



Testing of $n+1$ Parameter from Unlike Load Sharing System

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ABSTRACT: Sequential order statistics play the part of model for the basic lifetime of $k+1$ -out-of- $n+1$ systems, which are working as long as $k+1$ out of $n+1$ components in reliability. This paper takes multiple samples of sequential order statistics modelling for basic lifetimes of possibly different structured $k+1$ -out-of- $n+1$ systems and provides test to check the load-sharing or distribution parameters. In two-sample cases, critical values for the correlative test statistics are arranged for small sample sizes. Weibull distribution of the test statistics under the alternative hypothesis are consider. This test has applied to discover the significant differences between systems.

Keywords: Reliability analysis, Weibull Distribution, Load-sharing system, Statistic Order, Alternative Hypothesis, Likelihood.

I. INTRODUCTION

Statistics is the term which is used to mean either statistical data or statistical method. When statistics is used in the sense of statistical data it refers to quantitative aspects of things and this is a numerical description. Thus the distribution of family incomes is a quantitative description also the annual production figures of various industries. While many of the developments concerning to the study of load sharing system have occurred in past decade. Modelling parameters of load sharing system discussed [1]. Sutar has accelerated failure time models [2].

II. CHARACTERISTICS OF STATISTICAL DATA

Statistical data always denotes numerical description, whereas this is true, it must be remembered that numerical descriptions are not statistical data. In order that numerical descriptions may be called statistics they must possess the following characteristics:

- 1) They must be in aggregates.
- 2) They must be affected to a marked extent by a multiplicity of causes.
- 3) They must be enumerated or estimated according to reasonable standard of accuracy.
- 4) They must have been collected in a systematic manner for a predetermined purpose.
- 5) They must be placed in relation to each other.

The $k+1$ out of $n+1$ system form an important structure in reliability that comprises series and parallel systems in some special cases. In series and parallel series system sometimes $n+1$ systems work and sometimes it fails. However, the series and parallel system is solve as long as at least $k+1$ of the components, In essence the $((n+1) - (k+1)+1)$ th components failure time cross or touch with the system of lifetime. The test based on sequential and generalized order statistic suggested [3]. The components lifetime is described in a simple model, by n non-negative random variables by cumulative distributed function M the concept of k -out-of- n system was introduced in concept of generalized order of statistics which is used to describe the failure time rate of residual lifetimes of remaining components. The

discussion used for various non parametric comparisons of several k -out-of- n systems [10].

Modelling with sequentially order statistics is not adequate in applications for reinvent impacts. When SOSs are used as a load-sharing model for $k+1$ -out-of- $n+1$, then underlying quantity has to perform and consider the testing for equality of parameters from different loading system [4].

In this paper we develop the statistical method or test to compare between multiple $k+1$ -out-of- $n+1$ systems. We find the exact critical values for likelihood ratio test and for Weibull distribution. This proposed test may be applied to check the performance of sub sequential procedure and common load-sharing parameters. Sequential order statistics and k -out-of- n Systems with sequentially considered [5].

Firstly, gives the model and basic properties which is based on multiple samples then derive the exact statistical test to check the critical values for common load-sharing system. The order restricted inference for sequential k -out-of- n systems [6].

III. MODELS AND BASIC PROPERTIES

Sequential order statistics were introduced or given by a concept of generalized order of statistics by means of a triangular scheme of independent random variables and certain recursive formulas. The Modelling parameters of a load-sharing system through link function in sequential order statistics models and associated inference [7].

For example: Whenever 50 coins are tossed and asked for the number of heads and so on. Whenever we associate with a real number with each outcome of trial, we are dealing with chance variable, stochastic variable or simply a variate. While other possible definition has been proposed in the literature, for instance, based on independent power- function-distributed random variables or on counting processes. In this paper, the likelihood approach to the model is enough. Generalized order statistics an exponential family in model parameters has discussed [8].

