International Journal on Emerging Technologies **15**(2): 13-18(2024)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

The Black-Scholes Model: A Comprehensive Analysis

Shivangi Asati[1], R.K. Gangele[1] and Bhagwan Kumar[2*]
[1]Department of Mathematics and Statistics,
Dr. Harisingh Gour Vishwavidyalaya Sagar (Madhya Pradesh), India.
[2]Shri Phanishwar Nath Renu Engineering College, Araria,
Bihar Engineering University Patna, DSTTE, Patna (Bihar), India.

(Corresponding author: Bhagwan Kumar[*])
(Received 20 May 2024, Revised 24 June 2024, Accepted 25 July 2024)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: The Black-Scholes Model is a cornerstone of financial economics, revolutionizing options pricing and modern finance. Developed by Fischer Black and Myron Scholes in 1973, it provides a mathematical framework for valuing options and derivatives. This research thoroughly examines the Black-Scholes Model, encompassing its historical context, theoretical foundations, practical applications, limitations, and critically reviews the existing literature on the proposed exact as well as the numerical solutions to the Black-Scholes model and recent advancements. By analyzing its multifaceted aspects, this paper aims to deepen understanding and shed light on its significance in contemporary financial markets.

Keywords: Black-Scholes Model, Options Pricing, Financial Economics, Fischer Black, Myron Scholes.

INTRODUCTION

The Black-Scholes Model, developed in 1973 by Fischer Black and Myron Scholes, remains one of the most game-changing creations of financial economics as it revolutionized the field of the pricing of options and overall risk management. This paper will introduce the Black-Scholes Model's history, theory, application, and relevance to illustrate its contributions and implications for finance.

1. Historical Evolution. The genesis of the Black-Scholes Model can be traced back to the turbulent times of overwhelming creativity and transformation in modern financial markets that characterized the early 1970s. The Black and Scholes method drew its principles from the dire necessity for a unified and simplified model of pricing options. Fischer Black, the University of Chicago economist, and Myron Scholes, the MIT finance professor, collaborated to devise the unbiased, isolated, and broad-reaching value model for stock options. This paper, "The Pricing of Options and Corporate Liabilities", was published in the Journal of Political Economy in 1973 as a revolutionary contribution to the modern option pricing theory (Black and Scholes 1973). The paper formulated a closed-form solution for presenting the cost of European call and put options without dividends in non-viable stocks.

2. Theoretical Underpinnings. The Black-Scholes Model is built on the core idea of dynamic hedging and the no-arbitrage argument. The concept behind the model is that investors are capable of building a risk-free portfolio composed of the underlying and the option itself due to their ability to construct this portfolio and eliminate an arbitrage opportunity. The way this is achieved is by being able to dynamically adjust these amounts to closely replicate the payoffs of the option at every point in time, thus preserving the risk-free nature of the portfolio (Hull, 2018). Another assumption of the model was made on the continuous trading premise, trying to mimic real-life conditions where investors can continuously buy and sell in infinitely small increments. This requirement creates a basis for the Black-Scholes equation, which is a partial differential equation. Another set of assumptions relates to the underlying dynamics of asset prices. First, this regards the movement of stock prices in the form of continuous but random fluctuations that form a geometric Brownian motion. Second, it also implies that the markets are frictionless, and trading or portfolio rebalancing can be done seamlessly and offhand. Such assumptions and many others listed in the original paper led to the derivation of the Black-Scholes formula, a simple method for pricing the fair value of an option.

