*_{n-1})\right)}{s}\right|\right]$$

$$\leq \frac{k}{4}\left[\left|D_b\left(y^*_{n-1}, y^*_n\right)\right| + \left|D_b\left(y^*_{n-1}, y^*_n\right)\right| + \left|\frac{D_b(y^*_n, y^*_{n-1}) + D_b(y^*_{n-1}, y^*_n)}{s}\right|\right]$$

$$\leq \frac{k}{4}\left[2 + \frac{2}{s}\right] |D_b(y^*_{n-1}, y^*_n)|$$

$$\leq \frac{k}{4}\left[\frac{2s+2}{s}\right] |D_b(y^*_{n-1}, y^*_n)|$$

$$\leq \beta\, \lambda^{n-1} |D_b(y^*_0, y^*_1)| \quad (9)$$

where $\beta = \frac{k(2s+2)}{4s}$ and $0 < \beta < 1$

We prove that $\{y^*_n\}$ is a Cauchy sequence in $(Y, d^s_{D_b})$.

Now $d_{D_b}(y^*_n, y^*_{n+1}) = D_b(y^*_n, y^*_{n+1}) - D_b(y^*_n, y^*_n)$

$$\leq |D_b(y^*_n, y^*_{n+1})| + |D_b(y^*_n, y^*_n)|$$

$$\leq \lambda^n |D_b(y^*_0, y^*_1)| + \beta\, \lambda^{n-1} |D_b(y^*_0, y^*_1)|$$

$$\leq (\lambda^n + \beta\, \lambda^{n-1}) |D_b(y^*_0, y^*_1)|$$

$$\leq \lambda^n \left[1 + \frac{\beta}{\lambda}\right] |D_b(y^*_0, y^*_1)| \quad (10)$$

continuing this way,

Thus

$$d_{D_b}\left(y^*_{n+k-1}, y^*_{n+k}\right) = D_b\left(y^*_{n+k-1}, y^*_{n+k}\right) - D_b\left(y^*_{n+k-1}, y^*_{n+k-1}\right)$$

$$\leq \left|D_b\left(y^*_{n+k-1}, y^*_{n+k}\right)\right| - \left|D_b\left(y^*_{n+k-1}, y^*_{n+k-1}\right)\right|$$

$$\leq \left|D_b\left(y^*_{n+k-1}, y^*_{n+k}\right)\right| + \left|D_b\left(y^*_{n+k-1}, y^*_{n+k-1}\right)\right|$$

$$\leq \lambda^{n+k-1}\left[1 + \frac{\beta}{\lambda}\right] |D_b(y^*_0, y^*_1)|$$

Now using triangular inequality for d_{D_b}, Let n and m are two positive integer such that $m > n$.

Than

$$d_{D_b}(y^*_n, y^*_{n+k}) \leq d_{D_b}(y^*_n, y^*_{n+1}) + d_{D_b}(y^*_{n+1}, y^*_{n+2}) + \ldots\ldots\ldots\ldots + d_{D_b}(y^*_{n+k-1}, y^*_{n+k})$$

$$\leq (\lambda^n + \lambda^{n+1} + \ldots\ldots\ldots\ldots + \lambda^{n+k-1}) \left(1 + \frac{\beta}{\lambda}\right) |D_b(y^*_0, y^*_1)|$$

$$\leq \frac{\lambda^n}{1-\lambda}\left(1 + \frac{\beta}{\lambda}\right) |D_b(y^*_0, y^*_1)|$$

$$\Rightarrow \quad d_{D_b}(y^*_n, y^*_m) \leq \frac{\lambda^n}{1-\lambda}\left(1 + \frac{\beta}{\lambda}\right) |D_b(y^*_0, y^*_1)|$$

where n + k = m

Take limit $m, n \to \infty$, thus

$$\lim_{m,n\to\infty} d_{D_b}(y^*_n, y^*_m) = 0$$

(Since $\lim_{n\to\infty} \lambda^n = 0;\ 0 < \lambda < 1$)

$$\Rightarrow \quad \lim_{m,n\to\infty} d_{D_b}(y^*_n, y^*_m) = 0$$

and hence

$$\lim_{m,n\to\infty} d^s_{D_b}(y^*_n, y^*_m) = 0$$

Thus sequence $\{y^*_n\}$ in $(Y, d^s_{D_b})$ is a Cauchy sequence and (Y, D_b) is a complete DPbM Space,

From lemma (3.1), $(Y, d^s_{D_b})$ is as well complete.

