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Generalized Incomplete Trojan-Type Designs with Unequal Cell Sizes

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ABSTRACT: Generalized Incomplete Trojan-Type Designs are row-column designs in which each cell, corresponding to the intersection of row and column, contains more than one treatment and the rows are incomplete. In some experimental situations, it is not possible for the experimenter to have the row-column intersections of equal size. Generalized incomplete Trojan-type designs with unequal cell sizes are to be obtained for such situations. In this paper, a method of construction of a series of generalized incomplete Trojan-type design with unequal cell sizes has been developed.

Key Words: Row-column design, Trojan-type design, Semi-Latin square, Unequal cell size

I. INTRODUCTION

Row-column designs (RCDs) are used in experimental situations when the heterogeneity in the experimental material is due to two cross classified sources. Most commonly, the row-column designs have only one unit corresponding to the intersection of each row and column. However, there may be instances when the number of treatments is substantially large and a more general class of row-column designs is required wherein there is more than one unit in each row-column intersection.

For example, consider a sensory trial to make a comparative rating of 12 food products that are presented to 3 panel members. The constraint here is that each member can assess a maximum of 4 products in a session, more than which may cause assessor fatigue. The rating may vary from member to member and session to session and hence the panel members and sessions can be treated as row as well as column classifications. There are 3 rows and 3 columns, but the number of products to be rated is 12 and hence each row-column intersection has to be further divided into 4 sub-sessions or cells. Semi-Latin square, a generalized row-column design with *n* rows and *n* columns wherein the intersection of each row and each column contains a cell of *k* subdivions resulting in *nk* cells each row and each column, is appropriate for such an experimental situation. Thus, each row and each column is complete.

Darby and Gilbert [1] described Trojan squares based on sets of mutually orthogonal superimposed Latin squares as a special class of semi-Latin squares. These designs are shown to be maximally efficient designs for pair-wise treatment comparisons in the plots-within-blocks stratum by Bailey [2].

Some combinatorial properties of semi-Latin squares and related designs were discussed by Preece and Freeman [3]. Bailey [4, 2] obtained a range of semi-Latin and Trojan square designs and showed that the Trojan squares are the optimal choice of semi-Latin squares for pair-wise comparisons of treatment means.

Incomplete Trojan squares obtained by omitting a single row from a complete Trojan square have been discussed by Edmondson [5], which are of considerable practical utility, when there is a limitation of resources. Edmondson [6] constructed generalized incomplete Trojan square designs based on a set of cyclic generators.

Some methods of construction of semi-Latin squares were given by Bedford and Whitaker [7]. Further, Bailey and Monod [8] gave some efficient semi-Latin rectangles useful for plant disease experiments. Subsequently, Dharmalingam [9] used Trojan square design to obtain partial triallel crosses.

Jaggi *et al.* [10] defined Generalized Incomplete Trojan-Type Designs in which each cell, corresponding to the intersection of row and column, contains more than one treatment and the rows are incomplete. They have developed a method of constructing these designs. These designs are available for any number of treatments ≥ 6 with flexibility in choosing the cell size depending on the experimental resources available.

Many times, it is not possible for the experimenter to have the row-column intersections of equal size. Generalized incomplete Trojan-type designs with unequal cell sizes are suitable for such situations. Here, a method of construction of a series of generalized incomplete Trojan-type design with different cell sizes has been developed.

II. GENERALIZED INCOMPLETE TROJAN-TYPE DESIGNS

Definition (Jaggi *et al.*, [10]): Consider an array with m rows and n columns in which each intersection of row and column is divided into k sub-units forming a total of mnk sub-units in all. A Generalized Incomplete Trojan-Type design is an arrangement of v treatments in mnk sub-units, such that each treatment occurs in every column $\alpha (\geq 2)$ times, in every row β times ($\beta = 0$ or 1) and on every sub-unit γ times ($\gamma = 0$ or 1).

A general method of constructing generalized incomplete Trojan-Type designs with unequal cell sizes is explained in the subsequent section.

III. METHOD OF CONSTRUCTION OF GENERALIZED INCOMPLETE TROJAN-TYPE DESIGN

Let there be v treatments ($v > 5$) denoted by 1,2,…, v. Arrange the first s treatments $5 \leq s \leq (v-1)$ in the first row of the design.

