International Journal of Theoretical & Applied Sciences, 10(1): 53-65(2018)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Different Deterioration Rates of two Warehouse Inventory Model with Time and Price Dependent Demand under Inflation and Permissible Delay in Payments

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(Received 03 January, 2018, accepted 27 January, 2018)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: A two warehouse inventory model with different deterioration rates is developed. Demand is considered as function of price and time. Holding cost is considered as linear function of time. Inflation factor is also considered with permissible delay. Shortages are not allowed. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

Keywords: Two warehouse, Different deterioration, Time dependent demand, Price dependent demand, Inflation, Permissible Delay in Payments

I. INTRODUCTION

Inventory problems for deterioration items have been studied extensively by many researchers from time to time. Whitin [25] developed inventory model for fashion goods deteriorating at the end of prescribed storage period. Ghar and Schrader [7] developed an inventory model with a constant rate of deterioration. Shah and Jaiswal [19] considered an order level inventory model for items deteriorating at a constant rate. Alfares [2] developed an inventory model with a stock level demand rate and a variable holding cost with the assumption that holding cost increases with time spent in storage. The related works are found in (Nahmias [13], Raffat [16], Goyal and Giri [9], Ruxian *et al*. [17]).

Buzacott [4] considered inventory model by considering inflationary impacts into record. Su *et al.* [22] developed model under inflation for stock dependent consumption rate and exponential decay. Moon *et al*. [12] developed models for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effects of inflation and time value of money.

It is generally assumed that a supplier must be paid for items as and when the customer receives the items. But many times it happens that the supplier allows credit for some fixed time period in settling the payment for the product and is not charged any interest from the customer for that specified period. However, if he pays beyond that specified period, then the interest will be charged. Goyal [8] first considered the economic order quantity model under the condition of permissible delay in payments. Goyal's [8] model was extended by Aggarwal and Jaggi [1] for deteriorating items. An inventory model with varying rate of deterioration and linear trend in demand under trade credit was considered by Chang and Dye [5]. Teng *et al.* [23] developed an optimal pricing lot sizing model by considering price sensitive demand under permissible delay in payments. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao *et al.* [11]. Singh [21] developed an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account. Patel and Patel [15] developed an eoq model with linear demand under permissible delay in payments. A literature review on inventory model under trade credit is given by Chang *et al*. [6].

To take advantage of price discounts, many times retailer decides to buy goods exceeding his Own Warehouse (OW) capacity. Hence an additional warehouse is arranged known as Rented Warehouse (RW) which has better storage facilities with higher inventory carrying cost and low rate of deterioration. A two warehouse inventory model for deteriorating items with linear demand and shortages was developed by Bhunia [3]. Sana *et al*. [18] proposed two warehouse inventory model on pricing decision. Yu *et al* [26] gave two warehouse inventory model for deteriorating items with decreasing rental over time.

Tyagi [24] proposed a two warehouse inventory model with time dependent demand and variable holding cost. Sheikh and Patel [20] developed a two warehouse inventory model under linear demand and time varying holding cost. Parekh and Patel [14] developed deteriorating item inventory models for two warehouses with linear demand under inflation and permissible delay in payments. Jaggi *et al*. [10] gave replenishment policy for non-instantaneous deteriorating items in two storage facilities under inflation.

Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory model.

In this paper we have developed a two warehouse inventory model with different deterioration rates. Demand function is price and time dependent. Holding cost is time varying. Shortages are not allowed. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

II. ASSUMPTIONS AND NOTATIONS:

Notations: The following notations are used for the development of the model:

D(t) : Demand is a function of time and price ($a + bt - \rho p$, $a>0$, $0<b<1$, $\rho>0$)

HC(OW) : Holding cost is linear function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW.

HC(RW) : Holding cost is linear function of time t (x_2+y_2t, $x2>0$, $0<y_2<1$) in RW.