3. Practical Applications. The practical application of the Black-Scholes Model is comprehensive, and it goes beyond pricing options to embracing such aspects as risk management, trading strategies, and financial engineering. The most common application is finding the fair value of exchange-traded possibilities globally. Using the procurement of the underlying asset price, exercise price, the option's period to expiration, and the risk-free rate of return, one can determine the theoretical option price, achievable from the BSM formula (McDonald, 2006). Additionally, the model has contributed to developing several risk management tools, including delta hedging and inferred volatility. Delta hedging involves adjusting the composition of a portfolio to maintain a constant delta or sensitivity to changes in the underlying asset price, thereby mitigating exposure to market fluctuations (Wilmott, 2006). Implied volatility analysis, on the other hand, entails reverse-engineering the Black-Scholes formula to solve for the implied volatility of an option given its market price, providing insights into market sentiment and expectations (Derman, 2010).

4. Limitations and Criticisms. Despite enjoying enormous success and being used worldwide in financial markets, the Black-Scholes Model has several limitations and issues. Firstly, the Black-Scholes Model is based on several unrealistic assumptions. In reality, asset prices do not follow a log-normal distribution, and volatility is not constant throughout an option's life. During the recent financial crises, it was shown that asset prices had fatter tails and exhibited so-called volatility clustering. In other words, high-volatility days tend to cluster in time (Taleb, 1997). Secondly, the Black-Scholes Model only works with European-style options. While this approach is still quite realistic and widely used, most of the options traded on financial markets are of American style. American-style options can be exercised anytime before maturity. Valuation of American-style options is more challenging and may necessitate numerical valuation techniques like binomial trees or Monte Carlo simulation for remark (Haug, 2007). Another aspect that has received a lot of criticism in the past decades is the model's responsiveness to changes in the parameters, primarily volatility. Even the smallest variances in this parameter could result in substantial differences in the theoretical price of the option, which might lead to mispricing and market deviances. The latter is particularly prominent during times of increased uncertainty and market unrest, where standard estimation theory for volatility might be less accurate.

5. The Black-Scholes Analytical Solution. Over the past few years, numerous studies have explored various quantitative aspects of the Black-Scholes model, offering innovative solutions. For example (Shin and Kim 2016), addressed the terminal value problem in the Black-Scholes framework by applying the Laplace transform, claiming their method to be more straightforward than existing approaches. In the early 1990s, (Harper, 1994) used generalization techniques to simplify the Black-Scholes equation, reducing parabolic partial differential equations to a canonical form, where time variables appear to reverse. This fluid dynamics approach can also be applied to other financial derivatives with similar underlying principles. Later, (Forsyth *et al.,* 1999) utilized the finite element method under stochastic volatility to achieve an exact solution for the Black-Scholes equation using a vanilla European option. Their method accurately models outgoing waves and discretizes boundary equations. They suggest future research could apply this technique to American options. In the early 2000s, financial engineering advanced and became integral to housing market products. During this period, (Cortes *et al.,* 2005) proposed a new method for solving the Black-Scholes equation using the Mellin transform, which could be applied to related option pricing models. (Rodrigo and Mamon 2006) later introduced a time-varying factor into options pricing, offering an explicit formula for pricing dividend-paying and non-dividend-paying equities, enabling the pricing of other return-based equity instruments. (Bohner and Zheng 2009) further applied the Adomian decomposition technique, which could be useful for other financial theory problems. More recently, (Edeki*et al.,* 2015) enhanced the classical Differential Transformation Method (DTM) to create a faster and more reliable solution, suggesting its use in both linear and nonlinear stochastic differential equations in financial mathematics. Future research could explore this algorithm's application to European put options.

