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Some Fixed Point and Common Fixed Point Results in L-Space with Rational Contraction

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ABSTRACT : There are several Theorems are prove in L-space, using various type of mappings. In this paper, we prove some fixed point theorem and common fixed point. Theorems, in L-space using different, symmetric rational mappings.

Keywords: Fixed point, Commmon Fixed point, L-space, Continuous Mapping, Self Mapping, Weakly Compatible Mappings. 2000 Mathematics Subject Classification: 47H10, 54H25.

I. INTRODUCTION

It was shown by S. Kasahara [13] in 1976, that several known generalization of the Banach Contraction Theorem can be derived easily from a Fixed Point Theorem in an L-space. Iseki [10] has used the fundamental idea of Kasahara to investigate the generalization of some known Fixed Point Theorem in L-space.

Let N be the set of natural numbers and X be a nonempty set. Then L-space is defined to be the pair $(X, \to)$ of the set X and *a* subset $\to$ of the set $X^N \times X$, satisfying the following conditions:

$L_1 - if\ x_n = \ x \in X$ for all $n \in N$, then $(\{x_n\}_{n\in N}, x) \in \to$

$L_2 - if\,(\{x_n)_{n\in N}, x) \in \to$, then $(\{x_{n_i}\}_{i\in N}$

For every subsequence $\{x_{n_i}\}_{i\in N}$ of $\{x_n)_{n\in N}$

In what follows instead of writing $(\{x_n)_{n\in N}, x) \in \to$, we write $\{x_n)_{n\in N} \to x$ or $x_n \to x$ and read $\{x_n\}_{n\in N}$ converges to x. Further we give some definitions regarding L-space.

Definition 1. Let $(X, \to)$ be an L-space. It is said to be 'separated' if each sequence in x converges to at most one point of X.

Definition 2. A mapping f on $(X, \to)$ into an L-space $(X', \to')$is said to be 'continuous' if $x_n \to x$ implies $f(x_n) \to' f(x)$ for some subsequence $\{x_n)_{i\in N}$ for $\{x_n)_{n\in N}$.

Definition 3. Let d- be a non negative extended real valued function on $X \times X$: $0 \le d(x, y) \le \infty_i$ for all $x, y \in X$. The L-space is said to be d- complete if each sequence $\{x_n\}_{n\in N}$ in X with $\sum_{i=0}^{\infty} d(x_i, x_{i+1}) < \infty$ converges to the atmost one point of X.

In this context Kasahara S. proved a lemma, which as follows:

Lemma (S. Kasahara):

Let $(X, \to)$ be an L-space which is d- complete for a non negative real valued function d on $X \times X$. If $(X, \to)$ is separated then:

$d(x, y) = d(y, x) = 0$ implies, $x = y$ for all $x, y \in X$

During the past few years many great mathematicians Yeh [19], Singh [18], Pathak and Dubey [14], Sharma and Agrawa [17], Patel, Sahu and Sao [15], Patel and Patel [16], worked for L-space. In this chapter, we similar investigation for the study of Fixed Point Theorems in L-space are worked out. We find some more Fixed Point Theorem and Common Fixed Point Theorem in L-sapce.

Theorem 1

Let $(X, \to)$ be a separated L-space, which is d- complete for a non negative real valued function d on $X \times X$ with $d(x, x) = 0$, for each x in X. Let E, F and T be three continuous self mapping of X into itself, satisfying the following condition:

$1c_1:\ E(X) \subset T(X)$

and $F(X) \subset T(X)$, ET - TE, $FT = TF$

$$1c_2:\ d(Ex, Fy) \le \alpha \left[\frac{d(Tx,Ty)\{d(Tx,Ex) + d(Ty,Fy)\}}{d(Tx,Fy) + d(Ty,Ex)} \right]$$

$$+\beta[d(Tx,Ex) + d(Ty,Fy)]$$

$$+\gamma[d(Tx,Fy) + d(Ty,Ex)] + \delta.d(Tx,Ty)$$

For all x, y in X, where non negative α, β, γ, δ such that $0 \le \alpha + \beta + \gamma + \delta < 1$, with $Tx \ne Ty$. Then E, F, T have unique common fixed point.

