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Rigidity Effects on Surface Waves in Multilayered Media

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ABSTRACT: The effect of rigidity on surface waves in multi-layered medium have been studied in this paper. We have extended the research work which is based on the propagation of shear waves in a multilayered medium including a fluid-saturated porous layer with free surface [1]. The rigidity effect for a model consisting a transversely isotropic liquid-saturated porous layer between a non-homogeneous elastic half space and an elastic isotropic homogeneous layer have been studied and the dispersion relation for the considered problem have been derived. We have also discussed some particular cases for surface wave propagation with and without rigidity.

Keywords: Rigidity, Surface Waves, Dispersion Relation, Layered Media, Homogeneous Half Space

I. INTRODUCTION

The Earth is a layered medium and assumptions regarding the inner structure of the Earth may be evolved through analysing seismic waves that travel through the Earth and are calculated at seismic stations. Seismic waves are used by the Earth scientists to examine the depth of the layers consisted in the Earth. Due to slower attenuation of strength, surface wave causes plenty of destruction to the structure of the Earth in comparison to the body waves. It was Biot who first investigated the dispersion equation for surface wave propagation in a porous medium [2]. The distribution curves for Love wave propagation in a diagonally isotropic crustal layer with an irregularity along the boundary had been developed [3]. The dispersion equation for shear waves in a multilayered medium involving a liquid saturated porous layer developed by [1]. The propagation of torsional waves due to influence of rigidity and irregularity had been examined [4]. The Love wave propagation under a liquid saturated porous stratum through a rigid interface lying over an elastic half space under gravity studied with the help of a mathematical model [5]. Love wave propagation under the influence of rigid boundary in a non-homogeneous layer through an initially stressed half space studied and found that the heterogeneity was present in density as well as rigidity [6]. Love wave propagation in porous stratum due to the effect of rigid layer had been examined [7]. The effect of irregularity, inhomogenity and rigidity in fluid saturated porous layer over homogeneous and non-homogeneous half spaces had also been studied by many authors time to time like [8-15] and developed the corresponding frequency equations in terms of phase velocity wave number. Torsional ground waves in an inhomogeneous anisotropic layer lying between two non-homogeneous half-spaces discussed [16] and derived corresponding dispersion equation. The elastic wave propagation at imperfect boundary of micropolar- elastic solid and fluid saturated porous solid half-space had been studied and the dispersion equation had been derived [17]. In this paper, a mathematical model consisting intermediate transversely isotropic liquid-saturated porous layer resting on the non-homogeneous elastic half space and lying under an elastic isotropic homogeneous rigid boundary have been considered. The inhomogeneity present in the lower half space varies exponentially with depth. The dispersion relation has been derived for the surface waves and some particular cases have also been discussed.

II. PROBLEM FORMULATION

A model consisting a transversely isotropic fluid saturated porous stratum of thickness H resting on a non-homogeneous elastic half space and lying under an elastic isotropic homogeneous rigid layer of width 'h' have been considered. The Cartesian coordinate system (x, y, z) with z-axis vertically downward and x-axis is parallel to wave propagation have been taken. The components along the y-direction are zero due to two-dimensional x-z plane. Let the uppermost homogeneous rigid layer defined as the medium $M_1: -(h+H) \leq z \leq H,$ the intermediate layer be the medium $M_2: -H \leq z \leq 0$ and the half space as the medium $M_3: 0 \leq z \leq \infty$. The geometry of the proposed problem have been presented in Fig. 1.

III. GOVERNING EQUATIONS

The inhomogeneity is supposed to be changed exponentially according to depth, and is given by

$$\mu^*(z) = \mu_0^* e^{(mz)}$$
$$\rho^*(z) = \rho_0^* e^{(mz)} \quad (1)$$

where μ_0^* and ρ_0^* are the constant of shear modulus $\mu^*(z)$ and the mass density $\rho^*(z)$ respectively at the rigid boundary and m is any constant.

