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Optimization of Multi-objective Time-cost Trade off Problem with Various Risk Zones

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ABSTRACT : We have studied a problem for optimization of Multi-objective time-cost trade-off problem with various risk zones. Fuzzy logic theory is employed for economy of time and cost. Due to different uncertainties actual cost and time for a particular risk zone is not certainly known to service manager in advance. Therefore each project will differ in total time and cost because of uncertainties project manager can have different non-dominated solutions for his measure of accepted risks. This work can be employed in decision making to select the desired pareto front solution for different-cuts.

Keywords : Fuzzy logic, risk zones, Time-cost tradeoff operational risk Management (ORM), Non-dominated sorting genetic algorithm (NSGA), Genetic algorithm (GA).

I. INTRODUCTION

Time-cost trade off problem is one of the highly important issues in project accomplishment and has been ever taken into consideration by project managers. Time and cost as two critical objectives of construction project management, are not independent but intricately related. Time-cost optimization may be defined as a process to identify suitable construction activities for speeding up, and for deciding "by how much" so as to attain the best possible saving in both time and cost since there is a hidden trade off relationship between project time and cost, it might be difficult to predict whether the total cost (i.e. the direct and Indirect cost) would increase or decrease as a result of the schedule compression.

All projects have risks and uncertainties. In some cases, fro example in most research and development project the effect of such risk and uncertainties can be very significant. However many managers still did not employ project risk management process for their projects. In many cases they don't believe, that establishing and implementation of such process will be beneficial, since it is difficult to predict all potential risks and their affect of the project since different combinations of possible durations and costs at various risks can be associated with a project, the problem is which of these combinations are the best. Determining the best sets is the goal of time-cost optimization.

II. A FRAMEWORK OF THE EXISTING TECHNIQUE

Recent advances in decision support tools for product development and management make optimization for cost, time and to various risk zones. Mathematical programming methods convert the time cost trade off to mathematical models and utilize liners programming [5]. A wide variety of heuristics procedure were used to solve the time-cost trade off problem [2, 6, 8].

The successful development and application of meta-heuristic optimization algorithm for solving single objective optimization problems in recent years has attracted the attention of researches to investigate the possibility of their application to solve multi-objective optimization problems.

There exist numerous difficulties and complexities in applying meta heuristic algorithms to solve multi-objective problems and several researchers have engaged in appropriate use of these methods during past 2 decades. In this regards, different versions of Gas have successfully been applied to optimize time cost problem [1, 3, 4, 9].

Give the existence of asymmetric information in the real world and the importance of the form of risks for the selection of risk zones, it is certain that there are different risk strategies and levels of formality among which one can usefully differentiate. How ever, a total time-cost and risk of project may differ significantly because of these uncertainties.

In this paper, a new approach has been investigated in solving time-cost trade-off problem. Fuzzy logic theory is employ to consider effecting uncertainties in time, total direct and indirect cost of a construction project. To obtain appropriate solutions, genetic algorithm has been employed as an optimizer where uncertainties are considered through fuzzy logic representation project manager can also have different non-dominated solutions or pareto solutions which are dependent on his measure of accepted risk through applying cuts methods in fuzzy logic theory. A case-study through which considerable conclusions is drawn is finally investigated.

III. MAIN ELEMENTS OF THE BASIC CONCEPUAL FRAMEWORK

A. Definition and Key concepts

A fuzzy set is a class of objects with a continuum of grades of membership, such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one for example the class of animals clearly includes dogs, horses, bacteria, etc have an ambiguous status with respect to the class of animals.

Fuzzy numbers are a special kind of fuzzy set, which are normal and convex. The cut is a commonly used method to connect the principles of fuzzy sets with a collection of crisp set, which can in turn be fed into most of the existing system. The α-cut level set of A is the set.

$$A^a = \{[x, \mu_A(x)] \geq \alpha l x \in X\} \forall \alpha \in [0,1] \qquad \text{... (1)}$$

where X = range of possible values

$\mu_A(x)$ = membership function taking values from [0, 1] or grade of membership.