There for $\exists\, y^* \epsilon\, Y$ such that

$$\lim_{n\to\infty} d^s_{D_b}(y^*_n, y^*) = 0$$

Again from lemma (1), we have

$$\lim_{n\to\infty} D_b(y^*, y^*_n) = D_b(y^* y^*) = \lim_{m,n\to\infty} D_b(y^*_n, y^*_m) \quad (11)$$

Since $\lim_{m,n\to\infty} d_{D_b}(y^*_n, y^*_m) = 0$

Therefore $\lim_{m,n\to\infty} D_b(y^*_n, y^*_m) = 0$

By (11), we have

$$D_b(y^* y^*) = \lim_{n\to\infty} D_b(y^*_n, y^*) = 0$$

Following (S), (3.1) and (Db4), we have

$$D_b(y^*, W(y^*)) \leq s\left[D_b(y^*, y^*_n) + D_b\left(y^*_n, W(y^*)\right)\right] - D_b(y^*_n, y^*_n)$$

$$\leq s\left[D_b(y^*, y^*_n) + \left|D_b\left(y^*_n, W(y^*)\right)\right|\right] + |D_b(y^*_n, y^*_n)|$$

$$\leq s\left[D_b(y^*, y^*_n) + \frac{k}{4}\left[\left(|D_b\left(y^*_{n-1}, W(y^*_{n-1})\right)| + |D_b\left(y^*, W(h^*)\right)| + \frac{|D_b(W(y^*_{n-1}), y^*) + D_b(y^*_{n-1}, W(y^*)|}{s}\right)\right]\right] + |D_b(y^*_n, y^*_n)|$$

$$\leq s\left[D_b(y^*, y^*_n) + \frac{k}{4}\left[\left(|D_b(y^*_{n-1}, y^*_n)| + |D_b\left(y^*, W(y^*)\right)| + \frac{|D_b(y^*_n, y^*) + D_b(y^*_{n-1}, W(y^*)|}{s}\right)\right]\right] + |D_b(y^*_n, y^*_n)|$$

$$\left(1-\frac{ks}{4}\right) D_b\left(y^*,W(y^*)\right) \leq s\left[D_b(y^*,y_n^*)+\frac{k}{4}\left[\frac{|D_b\left(y_{n-1}^*,y_n^*\right)|+ |D_b(y_n^*,y^*)+D_b(y_{n-1}^*,W(y^*))|}{s}\right]\right] + |D_b(y_n^*,y_n^*)|$$

Letting $n \to \infty$

$$\left(1-\frac{ks}{4}\right) D_b\left(y^*,W(y^*)\right) \leq 0$$

$$[\text{ Using } \{y_n^*\} \to h^*]$$

$$D_b\left(y^*,W(y^*)\right) \leq 0$$

But also $\quad 0 = D_b(y^*,y^*) \leq D_b\,(y^*,W(y^*))$

Thus $\quad D_b\left(y^*,W(y^*)\right) = 0$

Since W has CCP. So,

$$D_b(y^*,y) \leq D_b(W(y^*),W(y))$$

$$D_b(y^*,y^*) \leq D_b(W(y^*),W(y^*))$$

$$\Rightarrow 0 = D_b(y^*,y^*) \leq D_b(W(y^*),W(y^*)) \quad (12)$$

By axiom (Db2), we have

$$D_b(W(y^*),W(y^*)) \leq D_b(y^*,W(y^*)) = 0 \quad (13)$$

This inequality (12) and (13)

$$D_b(W(y^*),W(y^*)) = 0$$

Thus

$$D_b\left(y^*,W(y^*)\right) = D_b\,(W(y^*),W(y^*)) = D_b(y^*,y^*)$$

By using (Db1), we have

$$y^* = W(y^*)$$

y^* is a fixed point of W.

III. CONCLUSION

In the above proved fixed point theorems on ordered dualistic partial b-metric space, a new mapping Modified Dominating Dualistic Kannan is used. This result is more generalized than the result of Nazam and Arsad [25] and considered as an extension of the same.

ACKNOWLEDGEMENT

We convey our sincere thanks to learned referee.

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