The basic equations for the considered mediums are as follow:

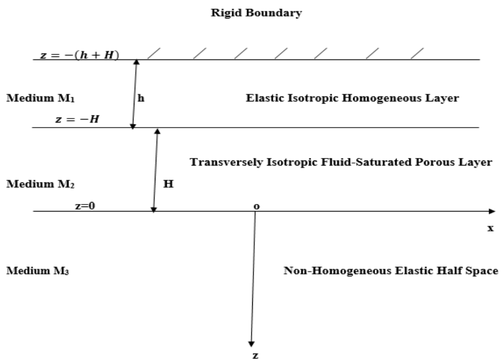

Fig. 1. Geometry of Surface Wave Propagation in Multilayered Media.

For Medium M_1

The equation of motion without body forces for medium M_1 is given by

$$\tau_{ij,j} = \rho_0 \ddot{v}_i \tag{2}$$

where ρ_0 is the density, v_i are displacement components at time t, and τ_{ij} are defined as the components of stress-tensor, comma indicate the differentiation with respect to the position x_k and dot is that of with respect to time t.

The stress-strain relations are given by [18]

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \tag{3}$$

where λ and μ are Lame's constants,

δ_{ij} is known the Kronecker delta and

$$e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \tag{4}$$

$$e_{kk} = div\, v. \tag{5}$$

For Medium M_2

The equation of motion without body forces, for medium M_2 are given by [2]

$$\sigma_{ij,j} = \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i + b_{ij}(\dot{U}_j - \dot{u}_j)$$

$$\sigma_{,j} = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i + b_{ij}(\dot{U}_j - \dot{u}_j) \tag{6}$$

where σ_{ij} are stress-tensor components of the solid frame, $\sigma = -pf$ is reduced pressure of the liquid (p is the pressure on the fluid, and f is porosity), and u_i are the components of displacement vector of the solid frame and U_i are that of fluid and σ is total surface unit. The parameter ρ_{11}, ρ_{12} and ρ_{22} are the dynamic coefficients and taken into consideration of the inertia effect of moving fluid, ρ_s and ρ_f are mass density of the solid and fluid respectively and related by the relation [2] as

$$\rho_{11}+\rho_{12} = (1-f)\rho_s,\ \rho_{12}+\rho_{22} = f\rho_f \tag{7}$$

and satisfy the inequalities

$$\rho_{11} > 0,\ \ \rho_{22} > 0,$$

$$e_{12} \le 0,\ \ e_{11}\rho_{22} - \rho_{12}^2 > 0 \tag{8}$$

where ρ_{12} is coupling parameter.

Following [19], the components of the flow opposition tensor b used for the transverse-isotropy are

$$b_{ij} = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{11} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \tag{9}$$

The constitutive relations, for medium M_2 are

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma \end{bmatrix} = \begin{bmatrix} 2N+A & A & F & 0 & 0 & 0 & M \\ A & 2N+A & F & 0 & 0 & 0 & M \\ F & F & 2C & 0 & 0 & 0 & Q \\ 0 & 0 & 0 & 2G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2N & 0 \\ M & M & Q & 0 & 0 & 0 & R \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \\ \varepsilon \end{bmatrix} \tag{10}$$

where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

$$\varepsilon_{kk} = div\, u, \qquad \varepsilon = div\, U \tag{11}$$

where A, C, F, G, M, N, Q, R are the materials constants.

For Medium M_3

The basic equations of motion in the absence of body forces for medium M3 are [20]

$$t_{ij,j} = \rho^* \ddot{w}_i \tag{12}$$

where t_{ij}, w_i and ρ^* are the stress, displacement vectors and density of fluids respectively.