The α represents the degree of risk that the managers is prepared to take. Since the value of α could. Severely influence the non-dominated solutions its choice should be carefully considered by decision makers. Fuzzy numbers are special kind of fuzzy set, which are normal and convex. A fuzzy set $x\tilde{A}$ is convex if.

$$\mu\tilde{A}[\lambda_{x1} + (1-\lambda)x_2] \geq \min[\mu\tilde{A}(x_1)\mu\tilde{A}[\mu\tilde{A}(x_2)],$$

$$x_1 x_2 \in x, \lambda \in [0,1]$$

Alternatively, a fuzzy set is convex if all α-level sets are convex.

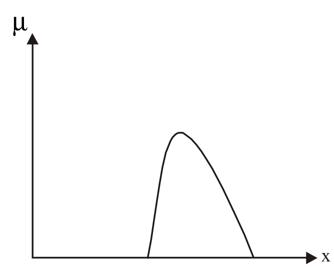

Fig. 1. Convex fuzzy set.

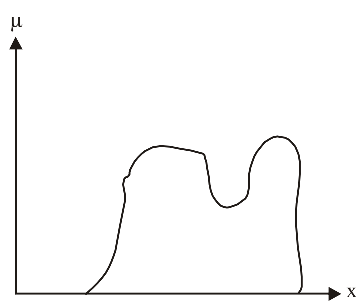

Fig. 2. Non-convex fuzzy set.

Genetic Algorithm (GA) : Over the last few years, scientists, engineers and economists have extensively used genetic algorithms (GA), to solve optimization problems involving single objective functions. During last few years several researchers have extended GA to solve multi objective problems. The basic operation of a genetic algorithm is simple. First a population of possible solution to a problem is developed. Then the better solutions are recombined with each other to form some new solutions for the next generation. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (crossover and possibly mutation) to from a new population. The new population is then used in the next iteration of the algorithm. Finally the new solutions are used to replace the poorer of the original solutions and the process is repeated. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or satisfactory fitness level has been reached for the population.

Non-dominated Sorting Genetic Algorithm (NSGA) : NSGA is a multi objective optimization algorithms and provides a trade-off between the various objectives considered, NSGA basically differs from simple genetic algorithm in the way the selection operator works. Crossover and mutation operator may be used without any modification. Before the selection is performed, the population is ranked on the basis of an individuals non-dominated. The non-dominated individuals present in the population are fires identified from the current population. Then, all these individuals are assumed to constitute the first non-dominated front in the population and assigned a large dummy fitness value. The same fitness value is assigned to give an equal reproductive potential to all these non-dominated individuals. These non-dominated individuals are ignored temporarily to process the rest of population in the same way to identify individuals for the second non-dominated front. These new set of points are then assigned a new dummy fitness value which is kept smaller than the minimum dummy fitness of the previous front. This process is continued until the entire population is classified into several fronts.

Time-cost Trade-off : The project under consideration was converted into a network model. This plan needs to be converted into working schedule for various activities by specifying their scheduled starting and completion time. While converting the plan into a be considered. The actual schedule of activities is very much dependent on the availability of resources particularly manpower. The reduction in normal time of completion will increase the total budget of the project.

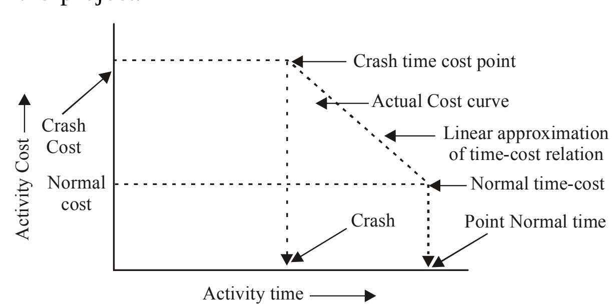

Fig. 3. The time-cost relationship can be visualised graphically.

