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Geometric ZWEIER Convergent Lacunary Sequence Spaces

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ABSTRACT: The The main purpose of this paper is to introduce geometric Zweier lacunary strong convergent sequence spaces $N_\theta^0[Z'(G)]$, $N_\theta[Z'(G)]$, $N_\theta^\infty[Z'(G)]$ consisting of all sequences $x=(x_k)$ such that $[Z(G)]x$ are in the spaces N_θ^0, N_θ and N_θ^∞ respectively, which are normed linear spaces. We also prove certain topological properties and inclusion relations by introducing their geometric Zweier lacunary statistical convergence.

Keywords: Lacunary sequence, Geometric sequence, Zweier Operator, Statistical Convergence

I. INTRODUCTION AND PRELIMINARIES

By ω, we denote the space of all real valued sequences and any subspace of ω is called a sequence space. Let l_∞, c and c_0 be the linear spaces of bounded, convergent and null sequences $x=(x_k)$ with real or complex terms, respectively. It is well known that a sequence space X with linear topology is called a K-space if and only if each of maps $p_n : X \to \mathbb{R}$ defined by $p_n(x)=x_n$ is continuous for all $n\in\mathbb{N}$. A K-space X is called FK-space if and only if X is a complete linear metric space. An FK- Space is a complete metric space for which convergence implies co-ordinate wise convergence. An FK-space whose topology is normable, is called a BK-space or a Banach co-ordinate space. For a sequence space X, the matrix domain X_A of an infinite matrix A is defined by

$$X_A=\{x=(x_k)\in\omega : Ax\in X\} \tag{1.1}$$

where the space X_A is the expansion or the contraction of the original space X [4] for more details.

By a lacunary sequence we mean an increasing integer sequence $\theta=(k_r)$ such that $k_0=0$ and $h_r := k_r-k_{r-1}\to\infty$ as $r\to\infty$. Here the intervals determined by θ will be denoted by $I_r := (k_{r-1},k_r]$.

Freedman *et al.*, [1] defined the space of lacunary convergent sequences N_θ as

$$N_\theta := \left\{ x=(x_i)\in\omega : \lim_{r\to\infty}\left(\frac{1}{h_r}\sum_{i\in I_r}|x_i-\ell|\right)=0, \text{for some } l \right\}. \tag{1.2}$$

which is a BK-space with the norm

$$\|x\|_{N_\theta}=\sup_r \frac{1}{h_r}\sum_{i\in I_r}|x_i| \tag{1.3}$$

for $l=0$ in equation (1.2), the space is denoted by N_θ^0. Also; $\left(N_\theta^0, \|\cdot\|_{N_\theta^0}\right)$ is a BK-space. Sengönül [11] introduced the spaces Z' and Z'_0 as the set of all sequences such that Z-transformations of them are in the spaces c and c_0 respectively, i.e.,

$$Z'=\{x=(x_k)\in\omega : Zx\in c\} \text{and}$$

$$Z'_0=\{x=(x_k)\in\omega : Zx\in c_0\},$$

where $Z'=(z_{nk})$, $n,k=0,1,2,.....$ with

$$z_{nk}=\begin{cases}\frac{1}{2}, k\le n\le k+1 \\ 1, \text{otherwise}\end{cases} \qquad (n,k\in N).$$

This matrix is called Zweier matrix. Türkmen and Başar [3] introduced geometric sequence spaces for $X=c,c_0,l_\infty,l_p$ as

$$\omega(G)=\{x=(x_k): x_k\in C(G), \text{for all } k\in\mathbb{N}\}$$

$$l_\infty(G)=\left\{x=(x_k)\in\omega(G) : \sup_{k\in\mathbb{N}}|x_k|^G<\infty\right\}$$

$$c(G)=\left\{x=(x_k)\in\omega(G) : G\lim_{k\to\infty}|x_k\ominus l|^G=1\right\}$$

$$c_0(G)=\left\{x=(x_k)\in\omega(G) : G\lim_{k\to\infty}x_k=1\right\}$$

$$l_p(G)=\left\{x=(x_k)\in\omega(G) : G\sum_{k=0}^{\infty}|x_k|_G^{p^G}<\infty\right\}$$

and the geometric complex number

$$\mathbb{C}(G) := \{e^z : z\in\mathbb{C}\}$$
$$=\mathbb{C}/\{0\}$$

where $(\mathbb{C}(G),\oplus,\odot)$ is a field with geometric zero 1 and geometric identity e, and we define the geometric addition, subtraction etc as follows:

- $x\oplus y=xy$
- $x\ominus y=x/y$
- $x\odot y=x^{\ln y}=y^{\ln x}$

- $x \oslash y \; or \; x/y \, G = x^{\frac{1}{\ln y}}, y \neq 1$
- $x^{2_G} = x \odot x = x^{\ln x}$
- $x^{pG} = x^{\ln^{p-1} x}$
- $\sqrt{x}^G = e^{(\ln x)^{1/2}}$
- $x^{-1_G} = e^{1/\log x}$
- $x \odot e = x \; and \; x \oplus 1 = x$
- $e^n \odot x = x^n = x \oplus x \oplus ....(\text{upto } n \text{ numbers of } x)$
- $|x|^G = \begin{cases} x, & if \;\; x > 1 \\ 1, & if \;\; x = 1 \\ \frac{1}{x}, & if \;\; x < 1 \end{cases}$
- $\sqrt{x^2 G}^G = |x|^G$
- $|e^y|^G = e^{|y|}$
- $|x \odot y|^G = |x|^G \odot |y|^G$
- $|x \oplus y|^G \leq |x|^G \oplus |y|^G$
- $|x \ominus y|^G \geq |x|^G \ominus |y|^G$
- $0_G \ominus 1_G \odot (x \ominus y) = y \ominus x, \; i.e, \text{ in short } \ominus (x \ominus y) = y \ominus x$

II. MAIN RESULTS

We introduce the geometric form of lacunary convergent sequence space N_θ as follows:

$$N_\theta^G = \left\{ x = (x_i) \in \omega(G) : G \lim_{r \to \infty} (1/h_r \, G \sum_{i \in I_r} |x_i \ominus \ell|^G = 1) \right\},$$

for some ℓ.

The space $N_\theta[Z'(G)]$ is a BK-space with the norm

$$\|x\|_{N_\theta^G}^G = \sup_r 1/h_r \, G \sum_{i \in I_r} |x_i|^G .$$

We define the geometric Z-transformations of the spaces c and c_0 as

$Z' = \{ x = (x_k) \in \omega(G) : Z(G) x \in c(G) \}$ and

$Z'_0 = \{ x = (x_k) \in \omega(G) : Z(G) x \in c_0(G) \}$

where $Z(G) = (z_{nk}(G))(n, k = 1, 2, ...)$ with

$$z_{nk}(G) = \begin{cases} e, & k \leq n \leq k+1 \\ 1, & \text{otherwise} \end{cases} \qquad (n, k \in N).$$

This matrix is called geometric Zweier matrix.

Geometric Zweier Lacunary Strong Convergence

Now we introduce the new geometric sequence spaces involving Zweier lacunary sequences of strictly positive real numbers, defined as follows:-

$$N_\theta[Z'(G)] = \left\{ x = (x_i) \in \omega(G) : G \lim_r 1/h_r \, G \sum_{i \in I_r} |e(x_i \oplus x_{i-1}) \ominus \ell|^G \right\}$$

$, \text{for some } l = 1,$ (2.1.1)

$$N_\theta^\infty[Z'(G)] = \left\{ x = (x_i) \in \omega(G) : \sup_r 1/h_r \, G \sum_{i \in I_r} |e(x_i \oplus x_{i-1})|^G < \infty \right\}$$

. (2.1.2)

Theorem 1. The space $N_\theta^\infty[Z'(G)]$ is a normed linear space with respect to the norm $\|x\|_{N_\theta^\infty[Z'(G)]}^G = \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i \oplus x_{i-1})|^G$.