The corresponding constitutive relations are

$$t_{ij} = \lambda^* \bar{e}_{kk}\delta_{ij} + 2\mu^* \bar{e}_{ij} \tag{13}$$

where λ^* and μ^* are the Lame's constants

$$\bar{e}_{ij} = \frac{1}{2}(w_{i,j} + w_{j,i}), \tag{14}$$

$$\bar{e}_{kk} = div\, w.$$

For wave changing harmonically in x-z plane, we take

$$v_1 = v_3 = 0, \qquad v_2 = v(x, z, t),$$

$$u_1 = u_3 = 0, \qquad u_2 = u(x, z, t),$$

$$U_1 = U_3 = 0, \qquad U_2 = U(x, z, t),$$

$$w_1 = w_3 = 0, \qquad w_2 = w(x, z, t). \tag{15}$$

The equations of motion (2), (6) and (12) with the help of Eqns. (3), (5), (9)-(11) and (13)-(14), respectively, take the form of

$$\mu\nabla^2 v = \rho_0 \ddot{v} \tag{16}$$

$$N\frac{\partial^2 u_2}{\partial x^2} + G\frac{\partial^2 u_2}{\partial z^2} = \rho_{11}\ddot{u} + \rho_{12}\ddot{U} - b_{11}(\dot{U} - \dot{u}) \tag{17}$$

$$\rho_{12}\ddot{u} + \rho_{22}\ddot{U} + b_{11}(\dot{U} - \dot{u}) = 0 \tag{18}$$

$$\mu^*\nabla^2 w + \mu_{,3}{}^* w_{,3} = \rho^* \ddot{w} \tag{19}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$, is the Laplacian operator.

IV. BOUNDARY CONDITIONS

The boundary conditions for the considered model are given as

(a) At the rigid surface $z = -(h + H)$, displacement components vanish i.e.,

$$v_o[x, z = -(h + H), t] = 0 \tag{20}$$

(b) At the interface z = – H, the displacement and stress components are continuous i.e.,

$$v\,(x, z = -H, t) = u\,(x, z = -H, t) \tag{21}$$

$$\tau_{32}(x, z = -H, t) = \sigma_{32}\,(x, z = -H, t) \tag{22}$$

(c) At the interface z = 0, the displacement and stress components are also continuous i.e.

$$u\,(x, z = 0, t) = w\,(x, z = 0, t) \tag{23}$$

$$\sigma_{32}\,(x, z = 0, t) = t_{32}(x, z = 0, t) \tag{24}$$

The Eqns. (16)-(19) with the help of (1) along with above boundary conditions are the governing equations for the considered model.

V. SOLUTION OF THE PROBLEM

For wave changing harmonically with time and propagate in x direction, so we can assume displacement in the form of

$$(v, u, U, w) = (v_o, u_o, U_o, w_o)e^{i(kx-\omega t)} \tag{25}$$

where v_o, u_o, U_o and w_o are the function of z only, ω is the angular frequency and k is the wave number.

The Eqns. (16)-(19) with the help of (1) and (25) give

$$\left(\frac{\partial^2}{\partial z^2} + \varkappa_1{}^2\right)v_o = 0, \tag{26}$$

$$\left(\frac{\partial^2}{\partial z^2} + \varkappa_2{}^2\right)\left(\frac{u_o}{U_o}\right) = 0, \tag{27}$$

$$\left(\frac{\partial^2}{\partial z^2} + m\frac{\partial}{\partial z} - \varkappa_3{}^2\right)w_o = 0, \tag{28}$$

where $\varkappa_1{}^2 = \frac{\omega^2 \rho_o}{\mu} - k^2,$

$\varkappa_2{}^2 = \xi^2 - \frac{k^2}{G}N,$

$\xi^2 = (F + iR)\frac{\omega^2}{G}$,

F = $\{(\rho_{11}\,\rho_{22} - \rho_{12}{}^2)\rho_{22} + (\rho_{11} - \rho_{22})b_{11}{}^2\}/(\rho_{22}\,\omega)^2 + b_{11}{}^2$,

R = $\{(-\rho_{22}{}^2 + \rho_{12}{}^2)b_{11}\omega\} / (\rho_{22}\,\omega)^2 + b_{11}{}^2$.

The quantity $\varkappa_2{}^2$ is complex due to the structure of ξ^2.

$\varkappa_3{}^2 = k^2 - \frac{\rho^*\omega^2}{\mu^*}$

where $\mu/\rho$and μ_0^*/ρ_0^* signify the velocities of waves for the upper layer and lower half space respectively.

