International Journal of Theoretical & Applied Sciences, 8(1): 7-11(2016)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Common Fixed Point Theorem Satisfying Generalized Contractive Conditions of Integral Type in Intuitionistic Fuzzy Metric Spaces

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(Received 11 January, 2016 accepted 25 February, 2016)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: In this paper, we prove some common fixed point results satisfying generalized contractive conditions of integral type in intuitionistic fuzzy metric spaces using the notion of occasionally weakly compatible maps. Our results extend and generalize known results of fixed point theorems in metric spaces, fuzzy metric spaces and intuitionistic fuzzy metric spaces.

Keywords: Intuitionistic fuzzy metric space, occasionally weakly compatible mapping.

Mathematics Subject classification: Primary 47H10 and Secondary 54H25.

I. INTRODUCTION

The concept of Fuzzy sets was initially investigated by Zadeh [16] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. Atanassov [3] introduced the concept of Intuitionistic fuzzy sets by generalizing the notion of fuzzy set by treating membership as a fuzzy logical value has to be consistent (in the sense $\gamma_A(x) + \mu_A(x) \geq 1$). $\gamma_A(x)$ *and* $\mu_A(x)$ denotes degree of membership and degree of non – membership, respectively. All results hold of fuzzy sets can be transformed intuitionistic fuzzy sets but converse need not be true. In 2004, Park [10] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm. Since the intuitionistic fuzzy metric space has extra conditions see [6],[15] modified the idea of intuitionistic fuzzy metric space and presented the new notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm. Branciari [4] gave a fixed point result for a single mapping satisfying Banach's contraction principle for an integral type inequality. The authors [2, 4, 5, 11,14] proved fixed point theorems using contractive conditions of integral type.

II. BASIC DEFINITIONS AND PRELIMINARIES

Definition 2.1. [13] A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is called a *t-norm* * satisfies the following conditions:
(i) * is continuous,
(ii) * is commutative and associative,
(iii) a * 1 =a for all a ∈ [0, 1],
(iv) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0,1].
Example 2.1. a * b = ab and a * b=min{a, b}.
Definition 2.2. [13] A binary operation $\Diamond:[0,1]\times[0,1]\rightarrow[0,1]$ is said to be continuous *t-conorm* if it satisfied the following conditions:
(i) ◊ is associative and commutative,
(ii) a ◊ 0 = a for all a ∈ [0,1],
(iii) ◊ is continuous,
(iv) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for each a, b, c, d ∈ [0,1]
Example 2.2. a ◊ b = min(a+b, 1) and a ◊ b = max(a, b)
Definition 2.3. [1] A 5- tuple (X, M, N, *, ◊) is called intuitionistic fuzzy metric space if X is an arbitrary non empty set, * is a continuous *t-norm,* ◊ continuous *t-conorm* and M, N are fuzzy sets on $X^2 \times [0,\infty]$ satisfying the following conditions:
For each x, y, z∈ X and t, s > 0
(IFM-1) M(x , y, t) + N(x, y, t) ≤ 1,
(IFM-2) M(x, y, 0) = 0, for all x, y in X,
(IFM-3) M(x, y, t) = 1 if and only if x=y,
(IFM-4) $M(x,y,t) = M(y,x,t)$,
(IFM-5) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,
(IFM-6) $M(x,y,.): [0,\infty] \rightarrow [0,1]$ is left continuous,
(IFM-7) $\lim_{t\rightarrow\infty} M(x,y,t) = 1$,
(IFM-8) $N(x,y,0) = 1$,
(IFM-9) $N(x,y,t) = 0$, if and only if x = y,
(IFM-10) $N(x,y,t) = N(y,x,t)$,
(IFM-11) $N(x,y,t) \Diamond N(y,z,s) \geq N(x,z,t+s)$,
(IFM-12) $N(x,y,.): [0,\infty] \rightarrow [0,1]$ is right continuous,
(IFM-13) $\lim_{t\rightarrow\infty} N(x,y,t) = 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The function M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non nearness between x and y with respect to t, respectively.

Remark 2.1. Intuitionistic Fuzzy Metric space, M(x, y,.) is non decreasing and N(x, y, .) is non increasing for all x, y ∈ [0, 1].