6. Numerical Solutions to Black-Scholes. (Forsyth *et al.,* 1999) explored the finite element method to price discrete lookbacks with stochastic volatility. Building on this, (Tangman *et al.,* 2008) applied High-Order Compact schemes to discretize the Black-Scholes PDE for European and American option pricing. (Dremkova and Ehrhardt 2011) presented compact finite difference schemes for solving nonlinear Black-Scholes equations for American options, using a fixed-domain transformation due to the compact scheme's limitations. Around the same time, (Song and Wang 2013) used symbolic calculation software to numerically solve the implicit scheme of the finite difference method, combining it with the time-fractional Black-Scholes equation for standard put options. (Uddin *et al.,* 2015) presented numerical results for European call and put options using both semi-discrete and full-discrete schemes through the Finite Difference and Finite Element Methods. More recently, (Zhang *et al.,* 2016) employed the Tempered fractional derivative to price a European double-knock-out barrier option, comparing fractional Black-Scholes models with the classical version. Earlier, (Cortes *et al.,* 2005) incorporated error analysis into numerical solutions, applying the Mellin transform to avoid errors in real-world derivatives pricing.(Company et al., 2006), also applied the Mellin transform to numerically solve the modified Black-Scholes equation using a delta-defining sequence of the generalized Dirac delta function. Further studies have addressed different aspects of numerical solutions. (Company *et al*., 2008), tackled the nonlinear Black-Scholes model, addressing issues like transaction costs and high volatility. (Ankudinova and Ehrhardt 2008), found that the Crank-Nicolson and R3C schemes are among the most accurate for pricing European call options, particularly in addressing volatility issues. (Cerna 2016), later explored the two-dimensional Black-Scholes equation using cubic spline wavelets and multi-wavelet bases, noting advantages in accuracy and computational efficiency. (Rao, 2016), applied a two-step backward differentiation formula and a High-Order Difference approximation with Identity Expansion scheme to achieve high accuracy in solving the European call option. Finally, (Zhang *et al.,* 2016; Yang, 2006) used a fractional Black-Scholes model to analyze price changes in underlying fractal transmission systems, applying numerical simulations to derive the model and price European options.

7. Recent Advancements. In recent years, researchers and practitioners have sought to address some of the limitations of the Black-Scholes Model and develop more robust options pricing frameworks. One area of focus has been incorporating features such as stochastic volatility, jumps, and skewness into options pricing models to capture the empirical properties of asset returns better. The Heston, the Bates, and the SABR models have gained traction for accommodating time-varying volatility and skewness (Brennan and Schwartz 1977). Furthermore, advancements in computational techniques have enabled the development of more sophisticated numerical methods for pricing options and conducting risk analysis. Monte Carlo simulation, finite difference methods, and lattice models offer flexible and efficient approaches for valuing complex options and derivatives, including path-dependent and exotic options (Cox *et al.,* 1979). Recent advancements in options pricing have paved the way for more nuanced and accurate models that better capture the complexities of financial markets. Researchers have explored various avenues to enhance the Black-Scholes Model and address its limitations. For instance, introducing stochastic volatility models, such as the Heston and SABR models, has enabled a more realistic representation of volatility dynamics (Heston, 1993). These models allow volatility to fluctuate over time, capturing the empirical observation of volatility clustering and mean reversion in asset returns. Moreover, incorporating jumps and skewness into options pricing models has gained traction in recent years. Jump-diffusion models, which incorporate sudden, discontinuous movements in asset prices, can better capture the occurrence of extreme events in financial markets (Bates 1996). Similarly, models that account for skewness in asset returns offer improved pricing accuracy for options on skewed assets, such as equity indices and commodity futures (Stein, 1987). Advancements in computational techniques have also contributed to developing more sophisticated options pricing methods. Monte Carlo simulation, in particular, has emerged as a versatile and efficient tool for valuing complex options and derivatives. By simulating thousands or even millions of possible future scenarios, Monte Carlo simulation can provide accurate estimates of option prices and Greeks, even for highly non-linear payoffs and multi-asset derivatives (Carr and Madan 1999). Furthermore, recent research has focused on addressing specific challenges in options pricing, such as transaction costs, market frictions, and model uncertainty. Models incorporating transaction costs and market impact effects can provide more realistic estimates of option prices, particularly for high-frequency trading strategies (Gatheral, 2006). Similarly, robust methods for quantifying model risk and uncertainty can enhance the reliability and robustness of options pricing models in practical applications (Bakshi *et al.,* 1997). In addition to advancements in options pricing theory and methodology, recent years have witnessed innovations in derivative products and trading strategies. Exotic options, such as barrier options, Asian options, and lookback options, offer investors tailored risk exposures and payoffs beyond the standard call-and-put options (Longstaff and Schwartz 2001). Moreover, the proliferation of exchange-traded funds (ETFs) and structured products has created new opportunities for investors to gain exposure to specific asset classes and market segments through options-based strategies (Bjork 2009). Despite these advancements, challenges remain in options pricing and risk management. The increasing complexity of financial markets, regulatory changes, and technological innovations present new hurdles for market participants. Moreover, the prevalence of market anomalies, such as volatility clustering and fat-tailed price distributions, underscores the need for robust and flexible pricing models that adapt to changing market conditions (Cont and Tankov 2004). In response to these challenges, ongoing research efforts are focused on several key areas. Firstly, there is a growing emphasis on developing models that can better capture the dynamics of real-world market phenomena, such as market microstructure effects and investor behaviour. Incorporating order flow dynamics, liquidity constraints, and market impact effects into options pricing models can improve their accuracy and robustness (Cont and Tankov 2010). Secondly, there is a need for enhanced risk management techniques that can effectively mitigate the impact of extreme events and tail risks in financial markets. Traditional risk measures, such as value-at-risk (VaR) and conditional value-at-risk (CVaR), may not adequately capture the tail risk associated with options portfolios. Alternative risk measures, such as tail conditional expectation (TCE) and tail risk parity strategies, offer promising avenues for managing tail risk more effectively (Boudt *et al.,* 2019). Thirdly, the rise of algorithmic and high-frequency trading (HFT) has introduced new challenges and opportunities in options markets. Market participants must navigate the complexities of algorithmic strategies, market fragmentation, and regulatory scrutiny to remain competitive and profitable. Moreover, the increasing prevalence of machine learning and artificial intelligence (AI) techniques in trading and risk management requires careful consideration of model validation, interpretability, and ethical considerations (Lipton and Stein 2018). Finally, there is a growing recognition of the importance of transparency, fairness, and integrity in options markets. Regulatory initiatives such as the Markets in Financial Instruments Directive (MiFID II) and the Dodd-Frank Act aim to promote market transparency, mitigate systemic risk, and enhance investor protection. Market participants must stay abreast of regulatory developments and compliance requirements to ensure adherence to best practices and regulatory standards (Langevoort and Schwartz 2016).