A : Ordering cost per order

c : Purchasing cost per unit

p : Selling price per unit

T : Length of inventory cycle

$I_0(t)$: Inventory level in OW at time t

$I_r(t)$: Inventory level in RW at time t

I_e : Interest earned per year

I_p : Interest paid in stocks per year

R : Inflation rate

Q : Order quantity

t_r : Time at which inventory level becomes zero in RW.

W : Capacity of own warehouse

θ : Deterioration rate in OW during $\mu_1<t<\mu_2$, $0<\theta<1$

θt : Deterioration rate in OW during $\mu_2 \le t \le T$, $0<\theta<1$

π : Total relevant profit per unit time.

Assumptions: The following assumptions are considered for the development of model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- OW has fixed capacity W units and RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory cost per unit in the RW is higher than those in the OW.
- Deteriorated units neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

At time t=0, Q units enters into the system of which W are stored in OW and rest (Q-W) are stored in RW. At time t_r level of inventory in RW reaches to zero because of demand and OW inventory remains W. During the interval (t_r,μ_1) inventory depletes in OW due to demand, during interval (μ_1, μ_2) inventory depletes in OW due to deterioration at rate θ and demand. During interval (μ_2, T) inventory in OW depletes due to joint effect of deterioration at rate θt and demand. By time T both the warehouses are empty.

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

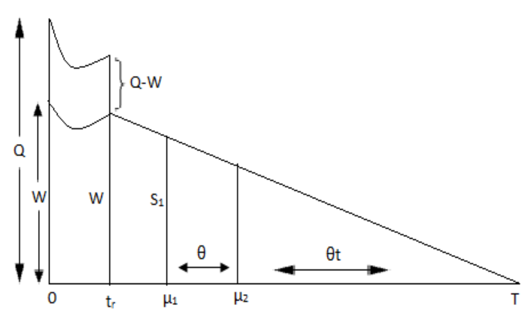


Fig. 1.

Hence, the inventory level at time t in RW and OW and governed by the following differential equations:

$$\frac{dI_r(t)}{dt} = -(a + bt - \rho p), \qquad 0 \le t \le t_r \quad (1)$$

$$\frac{dI_0(t)}{dt} = 0, \qquad 0 \le t \le t_r \quad (2)$$

$$\frac{dI_0(t)}{dt} = -(a + bt - \rho p), \qquad t_r \le t \le \mu_1 \quad (3)$$

$$\frac{dI_0(t)}{dt} + \theta I_0(t) = -(a+bt - \rho p), \qquad \mu_1 \le t \le \mu_2 \quad (4)$$

$$\frac{dI_0(t)}{dt} + \theta t I_0(t) = -(a+bt - \rho p), \qquad \mu_2 \le t \le T \quad (5)$$

with initial conditions $I_0(0) = W$, $I_0(\mu_1) = S_1$, $I_0(t_r)=W$, $I_r(0) = Q-W$, $I_r(t_r) = 0$ and $I_0(T) = 0$.
Solving equations (1) to (5) we have,

$$I_r(t) = Q - W - (a - \rho p)t - \frac{1}{2}bt^2 \quad (6)$$

$$I_0(t) = W \quad (7)$$

$$I_0(t)=S_1+(a-\rho p)\left(\mu_1-t\right)+\frac{1}{2}b\left(\mu_1^2-t^2\right) \quad (8)$$

$$I_0(t) = \left[\begin{array}{l} a\left(\mu_1-t\right)-\rho p\left(\mu_1-t\right)+\frac{1}{2}a\theta\left(\mu_1^2-t^2\right) \\ -\frac{1}{2}\rho p\theta\left(\mu_1^2-t^2\right)+\frac{1}{2}b\left(\mu_1^2-t^2\right) \\ +\frac{1}{3}b\theta\left(\mu_1^3-t^3\right)-a\theta t\left(\mu_1-t\right) \\ +\rho pt\left(\mu_1-t\right)-\frac{1}{2}b\theta t\left(\mu_1^2-t^2\right) \end{array}\right] + S_1\left(1+\theta(\mu_1-t)\right) \quad (9)$$