Proof:

Let $x_0 \in X$, since $E(X) \subset T(X)$ we can choose a point $x_1 \in X$, such that $Tx_1 = Ex_0$, also $F(X) \subset T(X)$, we can choose $x_2 \in X$ such that In general we can choose the point:

$$Tx_{2n+1} = Ex_{2n} \quad \text{... (1.1)}$$

$$Tx_{2n+2} = Fx_{2n+2} \quad \text{... (1.2)}$$

Now consider,

$$d(Tx_{2n+1}, Tx_{2n+2}) = d(Ex_{2n}, Fx_{2n+1})$$

From $1c_2$

$$d(Tx_{2n}, Fx_{2n+1}) \le \alpha\left[\frac{d(Tx_{2n+1})\{d(Tx_{2n}, Ex_{2n}) + d(Tx_{2n+1}, Fx_{2n+1})\}}{d(Tx_{2n}, Fx_{2n+1}) + d(Tx_{2n+1}, Ex_{2n})}\right]$$
$$+\beta[d(Tx_{2n}, Ex_{2n}) + d(Tx_{2n+1}, Fx_{2n+1})]$$
$$+\gamma[d(Tx_{2n}, Fx_{2n+1}) + d(Tx_{2n+1}, Ex_{2n})]$$
$$+\delta.d(Tx_{2n}, Tx_{2n+1})$$

$$d(Tx_{2n+1}, Tx_{2n+2}) \le \alpha\left[\frac{d(Tx_{2n}, Tx_{2n+1})\{d(Tx_{2n}, Tx_{2n+1}) + d(Tx_{2n+1}, Tx_{2n+2})\}}{d(Tx_{2n}, Tx_{2n+2}) + d(Tx_{2n+1}, Tx_{2n})}\right]$$
$$+\beta[d(Tx_{2n}, Tx_{2n+1}) + d(Tx_{2n+1}, Tx_{2n+2})]$$
$$+\gamma[d(Tx_{2n}, Tx_{2n+2}) + d(Tx_{2n+1}, Tx_{2n+1})]$$
$$+\delta.d(Tx_{2n}, Tx_{2n+1})$$

$$d(Tx_{2n+1}, Tx_{2n+2}) \le \left[\frac{\alpha+\beta+\gamma+\delta}{1-\beta-\gamma}\right].d(Tx_{2n}, Tx_{2n+1})$$

$$d(Tx_{2n+1}, Tx_{2n+2}) \le q.d(Tx_{2n}, Tx_{2n+1})$$

where $q = \left[\frac{\alpha+\beta+\gamma+\delta}{1-\beta-\gamma}\right] < 1;$

For n = 1, 2, 3, ...

Whether, $d(Tx_{2n+1}, Tx_{2n+2}) = 0$ or not

Similarly, we have

$$d(Tx_{2n+1}, Tx_{2n+2}) \le q^n.d(Tx_0, Tx_1)$$

For every positive integer n, this means that,

$$\sum_{i=0}^{\infty} d(Tx_{2i+1}, Tx_{2i+2}) < \infty$$

Thus the *d*- completeness of the space implies that, the sequence $(T^n x_0)_{n\in N}$ converges to some u in . so by (1.1) and (1.2):

$\{E^n x_0\}_{n\in N}$ and $\{F^n x_0\}_{n\in N}$ also converges to the some point u, respectively.