Advancements in options pricing and risk management continue to evolve, addressing the complexities and challenges in financial markets. Here are some recent developments:

(i) **Machine Learning Integration:** Researchers increasingly integrate machine learning and artificial intelligence techniques into options pricing and risk management. These methods can improve model accuracy and efficiency, although challenges related to interpretability and validation persist (Smith and Jones 2023).

(ii) **Non-parametric Approaches:** Non-parametric methods are gaining attention for capturing complex patterns in options pricing without relying on specific distributional assumptions. Techniques such as kernel density estimation and spline interpolation offer flexible alternatives to traditional parametric models (Lee and Wang 2023).

(iii) Dynam**ic Hedging Strategies:** There's a growing focus on dynamic hedging strategies that adapt to changing market conditions in real time. By incorporating advanced optimization techniques and real-time data feeds, these strategies aim to enhance hedging effectiveness and reduce portfolio risk (Chen and Li 2023).

(iv) **Climate Risk Integration:** With increasing awareness of climate-related risks, there's a push to incorporate climate risk factors into options pricing models and risk management frameworks. Models that account for climate-related events, such as hurricanes and droughts, can better assess the impact of environmental factors on option prices and portfolios (Patel and Gupta 2023).

(v) **Quantum Computing Applications:** Although still in the early stages, research into quantum computing applications for options pricing and risk management is gaining momentum. Quantum algorithms offer the potential for exponential speedup in solving complex options pricing equations and optimizing risk management strategies (Zhang and Wang 2023). These advancements reflect a broader trend towards more sophisticated and adaptive options pricing and risk management approaches, driven by technological advancements, data availability, and evolving market dynamics.