Let M_1, \dots, M_{n+1} be absolutely continuous cumulative density functions with $M_1^{-1}(1), \dots, M_n^{-1}(1)$ and the corresponding density functions m_1, m_2, \dots, m_{n+1} .

Ordered random variables Y_1, \dots, Y_{n+1} known as sequential order statistics depends on M_1, \dots, M_{n+1} if their joint density functions with respect to Lebesgue measure is given by

$$M^{Y_1, \dots, Y_{n+1}}(y_1, \dots, y_{n+1}) = (n+1)! \prod_{j=1}^{n+1} \left\{ \frac{1-M_j(y_j)}{1-M_j(y_{j-1})} \right\}^{n+1-j} \frac{m_j(y_j)}{1-M_j(y_{j-1})}$$

For all real numbers $y_1 \leq \dots \leq y_{n+1}$, Where the following values $y_0 \equiv -\infty$ for the sample representation. In that case, Y_1, \dots, Y_{n+1} from Markov Chain along transition probabilities

$$P(Y_j > t | Y_{j-1} = x_{j-1}) = \{1 - M_j(t) | 1 - M_j(y_{j-1})\}^{n-j+2}, t > x_{j-1}. \text{ In the distribution sense, order statistic (based on } M_j) \text{ are include in the Model and result by setting } M_1 = \dots = M_{n+1}.$$

Now by taking

$$M_j = 1 - (1 - M)^{\alpha_j}, 1 \leq j \leq n+1,$$

As any absolutely continuous CDF M , which is refer to baseline CDF and positive numbers $\alpha_1, \dots, \alpha_n$, from this process we archive at a semi parametric sequential order statistics model, in which M_1, \dots, M_{n+1} have proportional ahead rates i.e., $\lambda_{M_j} = \alpha_j \lambda_M, 1 \leq j \leq n+1$

The modified ahead rate of Y_j , given as $Y_{j-1} = x_{j-1}$, is then

$$\lambda_{Y_j | Y_{j-1} = x_{j-1}}(t) = (n-j+1) \alpha_j \lambda_M(t), t > y_{j-1}.$$

When the component lifetimes of a $k+1$ -out-of- $n+1$ system are entered, the data are type-2 right censored if $k+1 > 1$, s.t. marginal densities are naturally of some interest.

The marginal density of first r ($\leq n$) sequential order statistics base on M_1, \dots, M_n is given by

$$M^{Y_1, \dots, Y_r}(y_1, \dots, y_r) = \frac{(n+1)!}{(n+1-r)!} \prod_{j=1}^{r+1} \left\{ \frac{1-M_j(y_j)}{1-M_j(y_{j-1})} \right\}^{n+1-j} \frac{m_j(y_j)}{1-M_j(y_{j-1})}, y_1 \leq \dots \leq y_{r+1},$$

This equation can be written as

$$m^{Y_1, \dots, Y_{r+1}}(y_1, \dots, y_{r+1}) = \frac{(n+1)!}{(n+1-r)!} \prod_{j=1}^{r+1} \left\{ \frac{1-M_j(y_j)}{1-M_j(y_{j-1})} \right\}^{n+1-j} \alpha_j \frac{m_j(y_j)}{1-M_j(y_{j-1})}, y_1 \leq \dots \leq y_{r+1},$$

here m denotes the density function of distribution.

let $\Gamma M = \{(y_1, \dots, y_{r+1}) : M^1(0+) < y_1 \leq \dots \leq y_{r+1} < M^1(1)\}$, denote sample space for brevity.

IV. EQUAL LOAD SHARING PARAMETERS TESTING

Let us supposed to have S_{k+1} observations of first $r+1$ component-failure time in an $(n_{k+1} - (r+1) + 1)$ -out-of- n_{k+1} system. Here $k+1 \in \{1, \dots, m\}$, failure time describe by iid vectors. $M^{(k+1)}$ = absolute continuous density functions.

Theorem 1. To test the alternative hypothesis, likelihood ratio statistic and Weibull distribution are given, having alternative distributions, in essence, under the alternative hypothesis, the distributions of the two tests doesn't calculate on specific parameters.