Since E, F, T are continuous, there is a subsequence t of $\{T^n x_0\}_{n\in N}$ such that:

$$E[T(t)] \to E(u),\ T[E(t)] \to T(u),\ F[T(t)] \to F(u),$$
$$T[F(t)] \to T(u)$$

By $(1c_1)$ we have, $E(u) = F(u) = T(u)$ ... (1.3)

Thus, we can write

$T(Tu) = T(Eu) = E(Tu) = E(Eu) = E(Fu) = T(Fu) = F(Tu) = F(Eu) = F(Fu)$ ... (1.4)

By $1c_2$, (1.3) and (1.4) we have, if $E(u) \ne F(Eu)$

$$d[Eu, F(Eu)] \le \alpha\left[\frac{d[Tu, T(Eu)][d(Tu, Eu) + d\{T(Eu), F(Eu)\}]}{d[Tu, F(Eu)] + d[T(Eu), Eu]}\right]$$
$$+\beta[d(Tu, Eu) + d\{T(Eu), F(Eu)\}]$$
$$+\gamma[d\{Tu, F(Eu)\} + d\{T(Eu), Eu\}]$$
$$+\delta.d[Tu, T(Eu)]$$

$$d[Eu, F(Eu) \le (\beta+\gamma+\delta), d[Eu, F(Eu)]$$

Thus we get a contradiction,

Hence $Eu = F(Eu)$ ... (1.5)

From (1.4) and (1.5) we have

$$Eu = F(Eu) = T(Eu) = E(Eu)$$

Hence Eu is a common fixed point of E, F and T.

Uniqueness:

Let v is another fixed point of E, F and T different from u, then by $1c_2$ we have:

$$d(u, v) = d(Eu, Fv)$$

$$d(Eu, Fv) \le \alpha\left[\frac{d(Tu, Tv)\{d(Tu, Eu) + d(Tv, Fv)\}}{d(Tu, Fv) + d(Tv, Eu)}\right]$$
$$+\beta[d(Tu, Eu) + d(Tv, Fv)]$$
$$+\gamma[d(Tu, Fv) + d(Tv, Eu)]$$
$$+\delta.d(Tu, Tv)$$
$$d(u, v) \le (2\gamma+\delta).d(u, v)$$

Which contradiction.

Therefore u is unique fixed point of E, F and T in X.

Remark:

I. If we put $\alpha = \beta = \gamma = 0$ then we get result of Jungck [11] in Lspace.

II. If we put $\alpha = \gamma = \delta = 0$ then we get the result of Kannan [12] in L-space.

III. If we put $\alpha = \gamma = 0$ then we get the result of Chatterjee [5] in L-space.

Theorem 2

Let $(X, \to)$ be a separated L-space, which is *d*- complete for a non Negative real valued function d on $X \times X$ with $d(x, x) = 0$, for each x in X. Let E, F and T be three continuous self mapping of X into itselt, satisfying the following condition:

$2c_1$: $E(X) \subset T(X)$ and $F(X) \subset T(X)$

$ET = TE,\ FT = TF$

$2c_2$: $$d(E^r x, F^s y) \le \alpha\left[\frac{d(Tx, Ty)[d(TxE^r x) + d(TyFy)]}{d(TxF^s y) + d(TyE^r x)}\right]$$

$$+\beta[d(Tx,E^r x)+d(Ty,F^s y)]$$
$$+\gamma[d(Tx,F^s y)+d(Ty,E^r x)]$$
$$+\delta.d(Tx,Ty)$$

For all x, y in X, where non negative α, β, γ, δ such that $0 \le \alpha + \beta + \gamma + \delta < 1$ with $Tx \neq Ty$. If some positive integers r, s exists such that E^r, F^s and T are continuous. Then E, F, T have unique common fixed point.

Proof:

We have

$$E(X) \subset T(X) \text{ and } F(X) \subset T(X)$$
$$ET = TE,\ FT = TF$$

It follows that:

$$E^r(X) \subset T(X) \text{ and } F^s(X) \subset T(X)$$
$$E^rT = TE^r,\ F^sT = TF^s$$

By theorem (1) , there is a unique fixed point in X such that,

$$u = Tu = E^r u = F^s u \qquad \text{... (2.1)}$$

i.e u is the unique fixed point of T, E^r and F^s

Now $T(Eu) = E(Tu) = Eu = E(E^r u) = E^r(EU)$ ... (2.2)

And $T(Fu) = F(Tu) = Fu = F(F^s u) = F^s(Fu)$ ... (2.3)

Hence it follows that Eu is a common fixed point of E^r and T, similarly is Fu a common fixed point of T and F^s in X. The uniqueness of u from (2.1), (2.2) and (2.3) implies that:

$$u = Eu = Fu = Tu$$

This complete the proof of the theorem.