Example 2.3. [10] Let (X, d) be a metric space. Define a * b = ab and a ◊ b = min{1, a+b }, for all a, b ∈ [0, 1] and let M and N be fuzzy sets on X^2 x (0, ∞) defined as follows:

M(x, y, t) = $\frac{t}{t+d(x,y)}$ and N(x, y, t) = $\frac{d(x,y)}{t+d(x,y)}$ for all x, y ∈ X and all t > 0.

then (M, N) is called an intuitionistic fuzzy metric space on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.2. Note that the above examples holds even with the t- norm a * b = min{a, b} and t- conorm a ◊ b= max{a, b}and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t –conorm.

Definition 2.4. [1] A sequence $\{x_n\}$ in intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be Cauchy sequence if for all t > 0 and p >0,

$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ *and* $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$

The sequence $\{x_n\}$ converge to a point $x \in X$ if for all t > 0,

$\lim_{n\to\infty} M(x_n, x, t) = 1$ *and* $\lim_{n\to\infty} N(x_n, x, t) = 0$.

An Intuitionistic Fuzzy metric space (X, M, N, *, ◊) is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 2.5. [7] Two self mappings A and S of an Intuitionistic Fuzzy Metric space (X, M, N, *, ◊) are said to be compatible if and only if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) \to 1$ and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) \to 0$ for all t > 0 whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ *for some p in X as* $n \to \infty$.

Definition 2.6. [8] Two self mappings A and S of a metric space (X, d) are said to be occasionally weakly compatible (owc) if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

Lemma 2.1[11] Let (X,d) be a metric space. If f and g be self maps on X and let f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

We define above definitions and lemma in intutionistic fuzzy metric spaces as:

Definition 2.7. Two self mappings A and B of an Intuitionistic Fuzzy Metric space (X, M, N, *, ◊) are said to be weakly compatible if they commute at their coincidence point x, i. e. Ax = Bx implies ABx = BAx for some x in X.

Definition 2.8. Two self mappings A and S of an Intuitionistic Fuzzy metric space (X, M, N, *, ◊) are said to be occasionally weakly compatible (owc) if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

Lemma 2.2. [1] Let (X, M, N, *, ◊) Intuitionistic fuzzy metric space, If there exists k ∈ (0, 1) such that for all x, y ∈ X, M(x, y, kt) ≥ M(x, y, t) and , N(x, y, kt) ≤ N(x, y, t) for all t > 0, then x = y.

Lemma 2.3. Let (X, M, N, *, ◊) be an Intuitionistic Fuzzy metric space. f and g be self maps on X and let f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

III. MAIN RESULTS

Theorem 3.1. Let (X, M, N,*,◊) be an intuitionistic fuzzy metric space with continuous $t * norm$ and continuous $t - conorm$ ◊. Let A, B, S and T be self mappings of X. Let the pairs (A, S) and (B, T) be owc. For all $x, y \in X$, there exist non increasing, non decreasing continuous functions $\phi, \Psi: [0,1] \to [0,1]$ such that $\phi(t) > t, \Psi(t) < t$ *for all* $t \in (0,1)$. For every t > 0 there exist $k \in (0,1)$such that

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt \geq \phi\left(\int_0^{m(Ax,By,t)} \varphi(t)dt\right) \quad \ldots(3.1)$$

and

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt \leq \Psi\left(\int_0^{n(Ax,By,t)} \varphi(t)dt\right) \quad \ldots(3.2)$$

Where $\varphi: R^+ \to R^+$ is a lebesgue integrable mapping which is summable, non negative such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \; for\ each\ \ \varepsilon > 0$$

and

$$M(Ax, By, t) = \min\{M(Sx, Ty, t), M(By, Sx, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), \left(\frac{2.M(Sx, Ax, t)}{1 + M(By, Ty, t)}\right)\}$$

and

$$N(Ax,By,t) = \max\{N(Sx,Ty,t), N(By,Sx,t), N(Sx,Ax,t), N(By,Ty,t), M(Ax,Ty,t), \left(\frac{2.N(Sx,Ax,t)}{1+N(By,Ty,t)}\right)\}$$

for all $x, y \in X$ and t > 0. Then, there is a unique common fixed point of A, B, S and T.