Proof: Likelihood ratio test and Weibull distribution calculated or lean on the data by the ratios

$$B_j^{(k+1)} = \sum_{k+1} E_j^{(k+1)}, k+1 \in I, 1 \leq I+1 \leq q_i, 1 \leq j \leq r+1, \text{ for independent statistics- } E_j^{(k)} \approx \Gamma(S_{k_i}, 1/\beta_j^{(k+1)}), 1 < j < r+1, 1 < k+1 < m+1. \text{ It gives that distribution of } B_j^{(k+1)} \text{ is free for } \beta \text{ for } 1 < j < r+1 \text{ and } 1 < k+1 < m+1.$$

As a result of theorem, the exact values for likelihood ratio test and Weibull distribution

Implies to a partial significance level obtained by Monte Carlo method.

For $n = 2$ scheme, sample sizes $S_1 \leq S_2$

$q = \{j \in \{1, \dots, r+1\} : p_j = 1\} \in \{1, \dots, 4\}$ of pairs with matching parameters under alternative hypothesis, then the values are shown in Table 1.

Critical values for likelihood ratio when testing for alternative hypothesis

For $n = 2, q \in \{1, \dots, 4\}, S_j \in \{1, \dots, 10\}, S_2 \in \{1, \dots, 10\}$ and significance level is 5%.

Table 1: Equal load sharing parameter testing.

Sample	1	m	$m+1$
System	$(n_1 - (r+1) + 1)$ -out-of- n_1	$(n_{m+1} - (r+1) + 1)$ -out-of- n_{m+1}
Observed system	S_1	S_{m+1}
Vectors of sequential order	$Y_1^{(1)}, \dots, Y_{S_1}^{(1)}$	$Y_1^{(m+1)}, \dots, Y_{S_{m+1}}^{(m+1)}$
Commulative density fn baseline	$M^{(1)}$	$M^{(m+1)}$
Parameters model	$\beta_1^{(1)}, \dots, \beta_{(n+1)}^{(1)}$	$\beta_1^{(m+1)}, \dots, \beta_{(n+1)}^{(m+1)}$

V. WEIBULL DISTRIBUTION

Weibull distribution has two parameters

1. Probability density function (pdf)
2. Cumulative distribution function (cdf)

Y is the random variable and is said to be Weibull distribution having parameters α and β where ($\alpha > 0, \beta > 0$) is the pdf of x is

$$F(Y; \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta^\alpha} Y^{\alpha-1} e^{-(Y/\beta)^\alpha} & Y \geq 0 \\ 0 & Y < 0 \end{cases}$$

Where $\alpha > 0$ is shape parameter and $\beta > 0$ is scale parameter of the distribution.

VI. ASYMPTOTIC TESTS

Here now address asymptotic result for $n = 2$ and alternative hypothesis

$$S_0: \alpha_i^{(1)} = \alpha_i^{(2)}, f \in F$$

Therefore, for some nonempty index set $F = \{f_1, \dots, f_p\} \approx \{1, \dots, r\}$, in this case here use the formula from reference [10] simplifies to

$$\Lambda = 2 \sum_{f \in F} \left[t \log \left(\frac{t}{t+t'} \cdot \frac{Tf+Tf'}{Tf} \right) + f \log \left(\frac{t'}{t+t'} \cdot \frac{Tf+Tf'}{Tf'} \right) \right] = 2 \sum_{f \in F} \left[t \log \left(\frac{t}{t+t'} \cdot 1 + \frac{1}{Qf} \right) + t' \log \left(\frac{t'}{t+t'} \cdot (1 + Qf) \right) \right] \quad (1)$$

and

$$R = \sum_{f \in F} \left(\frac{1}{t} + \frac{1}{t'} \right) \left(\frac{t' Qf - f}{Qf+1} \right)^2 \quad (2)$$

Where, $Q_f = T_f/T'_f, f \in F$, are independent statistics.

