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Bianchi Type-II Bulk Viscous Cosmological Model in General Relativity

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ABSTRACT: Here, we investigated Bianchi type-II cosmological model with and without bulk viscosity. To get a deterministic model, it is assumed that (i) $3\xi\theta = \rho$, where ξ is the coefficient of bulk viscosity, θ is the scalar of expansion and ρ is the energy density (ii) relation between metric potential $A = B^n$. The physical and geometrical aspects of the model are discussed. Special case for n=1 is also discussed.

Keywords: Bianchi type-II space time, expansion scalar, shear scalar, bulk viscosity

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I. INTRODUCTION

Cosmic string plays an important role in structure formation in cosmology [1]. They arise when the symmetry between the strong and electroweak forces are broken due to the phase transition in the early universe ($t\sim10^{-36}$) [2] as the temperature goes down below some critical temperature ($T_{GUT}=10^{28}k$) as predicted by grand unified theories (GUT) [3-7]. It is believed that the vacuum strings give rise to density fluctuations sufficient enough for the formation of galaxies. The general statement of strings was initiated by Letelier [8-9]. These strings possess stress energy and are copier to the gravitational field.

Bianchi type-II models play an important role in current modern cosmology, for simplification and description of the large scale behavior of the actual universe. Kriori et al. [9] and Chakraborty and Nandi [10] have investigated cosmological models for Bianchi type-II, VIII and IX space times. Asseo and Sol [11] emphasized the importance of Bianchi type-II universe. Bali and Anjali [12] has investigated Bianchi type-I magnetized string cosmological model in general relativity. Patel, Maharaj and Leach [13] have studied the integrability of cosmic string in the context of Bianchi type-II, VIII and IX space times. Rao et al. [14] studied Exact Bianchi type-II, VIII and IX string cosmological models in Saez-Ballester theory of Gravitation. Singh and Agarwal [15] studied Bianchi type-II, VIII and IX models in scalar tensor theory under the assumption of a relationship between the cosmological constant (Λ) and scalar field (Ψ). Wang [16-19] also studied LRS Bianchi type-III cosmological models for a cloud string with bulk viscosity. Roy and Banerjee [20] dealt with LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Recently Wang [21] studied the Letelier model in the context of LRS Bianchi type-II space time. Tyagi and Keerti [22] investigated the Bianchi type-II bulk viscous string cosmological models in general relativity. Tiwari and Sonia [23] investigated the non-existence of shear in Bianchi type-III string cosmological models with bulk viscosity and time-dependent Λ term. Zhang et al. [24] studied the Friedmann cosmology on codimension-2 brane with time-dependent tension. Tiwari and Sonia [25] also investigated the Bianchi type-I string cosmological model with bulk viscosity and time-dependent Λ term.

II. THE METRIC AND FIELD EQUATION

Consider the Bianchi type-II space time

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2 \qquad \ldots (1)$$

Where A and B are function of t alone.

The energy momentum tensor T_i^j for the cloud of strings with bulk viscous fluid is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v^l_{;l}(g_i^j + v_i v^j) \qquad \ldots (2)$$

Where, $\rho = \rho_p + \lambda$, the proper energy density for cloud of string with particle attached to them. ξ is the coefficient of bulk viscosity, ρ_p is the particle energy density, λ is the string tension density, v^i is the four velocity vector of particles and x^i is the unit space like vector representing the direction of string satisfying

$$v_i v^i = -x_i x^i = -1 \qquad \ldots (3)$$

In a co-moving coordinate system, we have

$$v^i = (0,0,0,1), x^i = \left(\frac{1}{A},0,0,0\right) \qquad \ldots (4)$$

The Einstein's field equations for a system of strings are given by

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \qquad \ldots (5)$$

The line element (1) gives the following system of equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = \lambda + \xi\theta \qquad \ldots (6)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A} - \frac{3}{4}\frac{B^2}{A^4} = \xi\theta \qquad \ldots (7)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{B^2}{4A^4} = \rho \qquad \ldots (8)$$

Now there are three independent equations in five unknowns A, B, λ, ξ and ρ. Therefore, we need two extra conditions to solve the system completely.

(i) Relation between ξ(coefficient of bulk viscosity), θ(scalar of expansion) and ρ(energy density) [26]

i.e. $$3\xi\theta = \rho \qquad \ldots (9)$$

(ii) Expansion scalar is proportional to shear scalar, $\theta \propto \sigma$ which leads to [27,28,29]

$$A = B^n \qquad \ldots (10)$$

From eq. (7), (8), (9), we get

$$\frac{\ddot{A}}{A} + \frac{1}{3}\frac{\dot{A}^2}{A^2} - \frac{1}{3}\frac{\dot{A}\dot{B}}{AB} - \frac{1}{3}\frac{B^2}{A^4} = 0$$