Proof:-

1. $\|x\|_{N_\theta^\infty[Z'(G)]}^G \geq 1$

Now $\|x\|_{N_\theta^\infty[Z'(G)]}^G = \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i \oplus x_{i-1})|^G$

≥ 1

2. Suppose $\|x\|_{N_\theta^\infty[Z'(G)]}^G = 1$

$\Leftrightarrow \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i \oplus x_{i-1})|^G = 1$

$\Leftrightarrow e(x_i \oplus x_{i-1}) = 1$

$\Leftrightarrow (x_i \oplus x_{i-1}) = 1$

$\Leftrightarrow x_i . x_{i-1} = 1$

$\Leftrightarrow x_i = x_{i-1} = 1$

$\Leftrightarrow x_i = 1 \qquad \forall i$

$\Leftrightarrow x = (1,1,1,......) = 0_G$

3. $\|x \oplus y\|_{N_\theta^\infty[Z'(G)]}^G$

$= \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e\{(x_i \oplus x_{i-1}) \oplus (y_i \oplus y_{i-1})\}|^G$

$= \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i . x_{i-1}) \oplus (y_i . y_{i-1})|^G$

$= \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i . x_{i-1})|^G . |e(y_i . y_{i-1})|^G$

$= \left(\sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e\{(x_i . x_{i-1})\}|^G \right) . \left(\sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e\{(y_i . y_{i-1})\}|^G \right)$

$= \|x\|_{N_\theta^\infty[Z'(G)]}^G . \|y\|_{N_\theta^\infty[Z'(G)]}^G$

$= \|x\|_{N_\theta^\infty[Z'(G)]}^G \oplus \|y\|_{N_\theta^\infty[Z'(G)]}^G$

4. $\|\alpha \odot x\|_{N_\theta^\infty[Z'(G)]}^G$

$= \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e\{(\alpha \odot x_i) \oplus (\alpha \odot x_{i-1})\}|^G$

$= \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e\{|\alpha| \odot (x_i \oplus x_{i-1})\}|^G$

$= |\alpha| \odot \left(\sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i \oplus x_{i-1})|^G \right)$

$= |\alpha| \odot \|x\|_{N_\theta^\infty[Z'(G)]}^G$

Thus $\|.\|_{\Delta_G}^G$ is a norm on $\mathbb{C}(G)$.

Theorem 2. The space $N_\theta^\infty[Z'(G)]$ is a Banach space with respect to the norm $\|x\|_{N_\theta^\infty[Z'(G)]}^G = \sup_r \frac{1}{h_r} G \sum_{i \in I_r} |e(x_i \oplus x_{i-1})|^G$.

Proof: Let (x_n) be a Cauchy sequence in $N_\theta^\infty[Z'(G)]$, where

$x_n = \left(x_i^{(n)}\right) = \left(x_1^{(n)}, x_2^{(n)}, x_3^{(n)}, ...\right) \forall n \in \mathbb{N}$, and $x_i^{(n)}$ is the i^{th} co-ordinate of x_n. Then

$$\|x_n \ominus x_m\|^G_{N_\theta^\infty[Z'(G)]}$$

$$= \sup_r 1/h_r \, G \sum_{i \in I_r} \left| e\left\{ \left(x_i^{(n)} \oplus x_{i-1}^{(n)} \right) \ominus \left(x_i^{(m)} \oplus x_{i-1}^{(m)} \right) \right\} \right|^G \to 1$$

as m,n $\to \infty$

Hence we get $\left| x_i^{(n)} \ominus x_i^{(m)} \right|^G \to 1$ as n,m $\to \infty \, \forall i \in N$, since $\left| x_i^{(n)} \ominus x_i^{(m)} \right|^G \geq 1$.

Therefore for fixed i, the i-th co-ordinates of all sequences form a Cauchy sequence in $\mathbb{C}(G)$. Let $x_i^{(n)} = \left(x_i^{(1)}, x_i^{(2)}, x_i^{(3)}, ...\right)$ be a Cauchy sequence in $\mathbb{C}(G)$. Since $\mathbb{C}(G)$ is complete, $x_i^{(n)}$ converges to x_i (say) as

$x_n = \left(x_1^{(n)}, x_2^{(n)}, x_3^{(n)}, ..., x_k^{(n)} \right)$ converges to