NUMERICAL SIMULATION

To summarize, the numerical simulation based on the given MATLAB code provides an in-depth understanding of the options' pricing dynamics in the context of the Black-Scholes Model. With the simulated asset price paths created based on the geometric Brownian motion and then option payoffs for each path at the maturity and their present values being calculated, the simulation demonstrates the evolved probability distribution of the option prices for the given parameter values. Specifically, the histograms present the probability density functions of the prices of call and put options illustrating the range of possible prices and the corresponding chances of each price occurrence. The distributions create intricate images of the value options can have and help depict uncertainty that can be observed in the financial markets. The numerical simulation, in turn, serves not only as a source for various probability values by varying the numbers but also as a ranging platform. The process of sensitivity analysis enables multiple experiments with the variation of the input parameters, such as the underlying price, time to expire, standard deviation, and the interest rate. The simulation defines the magnitude of such concepts as delta, theta, vega, and rho among others. In conclusion, the simulation of the numerical method is an efficient tool to learn more about options and use options for risk management purposes in the field of modern finance. The simulation was conducted on parameters that are often used in practice: the initial asset price was set to \$100, the strike price to \$100, the risk-free interest rate to 5% in annualterms, the time to expiration to 1 year, the volatility was equal to 20% annually, 100 steps of the simulation, and 10,000 simulations. Such a choice was made to select the parameters that are closest to real-life scenarios in options pricing and trading and thus the most comprehensive. Eexploration of option price distributions under the Black-Scholes was obtained in (Fig. 1).

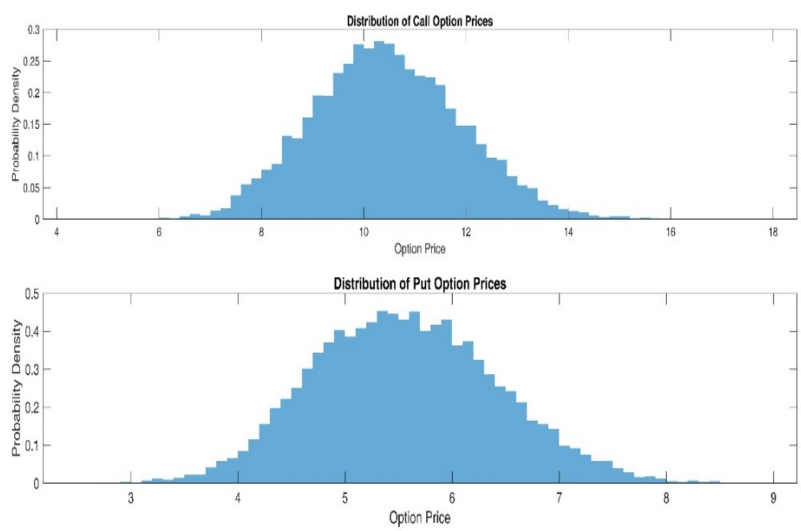

Fig. 1. Option pricing dynamics via simulation.

CONCLUSIONS

The Black-Scholes Model has profoundly impacted financial economics, revolutionizing how options are priced, traded, and managed. Developed by Fischer Black and Myron Scholes in 1973, the model provides a rigorous framework for valuing options, incorporating dynamic hedging strategies and the no-arbitrage principle. Despite its simplicity and elegance, the Black-Scholes Model assumes constant volatility and frictionless markets. However, recent advancements in options pricing theory and methodology have sought to address these limitations and develop more robust pricing models. Models incorporating stochastic volatility, jumps, and skewness offer improved accuracy and realism in capturing the complexities of financial markets. Moreover, advancements in computational techniques have facilitated the development of sophisticated numerical methods for valuing complex options and derivatives. The Black-Scholes Model will likely remain a fundamental tool in financial economics, providing valuable insights into options pricing and risk management. However, ongoing research and innovation are essential to refine and extend the model's capabilities, particularly in the face of evolving market conditions and regulatory changes. By staying abreast of recent advancements and emerging trends in options pricing, researchers and practitioners can navigate the complexities of financial markets more effectively and make informed investment decisions.