Remark:

(i) If $r = s = 1$ then we get Theorem 1.

Theorem 3

Let $(X, \rightarrow)$ be a separated L-space, which is d- complete for a non negative real valued function d on $X \times X$ with $d(x, x) = 0$, for each x in X. Let A, B, S and T be continuous self mapping of X into itself, satisfying the following condition:

$3c_1$: $A(X) \subseteq T(X)$ and $B(X) \subseteq T(X) As = SA$

$BT = TB$ and $T(X)$ or $S(X)$ are closed sub set of X

$3c_2$: $d(Ax, By) \le \alpha\, d(Sx, Ty) + \beta_{max}[d(Sx, Ax),$
$d(Ty, By), d(Sx, By), d(Ty, Ax)]$

For all x, y in X, where non negative such that $0 \le \alpha + \beta < 1$, then A, B, S, T have unique common fixed point in X.

Proof:

Let x_0 be an arbitrary point of X, since $A(X) \subseteq T(X)$ we can choose the point x_1 and y_0 in X such that,

$$Ax_0 = Tx_1 = y_0$$

Also $B(X) \subseteq S(X)$, we can choose the point x_2 and y_1 in X such that,

$$Bx_1 = Sx_2 = y_1$$

In general we can choose the points

$$Tx_{2n+1} = Ax_{2n} = y_{2n} \qquad \text{... (3.1)}$$

And $$Sx_{2n+2} = B_{2n+1} = y_{2n+1} \qquad \text{... (3.2)}$$

For all $n = 0, 1, 2, \ldots \ldots \ldots \ldots \ldots$

Now consider,

$$d(y_{2n}, y_{2n+1}) = d(Ax_{2n}, Bx_{2n+1})$$

From $3c_2$:

$$d(Ax_{2n}, Bx_{2n+1}) \le \alpha\, d(Sx_{2n}, Tx_{2n+1}) +$$
$$\beta_{\max}\begin{bmatrix} d(Sx_{2n}, Ax_{2n}), d(Tx_{2n+1}, Bx_{2n+1}) \\ d(Sx_{2n}, Bx_{2n+1}), d(Tx_{2n+1}, Ax_{2n}) \end{bmatrix}$$
$$d(y_{2n}, y_{2n+1}) \le \alpha\, d(y_{2n-1}, y_{2n}) +$$
$$\beta_{\max}\begin{bmatrix} d(y_{2n}, y_{2n-1}) \\ d(y_{2n+1}, y_{2n-1}), d(y_{2n}, y_{2n+1}) \end{bmatrix} \qquad \text{... (3.3)}$$

There arise three cases,

Case 1

If we take max is $d(y_{2n-1}, y_{2n})$, then (3.3) gives,

$$d(y_{2n+1}, y_{2n}) \le (\alpha + \beta) d(y_{2n-1}, y_{2n})$$

Case 2

If we take max is $d(y_{2n+1}, y_{2n})$, then (3.3) gives

$$d(y_{2n+1}, y_{2n}) \le \frac{\alpha}{1-\beta} d(y_{2n-1}, y_{2n})$$

Case 3

If we take max is $d(y_{2n+1}, y_{2n-1})$, then (3.3) gives

$$d(y_{2n+1}, y_{2n}) \le \frac{\alpha+\beta}{1-\beta} d(y_{2n-1}, y_{2n})$$

From the above Cases 1, 2, 3, we observe that,

$$d(y_{2n+1}, y_{2n}) \le q d(y_{2n-1}, y_{2n})$$

where $$q = \max\left[(\alpha+\beta), \frac{\alpha}{1-\beta}, \frac{\alpha+\beta}{1-\beta}\right] < 1$$