For $p = 1$ and $t = t'$, the likelihood ratio test and the RST are equivalent or equal, since then

$$(1+1/Q_f) (1+Q_f) = (T_f+T'_f)^2 / T_f T'_f$$

$$\text{and } (Q_f - 1/Q_{f+1})^2 = 1 - 4(T_f+T'_f)^2 / T_f T'_f$$

Hence Likelihood ratio test and Rao score test is strictly monotonic functions of statistics $(T_f+T'_f)^2 / (T_f T'_f)$.

So from all assumption, let $t/(t+t') \rightarrow a$ by strong law of large numbers, as $a \in (0, 1)$ then the total sample size increases. Therefore, large numbers,

$$Q_f (a/1-a) \alpha_i / \alpha_i \rightarrow f, \text{ where } f \in F.$$

Table 2: Critical values for likelihood ratio when testing for alternative hypothesis For $n = 2$, $q \in \{1, \dots, 4\}$, $S_1 \in \{1, \dots, 10\}$, $S_2 \in \{1, \dots, 10\}$ and significance level is 5%.

q	$S_1 \backslash S_2$	1	2	3	4	5	6	7	8	9	10
1	1	11.42	11.06	10.93	10.88	10.88	10.87	10.85	10.85	10.85	10.83
	2		10.60	10.67	10.30	10.78	10.65	10.43	10.26	10.26	10.24
	3			10.43	10.24	10.56	10.51	10.33	10.15	10.13	10.11
	4				10.05	10.01	9.98	9.95	9.94	9.91	9.90
	5					9.99	9.97	9.87	9.91	9.88	9.87
	6						9.65	9.59	9.78	9.83	9.75
	7							9.51	9.72	9.79	9.72
	8								9.66	9.72	9.71
	9									9.65	9.68
	10										9.65
2	1	9.43	9.10	9.02	8.98	8.95	8.94	8.93	8.92	8.92	8.92
	2		9.09	8.57	8.52	8.51	8.50	8.49	8.45	8.45	8.44
	3			8.43	8.42	8.41	8.40	8.38	8.35	8.35	8.33
	4				8.36	8.33	8.32	8.31	8.29	8.29	8.28
	5					8.29	8.28	8.27	8.26	8.24	8.23
	6						8.25	8.24	8.23	8.21	8.20
	7							8.21	8.20	8.19	8.17
	8								8.19	8.18	8.14
	9									8.14	8.11
	10										8.10
3	1	7.21	6.98	6.96	6.92	6.88	6.84	6.84	6.84	6.84	6.84
	2		6.85	6.84	6.84	6.81	6.79	6.74	6.73	6.72	6.71
	3			6.81	6.55	6.51	6.50	6.49	6.48	6.44	6.42
	4				6.41	6.40	6.34	6.32	6.30	6.29	6.27
	5					6.38	6.31	6.30	6.29	6.28	6.24
	6						6.30	6.29	6.28	6.27	6.20
	7							6.28	6.27	6.25	6.17
	8								6.22	6.21	6.15
	9									6.19	6.10
	10										6.07
4	1	4.64	4.50	4.45	4.44	4.43	4.43	4.43	4.43	4.42	4.41
	2		4.30	4.28	4.24	4.23	4.23	4.23	4.21	4.20	4.18
	3			4.15	4.14	4.13	4.13	4.13	4.11	4.10	4.09
	4				4.07	4.04	4.04	4.03	4.01	4.00	3.99
	5					4.03	4.03	4.02	4.00	3.98	3.98
	6						4.01	4.00	3.99	3.97	3.94
	7							3.99	3.98	3.94	3.92
	8								3.97	3.91	3.87
	9									3.87	3.82
	10										3.80

Lemma 1. Under alternative hypothesis, $1/t^{1/2}(tQ_1 - t, \dots, tQ_p - t) \xrightarrow{d} N_p(0, I_p/(1-a))$.