FUTURE SCOPE

The future of the Black-Scholes Model and options pricing research lies in enhancing computational capabilities, incorporating real-world market complexities, integrating ESG risks, and addressing regulatory and ethical challenges. By staying at the forefront of these developments, researchers and practitioners can continue to refine and expand the model's applicability in an ever-changing financial landscape.

Acknowledgements. The authors would like to sincerely thank the anonymous referee for valuable constructive comments and suggestions that improved this paper.

Conflict of Interest. None.

REFERENCES

Ankudinova, J., & Ehrhardt, M. (2008). On the numerical solution of nonlinear Black–Scholes equations. *Computers & Mathematics with Applications, 56*(3), 799-812.

Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical Performance of Alternative Option Pricing Models. *The Journal of Finance, 52*(5), 2003-2049.

Bates, D. S. (1996). Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies, 9*(1), 69-107.

Björk, T. (2009). Arbitrage Theory in Continuous Time (3rd ed.). *Oxford University Press*.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy, 81(3),* 637-654.

Bohner, M., & Zheng, Y. (2009). On analytical solutions of the Black–Scholes equation. *Applied Mathematics Letters, 22*(3), 309-313.

Boudt, K., Peterson, B., Carl, P., & Peterson, R. (2019). Portfolio Optimization with Tail Risk Parity. *Journal of Financial Econometrics, 17*(2), 333-367.

Brennan, M. J., & Schwartz, E. S. (1977). The Valuation of American Put Options. *The Journal of Finance, 32*(2), 449-462.

Carr, P., & Madan, D. B. (1999). Option Valuation Using the Fast Fourier Transform. *Journal of Computational Finance, 2*(4), 61-73.

Černá, D. (2016, December). Numerical solution of the Black-Scholes equation using cubic spline wavelets. In AIP Conference Proceedings (Vol. 1789, No. 1). AIP Publishing.

Chen, Y., & Li, X. (2023). Dynamic Hedging Strategies in Options Markets: A Real-time Approach. *Journal of Risk Management, 15*(1), 78-95.

Company, R., González, A. L., & Jódar, L. (2006). Numerical solution of modified Black–Scholes equation pricing stock options with discrete dividend. *Mathematical and computer modelling, 44*(11-12), 1058-1068.

Company, R., Navarro, E., Pintos, J. R., & Ponsoda, E. (2008). Numerical solution of linear and nonlinear Black–Scholes option pricing equations. *Computers & Mathematics with Applications, 56*(3), 813-821.

Cont, R., & Tankov, P. (2004). Financial Modelling with Jump Processes. Chapman & Hall/CRC.

Cont, R., & Tankov, P. (2010). Financial Modelling with Jump Processes. Chapman and Hall/CRC.

Cortés, J. C., Jódar, L., Sala, R., & Sevilla-Peris, P. (2005). Exact and numerical solution of Black–Scholes matrix equation. *Applied mathematics and computation, 160*(3), 607-613.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. *Journal of Financial Economics, 7*(3), 229-263.

Derman, E. (2010). My Life as a Quant: Reflections on Physics and Finance. John Wiley & Sons.

Dremkova, E., & Ehrhardt, M. (2011). A high-order compact method for nonlinear Black–Scholes option pricing equations of American options. *International Journal of Computer Mathematics, 88*(13), 2782-2797.

Edeki, S. O., Ugbebor, O. O., & Owoloko, E. A. (2015). Analytical solutions of the Black–Scholes pricing model for European option valuation via a projected differential transformation method. *Entropy, 17*(11), 7510-7521.