For $n = 1, 2, 3, \ldots \ldots \ldots$

Similarly we have,

$$d(y_{2n+1}, y_{2n}) \le q^n d(y_0, y_1)$$

For every positive integer n, this means that,

$$\sum_{i=0}^{\infty} d(y_{2i+1}, y_{2i}) < \infty$$

Thus the completeness of the space implies that the sequence $\{y_n\}_{n \in N}$ converges to the some u in X so by

(3.1) and (3.2) the sequence $\{A^n x_0)$, $\{B^n x_0\}$, $\{S^n x_0\}$, $\{T^n x_0\}$ also converges to the some points u respectively:

Since A, B, S, T are continuous, this implies

$$Tx_{2n+1} = Ax_{2n} = y_{2n} \to u \text{ as } n \to \infty$$

$$Sx_{2n+2} = B_{2n+1} = y_{2n+1} \to u \text{ as } n \to \infty$$

The pair (A, S) and (B, T) are weakly compatible which gives that, U is a common fixed point of A, B, S and T.

Uniqueness:

Let us assume that w is another fixed point of A, B, S, T in X different form u, *i.e.* $u \neq w$ then

$$Tu = Au = u \text{ and } Sw = Bw = w$$

From $3c_2$ we have,

$$d(u, w) < (\alpha + \beta)\, d(u, w)$$

Which contradiction.

Hence u is a unique common fixed point of A, B, S, T in X.

This complete the proof of the theorem.

Theorem 4

Let $(X, \to)$ be a separated L-space, which is d- complete for a non negative real valued function d on $X \times X$ with $d(x, x) = 0$, for each x in X.

Let E, F and T be three continuous self mapping of X into itself, satisfying the following condition:

$4c_1$: $E(X) \subset T(X)$ and $F(X) \subset T(X)$

$$ET = TE,\ FT = TF$$

$4c_2$:
$$\{d(Ex, Fy)\}^2 \leq \alpha d(Tx, Ex) d(Ty, Fy)$$
$$+\beta d(Tx, Fy) d(Ty, Ex)$$
$$+\gamma d(Tx, Ex) d(Ex, Ty) + \delta d(Tx, Ty) d(Ty, Fy)$$

For all x, y in X, where non negative α, β, γ, δ such that $0 \leq \alpha + \beta + \gamma + \delta < 1$, with $Tx \neq Ty$ then E, F, T have unique common fixed point.

Proof:

Let $x_0 \in X$, since $E(X) \subset T(X)$ we can choose a point $x_1 \in X$, such that $Tx_1 = Ex_0$, also $F(X) \subset T(X)$, we can choose $x_2 \in X$ such that $Tx_2 = Fx_1$.

In general we can choose the point:

$$Tx_{2n+1} = Ex_{2n} \qquad \text{... (4.1)}$$

$$Tx_{2n+2} = Fx_{2n+1} \qquad \text{... (4.2)}$$

For every $n \in N$, we have

$$[d(Tx_{2n+1}, Tx_{2n+2})]^2 = [d(Ex_{2n}, Fx_{2n+1})]^2$$

From $5c_2$

$$[d(Ex_{2n}, Fx_{2n+1})]^2 \leq \alpha d(Tx_{2n}, Ex_{2n}) d(Tx_{2n+1}, Fx_{2n+1})$$
$$+\beta(d(Tx_{2n}, Fx_{2n+1}) d(Tx_{2n+1}, Ex_{2n})$$
$$+\gamma d(Tx_{2n}, Ex_{2n}) d(Ex_{2n}, Tx_{2n+1})$$
$$+\delta d(Tx_{2n}, Tx_{2n+1}) d(Tx_{2n+1}, Fx_{2n+1})$$