$x = (x_1, x_2, x_3, ..., x_k, ...)$

$\Rightarrow G \lim_{n \to \infty} x_i^{(n)} = x_i, \forall i \in \mathbb{N}$

Further for each $\varepsilon > 1$, $\exists N = N(\varepsilon)$ such that $\forall n, m \geq \mathbb{N}$ we have

$$\sup_r 1/h_r \, G \sum_{i \in I_r} \left| e\left\{ \left(x_i^{(n)} \oplus x_{i-1}^{(n)} \right) \ominus \left(x_i^{(m)} \oplus x_{i-1}^{(m)} \right) \right\} \right|^G < \varepsilon$$

and

$$G \lim_{m \to \infty} G \sum_{i=1}^{\infty} \left| x_i^{(n)} \ominus x_i^{(m)} \right|^G < G \lim_{m \to \infty} G \sum_{i=1}^{\infty} \left| x_i^{(n)} \ominus x_i \right|^G < \varepsilon, \forall n \geq \mathbb{N}$$

since ε is independent of i. Hence we obtain $x_n \to x$ as $n \to \infty$. Now

$$\left| x_i \oplus x_{i-1} \right|^G = \left| x_i \ominus x_i^N \oplus x_i^N \ominus x_{i-1}^N \oplus x_{i-1}^N \oplus x_{i-1} \right|$$

$$= O(e)$$

$\Rightarrow x = (x_k) \in N_\theta^\infty \left[Z'(G) \right]$

$\Rightarrow N_\theta^\infty \left[Z'(G) \right]$ is a Banach space with continuous co-ordinates and it is a BK-Space. This completes the proof.

III. GEOMETRIC ZWEIER LACUNARY STATISTICAL CONVERGENCE

Fast [5] and Schoenberg [6] introduced independently the notion of statistical convergence. Let K be a subset of the set of natural numbers $\mathbb{N}$. Then the asymptotic density of K denoted by $\delta(k)$ is defined as $\delta(k) = \lim_n (1/n) \left| \{ k \leq n : k \in K \} \right|$, where the vertical bars denote the cardinality of the enclosed set. A number sequence $x = (x_k)$ is said to be statistically convergent to the number L if, for each $\varepsilon > 0$, the set $k(\varepsilon) = \{ k \leq n : |x_k - L| > \varepsilon \}$ has asymptotic density zero; that is, $\lim_n (1/n) \left| \{ k \leq n : |x_k - L| \geq \varepsilon \} \right| = 0$ this concept of statistical convergence from different aspects has been studied by various authors [5-10]. Here we write $S - \lim x = L$ or $x_k \to L(S)$. We use S to denote the set of all statistically convergent sequences. The idea of statistical convergence was introduced by Fast [10] and studied by several authors [5-10]. A sequence $x = (x_i)$ is said to be lacunary statistical geometric Zweier convergent to L if for $\varepsilon > 1$

$$S_\theta \left[Z'(G) \right] = \left\{ x = (x_i) \in \omega(G) : \lim_r 1/h_r \left| Z(G) K_\theta(\varepsilon) \right|^G = 1 \right\}$$

where, $Z(G) K_\theta(\varepsilon) = \left\{ i \in I_r : \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G \geq \varepsilon \right\}$.

If $x \in S_\theta \left[Z'(G) \right]$, then we write $x_i \to L\left(S_\theta \left[Z'(G) \right] \right)$.

Let $I_r^1 = \left\{ i \in I_r : \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G \geq \varepsilon \right\} = CK_\theta(\varepsilon)$ and

$I_r^2 = \left\{ i \in I_r : \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G < \varepsilon \right\}$.

IV. INCLUSION THEOREMS

In this section we first give some inclusion relations between the spaces $N_\theta\left(\left[Z'(G) \right] \right)$ and $S_\theta\left(\left[Z'(G) \right] \right)$ and show that they are equivalent for bounded sequences. We also study the inclusions $S\left(\left[Z'(G) \right] \right) \subseteq S_\theta\left(\left[Z'(G) \right] \right)$ and $S_\theta\left(\left[Z'(G) \right] \right) \subseteq S\left(\left[Z'(G) \right] \right)$ under certain restrictions on $\theta = \{k_r\}$.

Theorem 1. Let $\theta = \{k_r\}$ be a lacunary sequence; then

(i) $x_i \in L\left[N_\theta \left[Z'(G) \right] \right] \Rightarrow x_i \in L\left[S_\theta \left[Z'(G) \right] \right]$

(ii) $N_\theta \left[Z'(G) \right]$ is a proper subset of $S_\theta \left[Z'(G) \right]$.