Using eq. (10), we get

$$\dot{B} = \frac{1}{2\sqrt{n(n-1)}} B^{2(1-n)}$$

$$dt = 2\sqrt{n(n-1)} B^{2(n-1)} dB$$

Integrating, we get

$$\frac{\dot{B}}{B} = \frac{\dfrac{1}{2\sqrt{n(n-1)}} B^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}} \dfrac{B^{3-2n}}{3-2n} + C_1}$$

Taking usual transformations the line element (1) becomes,

$$ds^2 = -4\{n(n-1)\} T^{4(n-1)} dT + T^{2n}(dX^2 + dZ^2) + T^2(dY - XdZ)^2$$

III. SOME PHYSICAL AND GEOMETRICAL PROPORTIES

The energy density ρ, the string tension density λ, coefficient of bulk viscosity ξ, the scalar of expansion θ, the shear scalar σ, ρ_p the particle energy density are respectively given by

$$\rho = n(n+2)\left\{\frac{\dfrac{1}{2\sqrt{n(n-1)}} T^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}} \dfrac{T^{3-2n}}{3-2n} + C_1}\right\}^2 - \frac{1}{4}\left\{\frac{1}{2\sqrt{n(n-1)}} \frac{T^{3-2n}}{3-2n} + C_1\right\}^{2(1-2n)}$$

$$\lambda = -\frac{1-n}{2n}\left\{\frac{1}{2\sqrt{n(n-1)}} \frac{T^{3-2n}}{3-2n} + C_1\right\}^{2(1-2n)}$$

$$-2n(n+1)\left\{\frac{\dfrac{1}{2\sqrt{n(n-1)}} T^{2(1-n)}}{\left\{\dfrac{1}{2\sqrt{n(n-1)}} \dfrac{T^{3-2n}}{3-2n} + C_1\right\}}\right\}^2 + \left\{\frac{1}{2\sqrt{n(n-1)}} \frac{T^{3-2n}}{3-2n} + C_1\right\}^{2(1-2n)}$$

$$\xi = \frac{n(n+2)\left\{\dfrac{\dfrac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1}\right\}^2 - \dfrac{1}{4}\left\{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1\right\}^{2(1-2n)}}{3(2n+1)\dfrac{\dfrac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1}}$$

$$\theta = (2n+1)\frac{\dfrac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1}$$

$$\sigma = \frac{n-1}{\sqrt{3}}\frac{\dfrac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1}$$

$$\rho_p = n(n+2)\left\{\frac{\dfrac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1}\right\}^2 - \frac{1}{4}\left\{\frac{1}{2\sqrt{n(n-1)}}\frac{T^{3-2n}}{3-2n}+C_1\right\}^{2(1-2n)}$$

$$-\frac{1-n}{2n}\left\{\frac{1}{2\sqrt{n(n-1)}}\frac{T^{3-2n}}{3-2n}+C_1\right\}^{2(1-2n)} - 2n(n+1)\left\{\frac{\dfrac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\left\{\dfrac{1}{2\sqrt{n(n-1)}}\dfrac{T^{3-2n}}{3-2n}+C_1\right\}}\right\}^2$$

$$+\left\{\frac{1}{2\sqrt{n(n-1)}}\frac{T^{3-2n}}{3-2n}+C_1\right\}^{2(1-2n)}$$

$$\frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(2n+1)} = cons\tan t$$

IV. OBSERVATIONS

(i) The energy condition $\rho \geq 0$ in the presence of bulk viscous fluid leads to

$$n(n+2)\left\{\frac{\frac{1}{2\sqrt{n(n-1)}}T^{2(1-n)}}{\left[\frac{1}{2\sqrt{n(n-1)}}\frac{T^{3-2n}}{3-2n}+C_1\right]}\right\}^2-\frac{1}{4}\left\{\frac{1}{2\sqrt{n(n-1)}}\frac{T^{3-2n}}{3-2n}+C_1\right\}^{2(1-2n)}\geq 0$$

(ii) When $n=-\frac{1}{2}$, then scalar of expansion θ becomes zero.

(iii) $\frac{\sigma}{\theta}=\frac{n-1}{\sqrt{3}(2n+1)}=cons\tan t$, therefore model does not approach isotropy for large value of T.

(iv) Here it is observed that when T → 0, then the spacial, volume V → 0 and when T → ∞ then V → ∞. These results show that the universe starts expanding with zero volume and blows up at infinite past and future. The role of bulk viscosity in the cosmic evolution, especially as its early stages seems to be significant.

(v) As T → 0, the scalar of expansion θ tends to infinitely large and when T → ∞, the scalar of expansion θ → 0. Also at T → 0 shear scalar σ tends to infinity and when T → ∞ shear scalar σ tend to zero. The energy density ρ → ∞ when T → 0 and ρ → 0 when T → ∞, therefore the model describes a shearing, non-rotating, continuously expanding universe with a big-bang start.