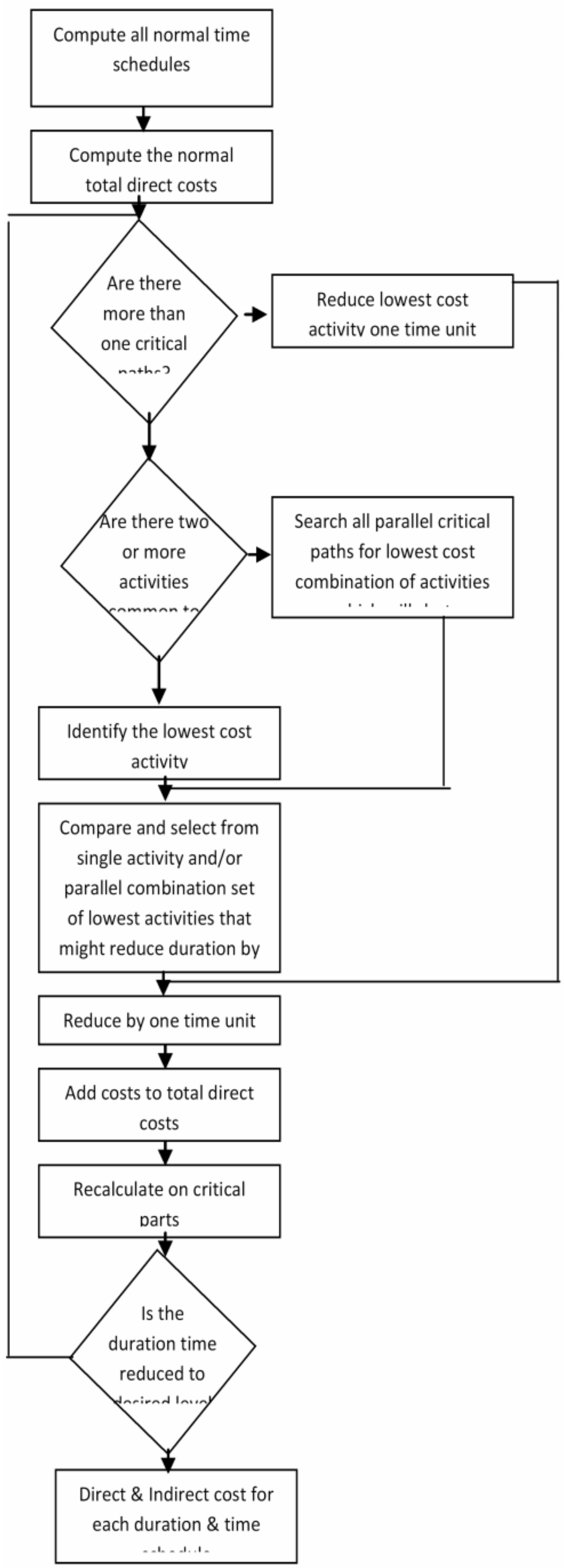

Fig. 4. Time cost trade-off

$$\text{Cost slope} \frac{\text{Crash cost} - \text{Normal cost}}{\text{Crash time} - \text{Normal time}}$$

IV. TOTAL COST OF THE PROJECT

The total cost of any project consist of the direct and indirect cost involved is its execution. The cost is directly dependent upon the amount of resources involved in the execution of the individual activities. It can be seen from the direct cost time relationship shown in fig 5

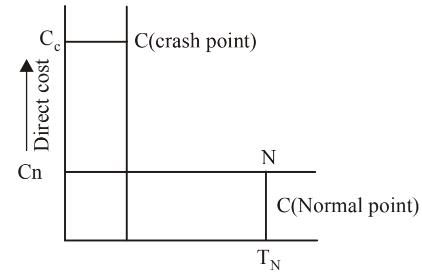

Fig. 5.

On the other hand, indirect cost is composed of the expenditure like the administrative expenses, license fee, insurance cost and taxes and does not depend on the progress of the project *i.e* longer the duration, the higher the indirect cost shown in Fig. 6.

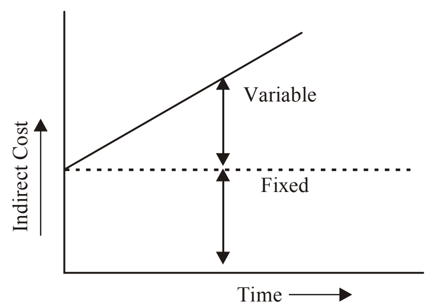

Fig. 6.