Proof. (a)Let $\varepsilon > 1$ and $x_i \to L\left(N_\theta \left[Z'(G) \right] \right)$, we can write

$$\frac{1}{h_r} G \sum_{i \in I_r} \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G \geq 1/h_r G \sum_{i \in I_r^1} \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G$$

$$\geq 1/h_r \left| Z(G) K_\theta(\varepsilon) \right|_G \varepsilon$$

It follows that $x_i \to L\left(S_\theta \left[Z'(G) \right] \right)$.

(b) Now to establish the inclusion $N_\theta \left[Z'(G) \right] \subseteq S_\theta \left[Z'(G) \right]$, let θ be given and define x_i to be $1, 2, ..., \left[\sqrt{h_r} \right]$ at the first $\left[\sqrt{h_r} \right]$ integers in I_r, and $x_i = 1$ otherwise.

Note that x is not bounded. As we have for every $\varepsilon > 1$,

$$1/h_r \, G \sum_{i \in I_r^1} \left| \left\{ \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G \ominus 1 \geq \varepsilon \right\} \right|^G \to 1$$

as $r \to \infty$

i.e., $x_i \to 1\left(S_\theta \left[Z'(G) \right] \right)$.

On the other hand,

$$1/h_r \, G \sum_{i \in I_r^2} \left| \left\{ \left| e(x_i \oplus x_{i-1}) \ominus L \right|^G \ominus 1 < \varepsilon \right\} \right|^G \neq 1$$

Hence x_i not $\to 1\left(N_\theta\left[Z'(G)\right]\right)$.

Theorem 2. (i) If

$$x \in N_\theta^\infty\left[Z'(G)\right] \text{and}\, x_i \to L\left(S_\theta\left[Z'(G)\right]\right)$$

$$\Rightarrow x_i \to L\left(N_\theta\left[Z'(G)\right]\right)$$

(ii) $S_\theta\left[Z'(G)\right] \cap N_\theta^\infty\left[Z'(G)\right] = N_\theta\left[Z'(G)\right] \cap N_\theta^\infty\left[Z'(G)\right]$

Proof: Suppose that $x_i \to L\left(S_\theta\left[Z'(G)\right]\right)$ and $x \in N_\theta^\infty\left[Z'(G)\right]$ say $\left|e\left(x_i \oplus x_{i-1}\right) \ominus L\right|^G \le M$ for all i.

Therefore we have for every $\varepsilon > 1$

$$1/h_r\, G \sum_{i \in I_r} \left|e\left(x_i \oplus x_{i-1}\right) \ominus L\right|^G$$

$$= 1/h_r\, G \sum_{i \in I_r^1} \left|e\left(x_i \oplus x_{i-1}\right) \ominus L\right|^G + 1/h_r\, G \sum_{i \in I_r^2} \left|e\left(x_i \oplus x_{i-1}\right) \ominus L\right|^G$$

$$\le M/h_r \left|Z(G) K_\theta(\varepsilon)\right|^G \oplus \varepsilon$$

Taking limit as $\varepsilon \to 1$, we get the result.

(ii) This is an immediate consequence of (i) and theorem 1.

Theorem 3. For any lacunary sequence $\theta, S\left(\left[Z'(G)\right]\right) \ominus G \lim x = L$ implies $S_\theta\left(\left[Z'(G)\right]\right) \ominus G \lim x = L$ if and only if $G \liminf_r q_r > e$, then there exists a bounded $S_\theta\left(\left[Z'(G)\right]\right)-$ summable sequence that is not $S\left(\left[Z'(G)\right]\right)-$summable (to any limit).