(vi) In the absence of bulk viscosity (when n = 1) i.e. for ξ = 0, we get

$\frac{\sigma}{\theta}=0$ therefore, model approach isotropy for large value of T.

REFERENCES

[1]. R.A. Sunyaev and Ya. B. Zeldovich, "The velocity of clusters of galaxies relative to the microwave background. The possibility of its measurement," *Mon. Not. R. Astro. Soc.* **192,** 663 (1980)

[2]. T.W.B. Kibble,"Topology of cosmic domains and strings", *J. Phys. A, math. Gen*. **9,** 1387-1398 (1976).

[3]. Ya. B. Zeldovich et al., "Cosmological consequences of a spontaneous breakdown of a discrete symmetry", *Zh. Eksp. Teor. Fiz*. **67**, 3-11 (1975).

[4] T.W.B. Kibble,"Some implications of a cosmological phase transition", *Phys. Rep*. **67,** 183-199 (1980).

[5]. A.E. Everett, "Cosmic strings in unified gauge theories", *Phys. Rev*. **24**, 858 (1981).

[6]. A. Vilenkin, "Cosmic strings", *Phys. Rev*. D **24**, 2082-2089 (1981).

[7]. P.S. Letelier, "String cosmologies",*Phys. Rev*. D **28**, 2414-2419 (1983).

[8] P.S. Letelier, "Clouds of strings in general relativity", *Phys. Rev*. D **20**, 1294-1302 (1979).

[9]. K.D. Krori et al., "Some exact solutions in string cosmology", *Gen. Relativ. Grav.* **22,** 123-130 (1990).

[10]. S. Chakraborty and G.C. Nandy, "Cosmological studies in Bianchi II, VIII spacetime

[11]. E. Asseo, H. Sol.,"Extra galactic magnetic fields", *Phys. Rep*. **148**, 307-436 (1987).

[12]. R. Bali and Anjali, "Bianchi type-I magnetized string cosmological models with bulk viscous fluid in general relativity", *Astrophys. Space Sci*. **302**, 201-206 (2006).

[13]. L.K. Patel et al., "Cosmic strings in Bianchi II, VIII and IX spacetimes: integrable Cases", *Pramana* **46**, 131-139 (1996).

[14]. U.V. M. Rao et al., “Exact Bianchi type-II, VIII and IX string cosmological models in Saez-Ballester theory of gravitation”, *Astrophys. Space Sci.* **314**, 73-77 (2008).

[15]. T. Singh and A. K. Agarwal, “Bianchi type II, VIII, IX in certain new theories of gravitation”, *Astrophys. Space Sci.***191**, 61-88 (1997).

[16]. X.X. Wang, “Bianchi type-I string cosmological models with bulk viscosity and magnetic field”, *Astrophys. Space Sci.* **293**, 433-440 (2004)

[17]. X.X. Wang, “Locally rotationally symmetric Bianchi type-I string cosmological Model with bulk viscosity”, *Chin. Phys. Lett*. **21**, 1205 (2004).

[18]. X.X. Wang, “Bianchi type-III string cosmological model with bulk viscosity in general relativity”,**22**, 29 (2005).

[19]. X.X. Wang, “Bianchi type-III string cosmological model with bulk viscosity and magnetic field”,**23**, 1702 (2006).

[20]. S.R. Roy and S. K. Banerjee, “Bianchi type-II string cosmological model in general relativity”, *Class. Quant. Grav.* **11,** 1943 (1995).

[21]. X.X. Wang, “Exact solutions for string cosmology”, **20**, 615 (2003).

[22]. A. Tyagi and Keerti Sharma, “Bianchi type-II bulk viscous string cosmological models in general relativity”, **49**, 1712 (2010).

[23]. R.K. Tiwari and Sonia Sharma, “Non-existence of shear in Bianchi type-III string cosmological models with bulk viscosity and time dependent Λ term”, *Chin. Phys. Lett.* **28**, 020401 (2011).

[24]. H. Zhang et al., “Friedmann cosmology on codimension-2 brane with time dependent tension”, arxiv:astro-ph/0504178v1 (2005).

[25]. R.K. Tiwari and Sonia Sharma, “Bianchi type-I string cosmological model with Bulk viscosity and time dependent Λ term”, **28,** 090401 (2011).

[26]. K.S. Adhav et al., “Non-existence of Bianchi type-V string cosmological model with bulk viscous fluid in general relativity”, *Rom. Journ. Phys.* **54**, 207 (2007).

[27]. R K. Tiwari and U.K. Dwivedi, “Kantowski-sachs cosmological models with time varying G and Λ”, Fizika B **18**, 181 (2010).

[28]. S.R. Roy, S. Narain and J.P. Singh, “Bianchi type-I cosmological models with perfect fluid and magnetic field”, *Aust. J. Phys.* **38**, 239 (1985).

[29] C.B. Collins et al., “Exact spatially homogeneous cosmologies”, *Gen. Relat. Gravit*. **10**, 805 (1980).