$$I_0(t) = \left[\begin{array}{l} a\left(T-t\right)-\rho p\left(T-t\right)+\frac{1}{6}a\theta\left(T^3-t^3\right) \\ -\frac{1}{6}\rho p\theta\left(T^3-t^3\right)+\frac{1}{2}b\left(T^2-t^2\right) \\ +\frac{1}{8}b\theta\left(T^4-t^4\right)-\frac{1}{2}a\theta t^2\left(T-t\right) \\ +\frac{1}{2}\rho p\theta t^2\left(T-t\right)-\frac{1}{4}b\theta t^2\left(T^2-t^2\right) \end{array}\right] \quad (10)$$

(by neglecting higher powers of θ)

Putting t = tr in equation (6), we get

$$Q = W + (a - \rho p)t_r + \frac{1}{2}bt_r^2 \quad (11)$$

Putting t = tr in equations (7) and (8), we get

$$I_0(t_r) = W \quad (12)$$

$$I_0(t_r)=S_1+(a-\rho p)\left(\mu_1-t_r\right)+\frac{1}{2}b\left(\mu_1^2-t_r^2\right) \quad (13)$$

So from equations (12) and (13), we have

$$S_1=W-(a-\rho p)\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^2-t_r^2\right) \quad (14)$$

Putting t = μ_2 in equations (9) and (10), we get

$$I_0(t) = \left[\begin{array}{l} a\left(\mu_1-\mu_2\right)-\rho p\left(\mu_1-\mu_2\right)+\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ +\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)-a\theta t\left(\mu_1-\mu_2\right) \\ +\rho pt\left(\mu_1-\mu_2\right)-\frac{1}{2}b\theta t\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + S_1\left(1+\theta(\mu_1-\mu_2)\right) \quad (15)$$

$$I_0(t) = \left[\begin{array}{l} a\left(T-\mu_2\right)-\rho p\left(T-\mu_2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ -\frac{1}{6}\rho p\theta\left(T^3-\mu_2^3\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right) \\ +\frac{1}{2}\rho p\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta t^2\left(T^2-\mu_2^2\right) \end{array}\right] \quad (16)$$

So from equations (15) and (16), we have

$$T = \frac{1}{b\left(\theta\mu_2^2-2\right)} \left(\begin{array}{l} -a\theta\mu_2^2 + \rho p\theta\mu_2^2 + 2a - 2\rho p \\ + \sqrt{\begin{array}{l} -4b\rho p\theta\mu_2^2 + 4b\theta\mu_2^2\rho pt_r - 2a\theta^2\mu_2^4\rho p \\ + 8a\theta\mu_2^2\rho p - 2b\theta^2\mu_2^2\rho p\mu_1^2 - 4b\theta^2\mu_2^2W\mu_1 \\ +4b\theta^2\mu_2^2\rho pt_r\mu_1+2ab\theta^2\mu_2^2\mu_1^2+2b\theta^2\mu_2^4\rho p \\ - 4ab\theta^2\mu_2^2t_r\mu_1 + 4ab\theta^2\mu_2^3t_r - 8bW\theta\mu_2 \\ - 4ab\theta\mu_1^2 + 4ab\theta\mu_2^2 -8b\rho pt_r +4b\theta\rho p\mu_1^2 \\ - 8b\theta\mu_2\rho pt_r\mu_1 + 8ab\theta t_r\mu_1 + 8b\theta\rho pt_r\mu_2 \\ - 8ab\theta t_r\mu_2 + 4b\theta^2\mu_2^3W - 2b^2\theta\mu_2^2t_r^2 \\ - 4\rho^2p^2\theta\mu_2^2 + a\rho^2p^2\theta^2\mu_2^4 + 8bW\theta\mu_1 \\ -4b\theta\mu_2^2W-4b\theta^2\mu_2^3\rho pt_r-4ab\theta\mu_2^2t_r+a^2\theta^2\mu_2^4 \\ - 4a^2\theta\mu_2^2 - 8a\rho p + b^2\theta^2\mu_2^6 + 8abt_r \\ - 2b^2\theta\mu_2^4 + 4a^2 + 4\rho^2p^2+ 4b^2t_r^2 + 8bW \end{array}} \end{array} \right) \quad (17)$$