Now, the solutions of differential Eqns. (26)-(28) with the help of (25) are

$$v(x,z,t) = (A_1 \cos\varkappa_1 z + A_2 \sin\varkappa_1 z)\, e^{i(kx-\omega t)} \tag{29}$$

$$u(x,z,t) = (A_3 \cos\varkappa_2 z + A_4 \sin\varkappa_2 z)\, e^{i(kx-\omega t)} \tag{30}$$

$$U(x,z,t) = (\overline{A_3} \cos\varkappa_2 z + \overline{A_4} \sin\varkappa_2 z)\, e^{i(kx-\omega t)} \tag{31}$$

$$w = A_5 e^{-\eta z} e^{i(kx-\omega t)} \tag{32}$$

where

$\eta = -\frac{m-\sqrt{m^2+4\varkappa_3{}^2}}{2}$ and A_1, A_2, A_3, A_4, A_5 are arbitrary constants.

By applying the boundary conditions (20)-(24) in Eqns. (29)-(30) and (32), we get a set of five homogeneous equations in five unknowns $A_1, A_2, A_3, A_4, \text{and } A_5$

$$A_1 \cos\varkappa_1(h+H) - A_2 \sin\varkappa_1(h+H) = 0 \tag{33}$$

$$(A_1 \cos\varkappa_1 H - A_2 \sin\varkappa_1 H) - (A_3 \cos\varkappa_2 H - A_4 \sin\varkappa_2 H) = 0, \tag{34}$$

$$\mu\varkappa_1(A_1 \sin\varkappa_1 H + A_2 \cos\varkappa_1 H) - G\varkappa_2(A_3 \sin\varkappa_2 H + A_4 \cos\varkappa_2 H) = 0 \tag{35}$$

$$A_3 - A_5 = 0 \tag{36}$$

$$G\varkappa_2 A_4 + \mu_0^*\eta A_5 = 0 \tag{37}$$

For non- trivial solution of above homogeneous system of Eqns. (33)-(37), we have

$$\begin{vmatrix} \cos\varkappa_1(h+H) & -\sin\varkappa_1(h+H) & 0 & 0 & 0 \\ \cos\varkappa_1(H) & -\sin\varkappa_1(H) & -\cos\varkappa_2(H) & \sin\varkappa_2(H) & 0 \\ \mu\varkappa_1\sin\varkappa_1(H) & \mu\varkappa_1\cos\varkappa_1(H) & -G\varkappa_2\sin\varkappa_2(H) & -G\varkappa_2\cos\varkappa_2(H) & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & G\varkappa_2 & \mu_0^*\eta \end{vmatrix} = 0 \tag{38}$$

On simplification, we get

$$\frac{\varkappa_2 G}{\mu\varkappa_1}\tan\varkappa_2 H \tan\varkappa_1 h = \mu_0^*\eta\left(\{\frac{\tan\varkappa_1 h}{\mu\varkappa_1} + \frac{\tan\varkappa_2 H}{G\varkappa_2}\} + 1\right) \tag{39}$$

Equation (39) is the required dispersion equation for the shear waves for the considered problem and that relates the phase velocity of propagation, inhomogeneity and rigidity parameter.

VI. PARTICULAR CASES

Case 1: If h = 0, equation (39) reduces to

$$\tan\varkappa_2 H = -G\varkappa_2/\,\mu_0^*\eta \tag{40}$$

which is the dispersion equation for shear waves for transversely isotropic fluid saturated porous layer lying over a non-homogeneous half space.

Case 2: If H = 0, Eqn. (39) become

$$\tan\varkappa_1 h = -\mu\varkappa_1/\mu_0^*\eta \tag{41}$$

The Eqn. (41) is the dispersion relation for shear waves in an elastic isotropic homogeneous layer lying over a non-homogeneous half space with rigid boundary.

VII. CONCLUSION

Surface wave propagation in multilayered media under the effect of rigidity had been studied and dispersion equation obtained analytically by using simple mathematical calculations. Some particular cases had also been discussed. Due to varied application of seismology, this paper may be very helpful for researchers as well as post graduate students.

CONFLICT OF INTEREST

Authors have no any conflict of interest.

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