The sum of the direct and indirect cost gives the total project cost. As the direct cost decreases with time and indirect cost increase with time, the total project curve with have a point where the total cost will be minimum shown in Fig. 7.

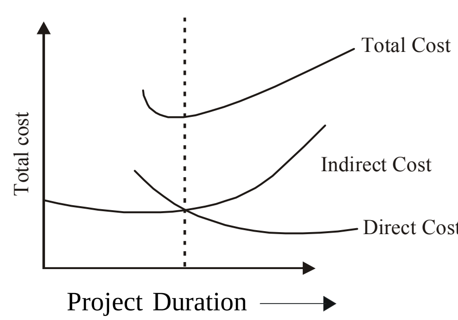

Fig. 7.

The total cost of the project

$$C = \sum_{i \in A} dc_i^{(k)} x_i^{(k)} + Tic_i^{(k)} \qquad \text{... (2)}$$

where $dc_i^{(k)}$ direct cost of activity i under the K^{th} option, $ic_i^{(k)}$ is the indirect cost per time under the K^{th} option.

Construction planners often face the challenge of optimum resources utilization to comprise between different and usually conflicting aspects of projects. Time, cost, risk of the project delivery are among the crucial aspects of each project. In another words, the actual cost of each option is not certainly known for the manager in advance. However, after project execution, they will be known. To apply time and cost of each option may be well considered employing fuzzy set theory so, the number presented for each activity time and cost is a fuzzy one.

A triangular fuzzy number $[\tilde{F} = (f_1, f_2, f_3)]$ may be assigned for the time or cost of that activity, Defining C_1, C_2 and C_3 as optimistic, pessimistic and the most probable time or cost for an activity, respectively. To solve time-cost trade off problem, some options can be chosen for implementation of each activity. For example if there exist 7 activities and 5 option for each activity, then 57 sets of solution will exist. Therefore, genetic algorithm is applied to obtain optimal solutions of the problem length of chromosome or number of genes equals to the number of activities and value of each gene is the option considered for fulfillment of the corresponding activity.

The basic operation of a genetic algorithm is simple, and a real value which is smaller than maximum number of each activity options is randomly chooses for each gene. So, a time and a cost value. Which is in the form of triangular fuzzy number are defined. When reading of all the genes value is terminated, fuzzy time and cost value are assigned to all the project activities summation of fuzzy times of a set of options in each path may be defined with a fuzzy number. Assume $\tilde{F}1$ and $\tilde{F}2$ are two fuzzy number and their α cuts are presented as $F1_a$ and $F2_a$. The sum of them two numbers would be as follows :

$$-F1_a = [F1_a^-, F1_a^+] \qquad \text{... (3)}$$

$$\text{F2a} = [F2_a^-, F2_a^+] \qquad \text{... (4)}$$

$$(F1 + F2)_a = [F1_a^- + F2_a^-, F1_a^+ + F2_a^+] \forall \alpha \in [0,1] \qquad \text{... (5)}$$

To produce the population of the next generation, non-dominated solutions from the previous generation are directly transferred to from part of the population for the new generation. Tournament selection is used to select parent chromosomes for crossover and mutation to complete the population size with reproduced offspring. The process will continue untill a desirable a set of non-dominated solutions is achieved.

Advantages of this model in comparison with similar one line on the method that, uncertainties in project cost are demonstrated aggregated and interpreted. In such a case it is important to study the trade off between completion time, risk involved in each resource option and cost of the project so, when project manager chooses his optimum solutions, he would face a total cost and corresponding membership function ahead that considerably help him to make appropriate decision based on his own level of risk acceptance.

The risk management process is define to minimize risk in order to reduce mishaps, preserve assets and safeguard the health and welfare operational risk management (ORM) is a decision making tool that helps to systematically identify risks and benefits and determine the best courses of action for my given situation.

V. PARAMETES USED IN THE PROPOSED MODEL

A. Risk factor zone and Risk factor value

As time and cost are always closely correlated, a lengthy scheduled will undoubtedly wreck the project cost benefit. All risk observed in the questionnaire can happer to any construction projects which makes it impossible to accurately predict the time required for various programme.

Table 1 : Rating Risk impact on a schedule on a three level scale.