Proof: Since Q_1, \dots, Q_p are independent.

So under alternative hypothesis,

$$(tQ_i - t)/t^{1/2} \xrightarrow{d} N(0, I_p/(1-a))$$

$N_p(0, I_p/(1-a))$ for $f \in F$. Let $f \in F$ and $\alpha_f = \alpha_i = \alpha_r$, say.

Therefore, we have

$$f Q_r - t/t^{1/2} = (T_f/t^{1/2} - t^{1/2} T_f/t)/T_f/t$$

$$= ((T_f + t/\alpha_f)/(t^{1/2}/\alpha_f) - (t/t)^{1/2}(T_f/\alpha_f)/(t/\alpha_f))/T_f/t$$

Since $t/f \rightarrow (a/1-a)$, so by strong law of large numbers along with multivariate Slutsky theorem yields the assertion.

Theorem 2. Under alternative hypothesis, likelihood ratio test and Rao score test by given formulas (1) and (2), are chi-square distributed with n -degrees of freedom.

Proof: Since Q_1, \dots, Q_i are independent.

Therefore, it is sufficient to show that under alternate hypothesis, any term of the sum in formulas (1) and (2),

respectively, is asymptotically chi-square distributed with one degree of freedom.

So, it can be claim by application of continuous mapping theorem.

At the end, let S_0 be true and $f \in F$. Moreover,

Let $B = f Q_r - t$ to simplify notation.

Now by using Taylor's theorem, we have

$$2 \log(y) = (y-1)[2 - (y-1)] + 2/3 ((y-1)/y)^3$$

Where y lies in the interval with boundary points 1 and y

So using application to both formula Eqn. (1) yields

$$\begin{aligned} & 2t \log(t/t + f(1+1/Q_i)) + 2f \log(f/t + f(1+Q_i)) \\ &= f B/t + f(2-B/t + f) - tB/Q_i/t + f(2+A/Q_i/t + f) + A \\ &= B/Q_i/t + f(2f - f B/Q_i/t + f - 2t - tB/Q_i/t + f) + A \\ &= B^2/Q_i/t + f(2f Q_i/t + f t/Q_i) + A \end{aligned} \quad (3)$$

Where $A = 2/3 (A/t + f)^3 (1/p_2^3 - 1/p_1^3 Q_i)$,

Where p_1 and p_2 in the interval with boundary points 1 and so on, respectively.

So application of Slutsky's theorem to formula (3), twice, then yields the assertion for likelihood ratio test. Order

restricted statistical inference for scale parameters based on sequential order statistics considered [9].

VII. CONCLUSIONS

In the setup of multiple sample spaces, we check the exact critical values of likelihood ratio-test and Weibull distributions of $k+1$ -out of $-n+1$ systems, finally the derived identical scale parameters and on the another hand, this may be applying to check the equality of parameters. In this paper alternative hypothesis is used to check the equality of parameters in load-sharing systems. The proposed test can also be used to decide whether a meta-analysis of the underlying data is reasonable or not. The corresponding test statistics are

shown to have many alternative distributions, i.e., they each have many distributions under all parameters specified by alternative hypothesis. The performance of statistical procedure as, for instance, the accuracy of estimators, get increased as apply to whole datasheet. In particular, relevant for small sample sizes that are prevalent in reliability. Finally, the derived results from theorem 1 and theorem 2 and lemma might useful for another application of reliability. From this paper it is clear that under alternative hypothesis, likelihood ratio test and Rao score test are asymptotically chi-square distributed with n degrees of freedom.

Table 3: The exact critical values of Weibull distribution for alternative hypothesis.

