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A Common Fixed Point Theorem for R-Weakly Commuting Maps in Fuzzy Metric Spaces using E.A.

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ABSTRACT: In the present paper, we are proving a common fixed point theorem using R-weakly commuting maps and property E.A. fuzzy in metric space which improves the result of Shrivastava *et al.* [4].

Mathematics Subject Classification: 47H10, 54H25

Keywords: Fuzzy Metric Spaces, R-weakly commuting maps, Common fixed point, Property E. A.

I. INTRODUCTION

Zadeh [5], introduced the concept of fuzzy set in 1965 and in the next decade were very productive for fuzzy mathematics and the recent literature has observed the fuzzy frication in at most every direction of Mathematics such as arithmetic topology graph theory, probability theory, Logic etc. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory mathematical programming modeling theory engineering sciences medical sciences (medical genetics nervous system) Image processing control theory, communication etc.

Kramosil and Michalek [1] introduced the concept of fuzzy metric space in 1975, which opened an avenue for further development of analysis in such spaces. Pant introduced the notation of R-weakly commutatively of mappings in metric spaces and proved some common fixed point theorem [2]. In this paper define R-weakly commuting of mapping in fuzzy metric spaces and prove the fuzzy version of pant's theorem. Before starting the main results first we write some definitions

II. PRELIMINARIES

Definition 2.1: [5] A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 2.2. [3]: A binary operation $M(x,\ y,\ 0)\ =\ 0$*: $[0.1]$ x $[0.1] \rightarrow [0,1]$ is continuous t-norm if * is satisfying the following conditions.

(1) * is commutative and associative.

(2) * is continuous.

(3) $a*1=a$, $\forall a \in [0,1]$

(4) $a*b \le c*d$ where $a \le c$ and $b \le d$ and $a,b,c,d \in [0,1]$

Example : $a*b=ab$, $a*b=min\{a,b\}$

Definition: 2.3 [1] A 3-taple $(X,M,*)$ is said to be a fuzzy metric space if X is an arbitrary set * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions; for all $x,y,z \in X$ and $s,t>0$.

1) $M(x,y,0)=0$

2) $M(x,y,t)=1$ for all $t > 0$ if and only if $x=y$

3) $M(x,y,t)=M(y,x,t)\neq 0$ for $t \neq 0$.

4) $M(x,y,t)*M(y,z,s) \le M(x,z,t+s)$

5) $M(x,y,\cdot):[0,\infty) \rightarrow [0,1]$ is continuous $\forall x,y,z \in X$ and $s,t>0$

6) $\lim_{x\to\infty} M(x,y,t)=1 \quad \forall x,\ y\ in\ X$.

Definition 2.4: A sequences $\{x_n\}$ in a fuzzy metric space $(X,M,*)$ is called Cauchy if $\lim_{n\to\infty} M(x_{n+p},s_n,t)=1$ for every $t>0$ and each $p>0$, $(X,M,*)$ is complete if every

Cauchy sequence in X converges in X . A sequence $\{x_n\}$ in X is convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for each $t > 0$.

Definition 2.5: Two mappings S and T of a fuzzy metric space $(X, M, *)$ into itself are said to be weakly commuting if

$M(STx, TSx, t)\ M(Sx, Tx, t)$ for each x in *X*.

Definition 2.6: Two mappings S and T of a metric space (X, d) into itself are said to be R-weakly commuting provided there exists some positive real number R such that

$$d(STx, TSx) \le Rd(Sx, Tx)$$

for each x in X .

Now we define R-weakly commuting maps in fuzzy metric spaces.

Definition 2.7: Two mappings S and T of a fuzzy metric space $(X, M, *)$ into itself are R-weakly commuting, provided there exists some positive real number R such that

$M(STx, TSx, t) \ge M(Sx, Tx, t/R)$

for all $x \in X$.

3. Property E.A.

Definition: Let f and g be two self mappings of a fuzzy metric space $(X, M, *)$. We say that A and S satisfy the property (E.A.) if there exists a sequence x_n such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, for some $t \in X$

4. The following theorem was proved by Pant [2].

Theorem 3.1: Let (X, d) be a complete metric space and let f and g be R – weakly commuting self – mapping of X satisfying the condition.

$d(fx, fy) \le r(d(gx, gy))$

Where $r: R_+ \to R_+$ is a continuous function such that $r(t) < t$ for each $t > 0$: If the range of f contains the range of g and, if either g , f is continuous. Then g and f have a unique common fixed point.

III. MAIN RESULT

In this section we present our main result.

Theorem 3.2: Let (X, M,*) be a fuzzy metric space with $t*t \ge t$ and let S and T be R –weakly commuting self mappings of X satisfying the property E.A. and the conditions: M(Sx, Sy, t) ≥ r(M (Tx, Ty, t) * M(Sx, Ty, t) * M(Sx, Tx, t)) ………(3.1)

Where r : [0, 1] → [0, 1] is continuous function such that r(t) > t for each 0 < t < 1 and r(1)=1. The sequence $\{x_n\}$ and $\{y_n\}$ in X are such that $x_n \to x$, $y_n \to y$, t>0 implies $M(x_n, y_n, t) \to M(x, y, t)$.

If the range of S or T is a complete subspace of X such that S(X) ⊂ T(X). Then S and T have a unique common fixed point.

Proof: Since S and Tare satisfy the property (E.A), there exists a sequence x_n in X such that

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = p$, for some p ∈ X.

Since S(X) ⊂ T(X) and p∈ T(X), there exists some point u in X such that

p = Tu where $p = \lim_{n\to\infty} Tx_n$.

If Su ≠ Tu, using (3.1) we get,

$M(Sx_n, Su, t)$

$\ge r(M(Tx_n, Tu, t) * M(Sx_n, Tu, t) * M(Sx_n, Tx_n, t))$

on letting $n \to \infty$ yields

M(Tu, Su, t) ≥ r(M (Tu, Tu, t) * M(Tu, Tu, t) * M(Tu, Tu, t))

M(Tu, Su, t) ≥ 1;

Hence Su = Tu.

Since S and T are R-weak commutating, there exists R > 0 such that M(STu, TSu, t) ≥ M(Su, Tu, t/R) = 1,

that is, STu = TSu and SSu = STu = TSu = TTu.

If Su ≠ SSu, using (3.1) and putting x=Sx, y=u, we get

M(SSx, Su, t)

≥ r(M (TSx, Tu, t) * M(SSx, Tu, t) * M(SSx, TSx, t))

M(SSx, Su, t)

≥ r(M (SSx, Su, t) * M(SSx, Su, t) * M(SSx, SSx, t))

M(SSx, Su, t) ≥ r(M (SSx, Tu, t) * M(SSx, Tu, t) * 1)

M(SSx, Su, t) > M (SSx, Tu, t) a contradiction.

Therefore, Su = SSu and Su = SSu = STu = TSu = TTu.

Hence Su is a common Sixed point of S and T.

The uniqueness follows from (3.1).

Since S(X) ⊂ T(X), the case when S(X) is a complete subspace of X is similar to the above case.

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