Scale	*Risk impact (R-I)*	*Risk on Schedule of Project*
01	Low	Over all project delay <5% less delay
02	Intermediate	Over all Project delay<5-25% (some delay)
03	Very high	Over all project delay>25% (delay)

Table 2 : Based upon the analysis the various risk zones have been classified.

Risk Factor Zone	*Risk Factor Value*
Zone-1	00-0.10
Zone-2	0.11-0.20
Zone-3	0.21-0.35
Zone-4	0.36-0.45
Zone-5	>0.60

B. Calculation Total direct cost of project

The total direct cost of project is calculated. It is necessary in this situation, to add costs of project activities to each other according to the options determined for them.

To demonstrate the pareto front in a time-cost coordinate system, fuzzy cost will be transformed to a crisp value through application of center of gravity defuzzifier. Therefore, if total fuzzy cost (F*) is covered by membership function A. The center of gravity defuzzifier defines the point C* as the centre of region which is covered by A for a = 1, the value of C will represent the total cost of the project is fully crisp environment.

$$F^* = \frac{\int F \mu_A df}{\int \mu_A df} \quad \text{... (6)}$$

C. Euclidion distance (dE)

The Euclidian distance (d_E) of each individual (I) from each non-dominate individual (NI) is calculated according to the following equation :

$$\forall INI : d_E(i, ni) = \sqrt{\left(\frac{F*i - F*n_i}{F^*_{\max} F^*_{\min}}\right) +} \left(\frac{T*i - F*n_i}{T_{\max}{}^* - T_{\min}{}^*}\right)^2 \quad \text{... (7)}$$

where $T_{\max}{}^*$ = Defuzzy maximum cost of population

$T^*_{\min}$ = defuzzy minimum cost of population.

F_i^* = defuzzy cost of individual

T^*n_i = defuzzy cost non-dominate individual in

$T^*_{\max}$ = Defuzzy maximum time of population

$T^*_{\min}$ = Defuzzy minimum time of population.

T_i^* = Defuzzy time of individual

T^*n_i = Defuzzy tiem non-dominate individual in

VI. CASE STUDY

To demonstrate the concept and test the performance of the proposed model, a simple case example was adopted from Zheng *et al.* (2004). It consist of 7 activities with different possible options. The cost data reported by Zheng *et al.* (2004) were assumed associate with the most probable condition with membership of 1; while for the minimum and maximum cost for any option cost value were assumed to from the triangular membership function.

The example has been solved for different values of cuts. Assuming $\alpha = 1$, and solve the model in an absolutely crisp space. In this case it is expected to obtain the same solution reported by previous researcher for a fully crisp problem.

The same problem was approached by Zheng et. al (2004) in a fully deterministic environment ($\propto$ = 1), employing modified Adaptive Weighted Approach (MAWA). Their results are shown in Table 2. As is clear, the proposed approach has dominated the solution for duration of 62 days, as well as providing one non-dominated solutions for 60 day project execution.

Model has the ability to easily calculate the total cost. The total project cost will be obtained by multi plying daily cost by project execution time and adding this product to the relevant direct cost. Assume that $\tilde{F}1$ and $\tilde{F}2$ are fuzzy number, the multiplication of these two numbers would be as follows :

$$\tilde{F} = \tilde{F}1 \& \tilde{F}2[F1_a^- \times F2_a^-],$$

$$\tilde{F}1_a^- \times F2_a^+ \times F1_a^+,$$

$$\tilde{F}1_a^+ \times F2_a^-, F1_a^+ \times F2_a^+,$$

$$\max[F1_a^- \times F2_a^-, F1_a^- \times F2_a^+, F1_a^- \times F2_a^-, F1_a^+ \times F2_a^+]$$

The assumed value for indirect cost is (410, 500, 720) dollass.

An ordered pair including time and cost of project termination is formed for each chromosome. The chromosomes than which no chromosome is formed having lower both time and cost, are named non-dominated chromosome. In order to find dominated and non-dominated solutions, it is necessary to make a comparison between the time and the cost of chromosomes. The cost and time of project termination for each alternative solution can be compared with other binaurally using fuzzy numbers comparison method.