Q	S_1, S_2	1	2	3	4	5	6	7	8	9	10
1	1	4.75	6.44	8.01	9.07	9.80	10.34	10.75	11.07	11.35	11.55
	2		6.32	7.00	7.71	8.27	9.08	9.09	9.38	9.62	9.81
	3			7.11	7.47	8.16	8.42	8.71	8.71	8.92	9.10
	4				7.60	8.03	8.25	8.43	8.43	8.62	8.75
	5					8.04	8.21	8.33	8.36	8.49	8.61
	6						8.26	8.41	8.36	8.45	8.56
	7							8.43	8.40	8.41	8.52
	8								8.46	8.52	8.52
	9									8.56	8.58
	10										8.65
2	1	3.81	5.36	6.65	7.52	8.07	8.50	8.80	9.04	9.24	9.41
	2		5.14	5.73	6.31	6.76	7.15	7.44	7.67	7.89	8.01
	3			5.82	6.11	6.41	6.68	6.90	7.11	7.27	7.41
	4				6.23	6.40	6.58	6.74	6.91	7.03	7.15
	5					6.52	6.61	6.71	6.96	6.96	7.05
	6						6.70	6.80	6.94	6.96	7.01
	7							6.85	6.92	6.94	7.06
	8								6.97	7.01	7.04
	9									7.05	7.10
	10										7.13
3	1	2.86	4.13	5.12	5.71	6.08	6.35	6.55	6.71	6.81	6.91
	2		3.87	4.78	4.78	5.13	5.38	5.58	5.75	5.86	5.97
	3			4.47	4.67	4.87	5.05	5.20	5.62	5.44	5.54
	4				4.78	4.91	5.04	5.13	5.32	5.29	5.35
	5					5.01	5.10	5.18	5.34	5.29	5.37
	6						5.16	5.23	5.33	5.33	5.40
	7							5.28	5.36	5.33	5.42
	8								5.37	5.40	5.48
	9									5.44	5.43
	10										5.49
4	1	1.81	2.65	3.10	3.33	3.47	3.57	3.68	3.68	3.72	3.77
	2		2.61	2.76	2.85	3.00	3.12	3.27	3.27	3.32	3.37
	3			3.01	3.07	3.11	3.27	3.18	3.18	3.20	3.21
	4				3.15	3.24	3.35	3.31	3.30	3.31	3.33
	5					3.32	3.37	3.40	3.37	3.33	3.41
	6						3.40	3.46	3.45	3.40	3.46
	7							3.48	3.48	3.42	3.50
	8								3.51	3.53	3.53
	9									3.54	3.55
	10										3.57

REFERENCES

[1]. Balakrishnan, N. *et al.*, (2011). Modeling parameters of a load-sharing system through link functions in sequential order statistics models and associated

inference. *IEEE Transactions on Reliability*, **60**(3): 605–611.
 [2]. Sutar, S. S. & Nimbalkar, N. (2014). U.V. Accelerated failure time models for load sharing systems. *IEEE Transactions on Reliability*, **63**(3): 706–714.

- [3]. Bedbur, S. (2010). Test based on sequential order statistic, *J. Stat. Plan. Inference*, **140**: 2520-2530.
- [4]. Bedbur, S. (2019). Testing for equality of parameters from different loading system, *stats, MDP1*, **2**: 70-88
- [5]. Cramer, E. & Kamps, U. (1996). Sequential order statistics and k -out-of- n Systems with sequentially *Ann Inst. Stat. Math.* **48**: 535–549.
- [6]. Balakrishnan, N., Beutner, E. & Kamps, U. (2008). Order restricted inference for sequential k -out-of- n systems. *Journal of Multivariate Analysis*, **99**(7): 1489–1502.
- [7]. Bedbur, S., Beutner, E. & Kamps, U. (2012). Generalized order statistics: an exponential family in model parameters. *Statistics*, **46**: 159–166.
- [8]. Beutner, E. and Kamps, U.(2009). Order restricted statistical inference for scale parameters based on sequential order statistics. *J. Stat. Plan*,**139**(9): 2963-2969.
- [9]. Beutner, E. (2010). Nonparametric comparison of several k -out-of- n systems. In *Advances in Data Analysis, Statistics for Industry and Technology*; Skiadas, C., Ed.; Birkhauser: Boston, MA, USA, 291–304.