Group these s treatments into n groups ($n \geq 2$) of sizes k_i, $i = 1, 2,\ldots, n$, ($\sum_{i=1}^{n} k_i = s$) such that at least one or all of the groups are of different size. Each group of treatments belongs to a cell in the first row of the design. Now develop (v-1) more rows of the design cyclically, column-wise, by adding 1 to the treatment in the previous row (reduced mod v). Thus, we get a generalized incomplete Trojan-type design with cell sizes k_i for v treatments in $m = v$ rows of size s and n columns of size vk_i. There are several ways to form the n cells in the first row and hence one can obtain a number of designs with different set of parameters for a given number of treatments, v.

Example 3.1: Let there be $v = 8$ treatments. Further let $s = 7$ and $n = 2$. Taking $k_1 = 5$ and $k_2 = 2$, the generalized incomplete Trojan-type design is obtained in 8 rows and 2 columns as:

Rows	Columns						
	i					ii	
i	1	2	3	4	5	6	7
ii	2	3	4	5	6	7	8
iii	3	4	5	6	7	8	1
iv	4	5	6	7	8	1	2
v	5	6	7	8	1	2	3
vi	6	7	8	1	2	3	4
vii	7	8	1	2	3	4	5
viii	8	1	2	3	4	5	6

The information matrix $\mathbf{C} = \begin{bmatrix} 5.0 & -1.3 & -0.6 & -0.4 & -0.4 & -0.4 & -0.6 & -1.3 \\ -1.3 & 5.0 & -1.3 & -0.6 & -0.4 & -0.4 & -0.4 & -0.6 \\ -0.6 & -1.3 & 5.0 & -1.3 & -0.6 & -0.4 & -0.4 & -0.4 \\ -0.4 & -0.6 & -1.3 & 5.0 & -1.3 & -0.6 & -0.4 & -0.4 \\ -0.4 & -0.4 & -0.6 & -1.3 & 5.0 & -1.3 & -0.6 & -0.4 \\ -0.4 & -0.4 & -0.4 & -0.6 & -1.3 & 5.0 & -1.3 & -0.6 \\ -0.6 & -0.4 & -0.4 & -0.4 & -0.6 & -1.3 & 5.0 & -1.3 \\ -1.3 & -0.6 & -0.4 & -0.4 & -0.4 & -0.6 & -1.3 & 5.0 \end{bmatrix}$

The information matrix of treatment effects and the variances pertaining to estimated elementary treatment contrasts under a four-way classified model were computed by writing a SAS code.

The elementary treatment contrasts are estimated with three types of variances, viz., 0.3231, 0.3685 and 0.3885. The average variance is calculated as 0.3646.

For 8 treatments, taking $s = 6$, $n = 2$, $k_1 = 4$ and $k_2 = 2$, the following design is obtained:

Rows	Columns					
	i				ii	
i	1	2	3	4	5	6
ii	2	3	4	5	6	7
iii	3	4	5	6	7	8
iv	4	5	6	7	8	1
v	5	6	7	8	1	2
vi	6	7	8	1	2	3
vii	7	8	1	2	3	4
viii	8	1	2	3	4	5

The information matrix $\mathbf{C} = \begin{bmatrix} 4.0 & -1.3 & -0.5 & -0.3 & 0.0 & -0.3 & -0.5 & -1.3 \\ -1.3 & 4.0 & -1.3 & -0.5 & -0.3 & 0.0 & -0.3 & -0.5 \\ -0.5 & -1.3 & 4.0 & -1.3 & -0.5 & -0.3 & 0.0 & -0.3 \\ -0.3 & -0.5 & -1.3 & 4.0 & -1.3 & -0.5 & -0.3 & 0.0 \\ 0.0 & -0.3 & -0.5 & -1.3 & 4.0 & -1.3 & -0.5 & -0.3 \\ -0.3 & 0.0 & -0.3 & -0.5 & -1.3 & 4.0 & -1.3 & -0.5 \\ -0.5 & -0.3 & 0.0 & -0.3 & -0.5 & -1.3 & 4.0 & -1.3 \\ -1.3 & -0.5 & -0.3 & 0.0 & -0.3 & -0.5 & -1.3 & 4.0 \end{bmatrix}$

The variances are 0.3976, 0.4857, 0.5405 and 0.5714 and average variance is 0.4884. Again, one can have a generalized incomplete Trojan-type design in 8 rows and 2 columns having two different cell sizes $k_1 = 4$ and $k_2 = 3$ as given below:

Rows	Columns						
	i				ii		
i	1	2	3	4	5	6	7
ii	2	3	4	5	6	7	8
iii	3	4	5	6	7	8	1
iv	4	5	6	7	8	1	2
v	5	6	7	8	1	2	3
vi	6	7	8	1	2	3	4
vii	7	8	1	2	3	4	5
viii	8	1	2	3	4	5	6

The information matrix $\mathbf{C} = \begin{bmatrix} 5.0 & -1.4 & -0.8 & -0.3 & 0.0 & -0.3 & -0.8 & -1.4 \\ -1.4 & 5.0 & -1.4 & -0.8 & -0.3 & 0.0 & -0.3 & -0.8 \\ -0.8 & -1.4 & 5.0 & -1.4 & -0.8 & -0.3 & 0.0 & -0.3 \\ -0.3 & -0.8 & -1.4 & 5.0 & -1.4 & -0.8 & -0.3 & 0.0 \\ 0.0 & -0.3 & -0.8 & -1.4 & 5.0 & -1.4 & -0.8 & -0.3 \\ -0.3 & 0.0 & -0.3 & -0.8 & -1.4 & 5.0 & -1.4 & -0.8 \\ -0.8 & -0.3 & 0.0 & -0.3 & -0.8 & -1.4 & 5.0 & -1.4 \\ -1.4 & -0.8 & -0.3 & 0.0 & -0.3 & -0.8 & -1.4 & 5.0 \end{bmatrix}$

The variances pertaining to estimated elementary treatment contrasts are 0.3221, 0.3744, 0.4268 and 0.4489 and the average variance is 0.3851. Another arrangement of 8 treatments is given below in 8 rows and 3 columns having three different cell sizes $k_1 = k_2 = 2$ and $k_3 = 3$:

Rows	Columns						
	i		ii		iii		
i	1	2	3	4	5	6	7
ii	2	3	4	5	6	7	8
iii	3	4	5	6	7	8	1
iv	4	5	6	7	8	1	2
v	5	6	7	8	1	2	3
vi	6	7	8	1	2	3	4
vii	7	8	1	2	3	4	5
viii	8	1	2	3	4	5	6

The information matrix $\mathbf{C} = \begin{bmatrix} 4.0 & -1.7 & -0.3 & 0.0 & 0.0 & 0.0 & -0.3 & -1.7 \\ -1.7 & 4.0 & -1.7 & -0.3 & 0.0 & 0.0 & 0.0 & -0.3 \\ -0.3 & -1.7 & 4.0 & -1.7 & -0.3 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.3 & -1.7 & 4.0 & -1.7 & -0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.3 & -1.7 & 4.0 & -1.7 & -0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.3 & -1.7 & 4.0 & -1.7 & -0.3 \\ -0.3 & 0.0 & 0.0 & 0.0 & -0.3 & -1.7 & 4.0 & -1.7 \\ -1.7 & -0.3 & 0.0 & 0.0 & 0.0 & -0.3 & -1.7 & 4.0 \end{bmatrix}$

The variances are 0.4055, 0.5972, 0.7247 and 0.7660 giving an average variance of 0.6030.A generalized incomplete Trojan-type design in 8 rows and 2 columns having cell sizes $k_1 = 3$ and $k_2 = 2$ can be obtained for 8 treatments as:

Rows	Columns				
	i			ii	
i	1	2	3	4	5
ii	2	3	4	5	6
iii	3	4	5	6	7
iv	4	5	6	7	8
v	5	6	7	8	1
vi	6	7	8	1	2
vii	7	8	1	2	3
viii	8	1	2	3	4

The information matrix $\mathbf{C} = \begin{bmatrix} 3.0 & -1.2 & -0.3 & 0.0 & 0.0 & 0.0 & -0.3 & -1.2 \\ -1.2 & 3.0 & -1.2 & -0.3 & 0.0 & 0.0 & 0.0 & -0.3 \\ -0.3 & -1.2 & 3.0 & -1.2 & -0.3 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.3 & -1.2 & 3.0 & -1.2 & -0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.3 & -1.2 & 3.0 & -1.2 & -0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.3 & -1.2 & 3.0 & -1.2 & -0.3 \end{bmatrix}$

There are 4 types of variances, viz., 0.5355, 0.7506, 0.9072 and 0.9558 and the average variance had turned out to be 0.7632.

It can be seen that for a fixed total row size, if the cells are formed such that there are less number of cells with bigger cell sizes, the variance is less. The more the number of cells, the more is the variance for a given row size.

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