Minimum of Euclidian distance is considered as fitness function for each chromosome. Basically, non-dominated solutions have fitness as equal to zero and the others belong more or less fitness values in proportion to their distance from non-dominate solution. Determination of (i.e.-accepting different risk percentage), would lead to different parent solution.

Table 3 : Fuzzy time and cost data for the test problem.

Activity	*Preceding activity*	*Resource option*	*Fuzzy time Duration (in day)*	*Fuzzy cost (Dollars)*			*Identified Risk Zones*				
				Cost-1	Cost-2	Cost-3	Zone-1	Zone-2	Zone-3	Zone-4	Zone-5
Initialization for the site Preparation		1	14	20500	23000	226750	0.08	-	-	-	-
		2	20	16200	18000	21200	-	-	0.35	-	-
		3	24	23000	18000	12000	-	-	0.32	-	-
Development or acquisition or rebars and form	1	1	2	15	28700	3000	3580	0.09	-	-	-
		2	18	2260	2400	2900	-	0.18	-	-	-
		3	20	1500	1800	1900	-	-	0.33	-	-
		4	30	1080	1200	620	-	-	-	0.42	-
		5	60	600	700	-	-	-	-	-	0.59
Excavation	1	1	15	4100	4400	4850	0.08	-	-	-	-
		2	22	3250	4000	3750	-	-	0.32	-	-
		3	33	2800	3300	3850	-	-	-	-	0.58
Pre-cast Concrete girders	1	1	12	44000	45500	49550	-	0.19	-	-	-
		2	16	33500	30000	35000	-	-	-	0.14	-
		3	20	28500	35000	45550	-	-	-	-	0.59
Pour foundation and piers	2, 3	1	22	18000	20000	23000	0.08	-	-	-	-
		2	24	16000	17500	22000	-	-	0.32	-	-
		3	28	13500	15000	12000	-	-	-	0.42	-
		4	30	9750	10000	11900	-	-	-	-	0.59
Deliver pre cast girder	4	1	14	38000	40000	48500	0.08	-	-	-	-
		2	18	30000	40000	49000	-	-	-	0.43	-
		3	24	16900	19000	21050	-	-	-	-	0.58
Erect girders	5, 6	1	9	28500	35500	36500	0.09	-	-	-	-
		2	15	22000	24000	26500	-	-	-	0.43	-
		3	18	21000	32000	23250	-	-	-	-	0.58

Table 4 : The comparison between the proposed model and the different model considering different parameters.

Solution	1	2	3	4
Time (days)	60	60	60	60
Cost (&)	16550	155500	173500	173000
Risk	0.136	0.1835	0.1542	0.1492
Resource option	1111111	1112111	1121211	1211211
Model	MOON	MOACO	Proposed Model	MAWA

Table 5 : Pareto solution of different -cut.

-cut Pareto	*Time (days)*	*Defuzzy cost dollars*	*Cost (dollars)*			*Utilizations optima ion for each activity*						
			1	2	3	1	2	3	4	5	6	7
1	60	178900	159570	173500	203630	1	1	1	1	1	3	1
2	61	178840	158440	173000	205080	1	1	1	3	1	2	1
3	62	176730	155960	171000	203230	1	1	1	3	2	2	1
4	63	167893.33	148050	162500	193130	1	1	1	2	2	3	1
5	66	167296.67	147810	161500	192580	1	1	1	2	3	3	1
6	67	162980	143110	157000	188830	1	1	1	3	3	3	1
7	68	157953.33	138580	152500	182780	1	1	1	3	4	3	1
8	74	154460	134900	149500	178980	1	1	1	3	4	3	1
9	77	153780	134960	149000	177380	1	1	1	3	4	3	2
10	78	151620	133780	146500	174580	3	1	1	3	4	3	3