Proof: As the pair (A, S) and (B, T) are owc, so there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that $Ax = By$. By (3.1), (3.2) we have,

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt \geq \phi(\int_0^{\min\{M(Sx,Ty,t),M(By,Sx,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),\left(\frac{2.M(Sx,Ax,t)}{1+M(By,Ty,t)}\right)\}} \varphi(t)dt)$$

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt \geq \phi(\int_0^{\min\{M(Ax,By,t),M(By,Ax,t),M(Ax,Ax,t),M(By,By,t),M(Ax,By,t),\left(\frac{2.M(Ax,Ax,t)}{1+M(By,By,t)}\right)\}} \varphi(t)dt)$$

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt \geq \phi(\int_0^{\min\{M(Ax,By,t),M(By,Ax,t),1,1,M(Ax,By,t),1\}} \varphi(t)dt)$$

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt \geq \phi(\int_0^{M(Ax,By,t)} \varphi(t)dt)$$

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt > \int_0^{M(Ax,By,t)} \varphi(t)dt$$

and

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt \leq \Psi(\int_0^{\max\{N(Sx,Ty,t),N(By,Sx,t),N(Sx,Ax,t),N(By,Ty,t),N(Ax,Ty,t),\left(\frac{2.N(Sx,Ax,t)}{1+N(By,Ty,t)}\right)\}} \varphi(t)dt)$$

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt \leq \Psi(\int_0^{\max\{N(Ax,By,t),N(By,Ax,t),N(Ax,Ax,t),N(By,By,t),N(Ax,By,t),\left(\frac{2.N(Ax,Ax,t)}{1+N(By,By,t)}\right)\}} \varphi(t)dt)$$

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt \leq \Psi(\int_0^{\max\{N(Ax,By,t),N(By,Ax,t),0,0,N(Ax,By,t),0\}} \varphi(t)dt)$$

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt \leq \Psi(\int_0^{N(Ax,By,t)} \varphi(t)dt)$$

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt < \int_0^{N(Ax,By,t)} \varphi(t)dt$$

therefore by lemma 2.2, $Ax = By$ $i.e.$ $Ax = Sx = By = Ty$. Suppose that there is another point z such that $Az = Sz$ then by inequality (3.1),(3.2), we have $Az = Sz = By = Ty$ *so* $Ax = Az$ *and* $w = Ax = Sx$ is the unique point of coincidence of A and S. By lemma 2.3, w is only common fixed point of A and S. Similarly, there is a unique point z in X such that $z = Bz = Tz$. We now show that $z = w$. By (3.1),(3.2), we have,

$$\int_0^{M(w,z,kt)} \varphi(t)dt = \int_0^{M(Aw,Bz,kt)} \varphi(t)dt$$

$$\geq \phi(\int_0^{\min\{M(Sw,Tz,t),M(Bz,Sw,t),M(Sw,Aw,t),M(Bz,Tz,t),M(Aw,Tz,t),\left(\frac{2.M(Sw,Aw,t)}{1+M(Bz,Tz,t)}\right)\}} \varphi(t)dt)$$

$$\geq \phi(\int_0^{\min\{M(w,z,t),M(z,w,t),M(w,w,t),M(z,z,t),M(w,z,t),\left(\frac{2.M(w,w,t)}{1+M(z,z,t)}\right)\}} \varphi(t)dt)$$

$$\geq \phi(\int_0^{\min\{M(w,z,t),M(z,w,t),1,1,M(w,z,t),1\}} \varphi(t)dt)$$

$$\geq \phi(\int_0^{M(w,z,t)} \varphi(t)dt)$$

$$> \int_0^{M(w,z,t)} \varphi(t)dt$$

and

$$\int_0^{N(w,z,kt)} \varphi(t)dt = \int_0^{N(Aw,Bz,kt)} \varphi(t)dt$$

$$\leq \Psi(\int_0^{\min\{N(Sw,Tz,t),N(Bz,Sw,t),N(Sw,Aw,t),N(Bz,Tz,t),N(Aw,Tz,t),\left(\frac{2.N(Sw,Aw,t)}{1+N(Bz,Tz,t)}\right)\}} \varphi(t)dt)$$

$$\leq \Psi(\int_0^{\min\{N(w,z,t),N(z,w,t),N(w,w,t),N(z,z,t),N(w,z,t),\left(\frac{2.N(w,w,t)}{1+N(z,z,t)}\right)\}} \varphi(t)dt)$$

$$\leq \Psi(\int_0^{\min\{N(w,z,t),N(z,w,t),0,0,N(w,z,t),0\}} \varphi(t)dt)$$

$$\leq (\int_0^{N(w,z,t)} \varphi(t)dt)$$

$$< \int_0^{N(w,z,t)} \varphi(t)dt$$

Therefore by lemma 2.2, we have $w = z$, hence z is a common fixed point of A, B, S and T. For uniqueness, let u be another common fixed point of A, B, S and T. Then by (3.1),(3.2), we have