Proof: Suppose first that $G \liminf_r q_r > e$; then, there exits $\delta > e$ such that $q_r \ge e \oplus \delta$ for sufficiently large r. which implies that

$$\frac{h_r}{k_r} \ge \frac{\delta}{\delta \oplus e} \text{and} \frac{k_r}{h_r} \ge \frac{\delta \oplus e}{\delta}.$$

If $x_i \to L\left(S\left[Z'(G)\right]\right)$ then for every $\varepsilon > 1$ and sufficiently large r, we have

$$1/k_r \left|\left\{k \le k_r : \left|e(x_i \oplus x_{i-1}) \ominus L\right|^G \ge \varepsilon\right\}\right|^G$$

$$\ge 1/k_r \left|\left\{k \le I_r : \left|e(x_i \oplus x_{i-1}) \ominus L\right|^G \ge \varepsilon\right\}\right|^G$$

$$\ge \frac{\delta}{\delta \oplus e} 1/h_r \left|\left\{k \le I_r : \left|e(x_i \oplus x_{i-1}) \ominus L\right|^G \ge \varepsilon\right\}\right|^G$$

this proves the sufficiency. Conversely, suppose that $G \liminf_r q_r > e$. Proceeding as in [2] we can select a subsequence $\left\{k_{r(j)}\right\}$ of lacunary sequence θ such that

$$\frac{k_{r(j)}}{k_{r(j)-1}} < e \oplus \frac{e}{j} \text{and} \frac{k_{r(j)-1}}{k_{r(j-1)}} > j, \quad \text{where } r(j) \ge r(j-1) \oplus e^2$$

Now define a bounded sequence x by $x_i(G) = e$ if $i \in I_{r(j)}$ for some $j = 1,2,3,....$ and $x_i(G) = 1$. Otherwise it is shown in [2] that $x \notin N_\theta\left[Z'(G)\right]$ but $x \in \left|\sigma_1\right|^G$. The above Theorem 4.2 (i) implies that $x \notin S_\theta\left[Z'(G)\right]$ but it follows from Theorem 4.1, $x \in S\left[Z'(G)\right]$. Hence $S\left[Z'(G)\right] \subseteq S_\theta\left[Z'(G)\right]$

V. CONCLUSIONS

Our results generalize the results of Turkmen, C. and Başar, F. [3], Şengönül, M. [11], Singh, S. and Dutta, S. [12], Kadak, U. [13], and many others. As a future work we will study certain matrix transformation, inclusion relations and $\alpha-, \beta-$ and $\gamma-$ of these spaces. Further the present results can be extended to the m-th order difference sequence spaces.

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REFERENCES

[1]. Freedman, A. R., (1981). Some Cesάro-type summability spaces. *Pacific J. Math.*, 35: 293-305.

[2]. Freedman, A. R., Sember, J. J. & Raphael, M., (1978). Some Cesάro type summability spaces. *Proc. London Math. Soc*, **37**: 508-520.

[3]. Türkmen, C. & Başar, F., (2012). Some Basic Results on the sets of Sequences with Geometric Calculus. *Commun. Fac. Fci. Univ. Ank. series A1.*, 2: 17-34.

[4]. Başar, F. (2012). Summability Theory and its applications. Bertham Science Publishers, e-books, Monographs, xi+405pp, Istanbul , ISB:978-1-60805-252-3.

[5]. Fast, H., (1951). Sur la convergence statistique. *Colloq. Math.*, 2: 241-244.

[6]. Schoenberg, J., (1959). The integrability of certain functions and related summability methods. *Amer. Math. Monthly*, **66**: 361-375.

[7]. Maddox, I. J., (1970). Elements of Functional Analysis. Cambridge University Press.

[8]. Connor, J., (1988). The statistical and strong p-Cesάro convergence of sequences. *Analysis*, **8:** 47-63.

[9]. Fridy, J. A., (1985). On statistical convergence. *Analysis*, **5**: 301-313.

[10]. Fridy, J. A. & Miller, H. I., (1991). A matrix characterization of statistical convergence. *Analysis*, **11**: 55-66.

[11]. Şengönül, M., (2005) On the Zweier Space. *Demonstration Mathematica,* Vol. **XL**(1**)**:181-186.

[12]. Singh, S. and Dutta, S., (2019). On certain generalized mth order geometric difference sequence spaces. *Far East Journal of Mathematical Sciences.* **116**(1): 83-100.

[13]. Kadak, U., (2015). Generalized Lacunary Statistical Difference Sequence Spaces of Fractional Order. *International Journal of Mathematics and Mathematical Sciences,* Article ID 984283, 2015: 6 pages.