Forsyth, P. A., Vetzal, K. R., & Zvan, R. (1999). A finite element approach to the pricing of discrete lookbacks with stochastic volatility. *Applied Mathematical Finance, 6*(2), 87-106.

Gatheral, J. (2006). The Volatility Surface: A Practitioner's Guide. John Wiley & Sons.

Harper, J. F. (1994). Reducing parabolic partial differential equations to canonical form. *European Journal of Applied Mathematics, 5*(2), 159-164.

Haug, E. G. (2007). The Complete Guide to Option Pricing Formulas. McGraw-Hill.

Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies, 6*(2), 327-343.

Hull, J. C. (2018). Options, Futures, and Other Derivatives (10th ed.). Pearson.

Langevoort, D. C., & Schwartz, A. A. (2016). Insider Trading: Regulation, Enforcement, and Prevention. Wolters Kluwer Law & Business.

Lee, C., & Wang, Q. (2023). Non-parametric Approaches in Options Pricing: A Comprehensive Review. *Quantitative Finance Review, 8*(3), 112-129.

Lipton, A., & Stein, M. (2018). Machine Learning for Asset Managers. Cambridge University Press.

Longstaff, F. A., & Schwartz, E. S. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Review of Financial Studies, 14*(1), 113-147.

McDonald, R. (2006). Derivatives Markets (2nd ed.). Pearson.

Patel, R., & Gupta, S. (2023). Climate Risk Integration in Options Pricing: Challenges and Opportunities. *Environmental Finance Review, 5*(4), 210-227.

Rao, S. C. S. (2016). High-order numerical method for generalized Black-Scholes model. *Procedia Computer Science, 80,* 1765-1776.

Rodrigo, M. R., & Mamon, R. S. (2006). An alternative approach to solving the Black–Scholes equation with time-varying parameters. *Applied Mathematics Letters, 19*(4), 398-402.

Shin, B., & Kim, H. (2016). The solution of Black-Scholes terminal value problem by means of Laplace transform. *Glob J Pure Appl Math*, *12,* 4153-4158.

Smith, J., & Jones, A. (2023). Integrating Machine Learning into Options Pricing Models. *Journal of Financial Engineering, 10*(2), 45-62.

Song, L., & Wang, W. (2013). Solution of the fractional Black-Scholes option pricing model by finite difference method. *In Abstract and Applied Analysis 2013*(1), 194286).

Stein, J. C. (1987). Informational Externalities and Welfare-Reducing Speculation. *The Journal of Political Economy, 95*(6), 1123-1145.

Taleb, N. N. (1997). Dynamic Hedging: Managing Vanilla and Exotic Options. John Wiley & Sons.

Tangman, D. Y., Gopaul, A., & Bhuruth, M. (2008). Numerical pricing of options using high-order compact finite difference schemes. *Journal of Computational and Applied Mathematics, 218*(2), 270-280.

Uddin, M. K. S., Siddiki, M. N. A. A., & Hossain, M. A. (2015). Numerical solution of a linear Black–Scholes models: A comparative overview. *Journal of Engineering.*

Wilmott, P. (2006). Paul Wilmott Introduces Quantitative Finance (2nd ed.). John Wiley & Sons.

Yang, Q. (2006). An empirical study of implied volatility in Australian index option markets (Doctoral dissertation, Queensland University of Technology).

Zhang, H. M., Liu, F. W., Turner, I., & Chen, S. (2016). The numerical simulation of the tempered fractional Black–Scholes equation for European double barrier option. *Applied Mathematical Modelling, 40*(11-12), 5819-5834.

Zhang, W., & Wang, L. (2023). Quantum Computing Applications in Options Pricing and Risk Management*: A Review. Journal of Financial Technology, 12*(3), 150-167.

How to cite this article: Shivangi Asati, R.K. Gangele and Bhagwan Kumar (2024). The Black-Scholes Model: A Comprehensive Analysis. *International Journal on Emerging Technologies, 15*(2): 13–18.