$$\int_0^{M(z,u,kt)} \varphi(t)dt = \int_0^{M(Az,Bu,kt)} \varphi(t)dt$$

$$\geq \phi(\int_0^{\min\{M(Sz,Tu,t),M(Bu,Sz,t),M(Sz,Az,t),M(Bu,Tu,t),M(Az,Tu,t),\left(\frac{2.M(Sz,Az,t)}{1+M(Bu,Tu,t)}\right)\}} \varphi(t)dt)$$

$$\geq \phi(\int_0^{\min\{M(z,u,t),M(u,z,t),M(z,z,t),M(u,u,t),M(z,u,t).\left(\frac{2.M(z,z,t)}{1+M(u,u,t)}\right)\}} \varphi(t)dt)$$

$$\geq \phi(\int_0^{\min\{M(z,u,t),M(u,z,t),1,1,M(z,u,t),1\}} \varphi(t)dt)$$

$$\geq \phi(\int_0^{M(z,u,t)} \varphi(t)dt)$$

$$> \int_0^{M(z,u,t)} \varphi(t)dt$$

and

$$\int_0^{N(z,u,kt)} \varphi(t)dt = \int_0^{N(Az,Bu,kt)} \varphi(t)dt$$

$$\leq \Psi(\int_0^{\min\{N(Sz,Tu,t),N(Bu,Sz,t),N(Sz,Az,t),N(Bu,Tu,t),N(Az,Tu,t).\left(\frac{2.N(Sz,Az,t)}{1+N(Bu,Tu,t)}\right)\}} \varphi(t)dt)$$

$$\leq \Psi(\int_0^{\min\{N(z,u,t),N(u,z,t),N(z,z,t),N(u,u,t),N(z,u,t),\left(\frac{2.N(z,z,t)}{1+N(u,u,t)}\right)\}} \varphi(t)dt)$$

$$\leq \Psi(\int_0^{\min\{N(z,u,t),N(u,z,t),0,0,N(z,u,t),0\}} \varphi(t)dt)$$

$$\leq (\int_0^{N(z,u,t)} \varphi(t)dt)$$

$$< \int_0^{N(z,u,t)} \varphi(t)dt$$

Therefore, by lemma 2.2, we have $z = u$. Hence z is unique common fixed point of A, B, S and T.
This complete the proof.

Theorem 3.2. Let (X, M, N,*,◊) be an intuitionistic fuzzy metric space with continuous $t * norm$ and continuous $t - conorm$ ◊. Let A, B, S and T be self mappings of X. Let the pairs (A, S) and (B, T) be owc. For all $x, y \in X$, there exist non increasing, non decreasing continuous functions $\phi, \Psi: [0,1] \to [0,1]$ such that $\phi(t) > t, \Psi(t) < t \; for \; all \; t \in (0,1)$. For every t > 0 there exist $k \in (0,1)$such that

$$\int_0^{M(Ax,By,kt)} \varphi(t)dt \geq \phi\left(\int_0^{m(Ax,By,t)} \varphi(t)dt\right)$$

and

$$\int_0^{N(Ax,By,kt)} \varphi(t)dt \leq \Psi\left(\int_0^{n(Ax,By,t)} \varphi(t)dt\right)$$

Where $\varphi: R^+ \to R^+$ is a lebesgue integrable mapping which is summable, non negative such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \; for \; each \;\; \varepsilon > 0$$

and

$$M(Ax, By, t) = \min\{M(Sx, Ty, t), M(By, Sx, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), \left(\frac{2.M(Sx, Ax, t)}{1 + M(By, Ty, t)}\right)\}$$

and

$$N(Ax, By, t) = \max\{N(Sx, Ty, t), N(By, Sx, t), N(Sx, Ax, t), N(By, Ty, t), M(Ax, Ty, t), \left(\frac{2.N(Sx, Ax, t)}{1 + N(By, Ty, t)}\right)\}$$

for all $x, y \in X$ and t > 0. And$\phi, \Psi: [0,1]^6 \to [0,1]$ such that $\phi(t, t, 1,1, t, 1) < t$, $\Psi(t, t, 0,0, t, 0) < t$ for all $t \in (0,1)$. Then, there is a unique common fixed point of A, B, S and T.

Proof: The proof follows on the lines of theorem 3.1.

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