$$[d(Tx_{2n+1}, Tx_{2n+2})]^2 \leq \alpha d(Tx_{2n}, Tx_{2n+1}) d(Tx_{2n+1}, Tx_{2n+2})$$
$$+\beta d(Tx_{2n}, Tx_{2n+2}) d(Tx_{2n+1}, Tx_{2n+1})$$
$$+\gamma d(Tx_{2n}, Tx_{2n+1}) d(Tx_{2n+1}, Tx_{2n+1})$$
$$+\delta d(Tx_{2n}, Tx_{2n+1}) d(Tx_{2n+1}, Tx_{2n+2})$$
$$d(Tx_{2n+1}, Tx_{2n+2}) \leq (\alpha + \delta) d(Tx_{2n}, Tx_{2n+1})$$

For $n = 1, 2, 3, \ldots \ldots \ldots$

Whether $d(Tx_{2n+1}, Tx_{2n+2}) = 0$ or not

Similarly, we have

$$d(Tx_{2n+1}, Tx_{2n+2}) \leq (\alpha + \delta)^n d(Tx_0, Tx_1)$$

For every positive integer n, this means that,

$$\sum_{i=0}^{\infty} d(Tx_{2i+1}, Tx_{2i+2}) < \infty$$

Thus the d- completeness of the space implies that, the sequence $\{T^n x_0\}_{n \in N}$ converges to some u by (4.1) and (4.2):

$\{E^n x_0\}_{n \in N}$ and $\{F^n x_0\}_{n \in N}$ also converges to the some point respectively.

Since E, F, T are continuous, there is a subsequence t of $\{T^n x_0\}_{n \in N}$ such that,

$$E[T(t)] \to E(u), T[E(t)] \to T(u)$$

$$F[T(t)] \to F(u), T[F(t)] \to T(u)$$

By $(4c_1)$ we have,

$$E(u) = F(u) = T(u) \qquad \text{... (4.3)}$$

Thus,

$$T(Tu) = T(Eu) = E(Tu) = E(Eu) = E(Fu)$$
$$= T(Fu) = F(Tu) = F(Eu) = F(Fu) \qquad \text{... (4.4)}$$

By $4c_2$, (4.3) and (4.4) we have,

$$E(u) \neq F(Eu)$$

$$[d\{Eu, F(Eu)\}]^2 \leq \alpha d(Tu, Eu) d[\{T(Eu), F(Eu)\}]$$
$$+\beta d[Tu, F(Eu)] d[T(Eu), Eu]$$
$$+\gamma d(Tu, Eu) d[Eu, T(Eu)]$$
$$+\delta d[Tu, T(Eu)] d[Tu, F(Eu)]$$
$$d[Eu, F(Eu)] \leq 0$$

Thus we get a contradiction.

Hence $Eu = F(Eu)$ ... (4.5)

From (4.4) and (4.5) we have

$$Eu = F(Eu) = T(Eu) = E(Eu)$$

Hence Eu is a common fixed point of E, F and T.

Uniqueness:

Let v is another fixed point of E, F and T different from then by $1c_2$ we have,

$$[d(u,v)]^2 = [d(Eu,Fv)]^2$$

$$[d(Eu,Fv)]^2 \leq \alpha d(Tu,Eu)d(Tv,Fv)$$

$$+\beta d(Tu,Fv)d(Tv,Eu)$$

$$+\gamma d(Tu,Eu)d(Eu,Tv)$$

$$+\delta(Tu,Tv)d(Tv,Fv)$$

$$d(u,v) \leq \beta d(u,v)$$

Which contradiction.

Therefore u is unique fixed point of E, F and T in X.

Remarks:

(*i*) If we put $\alpha = \gamma = \delta = 0$ and $E = F$ then we get the result of Jungek [11].

(*ii*) If we put $\gamma = \delta = 0$ then we get the result of Pathak and Dubey [14].

(*iii*) If we put $\gamma = \delta = 0$ and $E = F$ then we get the result of Yeh [19].

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