11	83	151486.67	133180	147000	174280	2	1	1	3	4	3	1
12	84	148126.67	130100	143500	170780	3	1	1	3	4	3	3
13	87	147446.67	130160	143000	169180	3	1	1	3	4	3	2
1	60	176200	166535	173500	188565	1	1	1	1	1	3	3
2	61	175920	165720	173000	189040	1	1	1	3	1	2	1
3	62	173865	163480	171000	187115	1	1	1	3	2	2	1
4	63	161596.67	155275	162500	177815	1	1	1	2	3	3	1
5	66	164358.33	154655	161500	177040	1	1	1	2	3		
6	67	159990	150055	157000	172915	1	1	1	3	4		
7	68	155226.67	145540	152500	167640	1	1	1	3	4	3	1
8	74	151980	142200	149500	164240	1	1	1	3	4	3	2
9	77	151390	141980	149000	163190	1	1	1	3	4	3	3
10	78	149060	140140	146500	160540	3	1	1	3	4	3	1
11	84	145813.33	136800	143500	157140	3	1	1	3	1	3	2
12	87	145223.33	136580	143000	156090	3	1	1	3	1	3	3
1	60	173500	173500	173500	173500	1	1	1	1	2	3	1
2	61	173000	173000	173000	173000	1	1	1	3	2	2	1
3	62	171000	171000	171000	171000	1	1	1	3	3	2	1
4	63	162500	162500	162500	162500	1	1	1	2	3	3	1
5	66	161500	161500	161500	161500	1	1	1	2	4	3	1
6	67	157000	157000	157000	157000	1	1	1	3	4	3	1
7	68	152500	152500	152500	152500	1	1	1	3	4	3	1
8	74	149500	149500	159500	149500	1	1	1	3	4	3	2
9	77	14900	149000	149000	149000	1	1	1	3	4	3	3
10	78	146500	146500	146500	146500	3	1	1	3	4	3	1
11	84	143500	143500	143500	143500	3	1	1	3	4	3	2
12	87	143000	143000	143000	143000	3	1	1	3	4	3	3

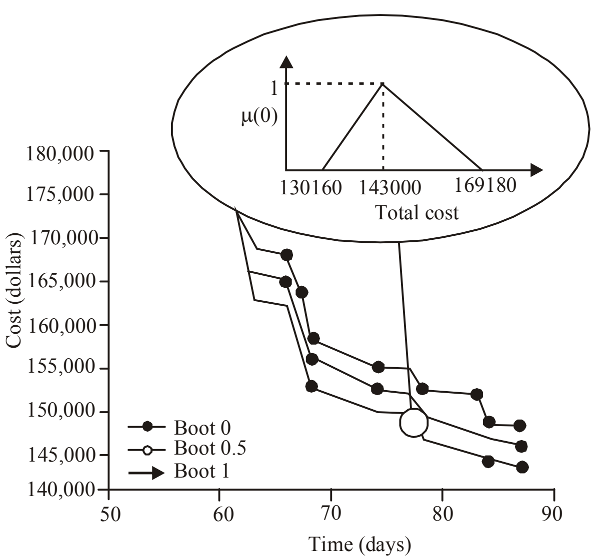

Fig. 8. Pareto fronts for different a cuts.

Fuzzy costs related to $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$ have been transformed to crisp values by centre of gravity defuzzifire and pareto fronts in a time cost coordinate system and shown if Fig. 2 Table. The lower the $\propto$ the higher the total cost for any given time, has been resulted. However, assuming bigger values for $\propto$ associate, with the high risk acceptance, which results in the lower cost with quite the larger range of changes. This is the major benefit of this model application.

VII. CONCLUSION

Our study needs to be carried elaborately into the interaction of various risk and influence of inter-acting risk factors on building construction project. The novelty concept of risk with time-cost trade off using the proposed model is introduced into construction management research. The model adopts fuzzy sets to simulate the degree of uncertainty of the input data. To find dominated and non-dominated solution, fuzzy numbers comparison is applied because cost are fuzzy numbers. Non-dominated sorting Genetic algorithm (NSGA) is used for extraction the pareto front. The project manager can apply his own risk acceptance level to obtain a new pareto front with new non-project manager can apply his own risk acceptance level to obtain a new pareto front with new non-project manager can apply his own risk acceptance level to obtain a new pareto front with new non-dominated solutions using cuts property. For the lower risk, the higher time and cost would be accrued for project execution project manager can apply different risk acceptance level for direct cost and indirect separately.

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