From equation (17), we see that T is a function of W and t_r, so T is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant profit(π), include the following elements:

(i) Ordering cost (OC) = A (18)

$$\text{(ii) } HC(OW)=\int_0^{t_r}(x_1+y_1t)e^{-Rt}I_0(t)\,dt+\int_{\mu_2}^{T}(x_1+y_1t)e^{-Rt}I_0(t)\,dt \quad (19)$$

$$\text{(iii) } HC(RW)=\int_0^{t_r}(x_2+y_2t)e^{-Rt}I_r(t)\,dt \quad (20)$$

$$\text{(iv) } DC=c\left(\int_{\mu_1}^{\mu_2}\theta e^{-Rt}I_0(t)\,dt+\int_{\mu_2}^{T}\theta te^{-Rt}I_0(t)\,dt\right) \quad (21)$$

$$\text{(v) } SR=p\left(\int_0^{T}(a+bt-\rho p)e^{-Rt}dt\right) \quad (22)$$

To determine the interest earned, there will be two cases i.e. Case I: ($0\le M\le T$) and Case II: ($M>T$).

Case I: ($0\le M\le T$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T. So

(vi) Interest earned per cycle:

$$IE_1=pI_e\int_0^{M}(a+bt-\rho p)te^{-Rt}dt \quad (23)$$

Case II: ($0\le T\le M$):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(vii) Interest earned up to the permissible delay period is:

$$IE_2=pI_e\left[\int_0^{T}(a+bt-\rho p)te^{-Rt}dt+(a+bT-\rho p)T(M-T)\right] \quad (24)$$

To determine the interest payable, there will be five cases i.e.

Interest payable per cycle for the inventory not sold after the due period M is

Case I: ($0\le M\le t_r$):

$$\text{(viii) } IP_1=cI_p\int_M^{T}I(t)e^{-Rt}dt$$

$$=cI_p\left(\int_M^{t_r}I_r(t)e^{-Rt}dt+\int_M^{t_r}I_0(t)e^{-Rt}dt+\int_{t_r}^{\mu_1}I_0(t)e^{-Rt}dt\right)+cI_p\left(\int_{\mu_1}^{\mu_2}I_0(t)e^{-Rt}dt+\int_{\mu_2}^{T}I_0(t)e^{-Rt}dt\right) \quad (25)$$

Case II: ($t_r\le M\le \mu_1$):

$$\text{(ix) } IP_2=cI_p\int_M^{T}I(t)e^{-Rt}dt=cI_p\left(\int_M^{\mu_1}I_0(t)e^{-Rt}dt+\int_{\mu_1}^{\mu_2}I_0(t)e^{-Rt}dt+\int_{\mu_2}^{T}I_0(t)e^{-Rt}dt\right) \quad (26)$$

Case III: ($\mu_1\le M\le \mu_2$):

$$\text{(x) } IP_3=cI_p\int_M^{T}I(t)e^{-Rt}dt=cI_p\left(\int_M^{\mu_2}I_0(t)e^{-Rt}dt+\int_{\mu_2}^{T}I_0(t)e^{-Rt}dt\right) \quad (27)$$

Case IV: ($\mu_2\le M\le T$):

$$\text{(xi) } IP_4=cI_p\int_M^{T}I(t)e^{-Rt}dt \quad (28)$$

Case V: (M>T):

(xii) $IP_5 = 0$ (29)

(by neglecting higher powers of b and R)

The total profit (π_i), i=1,2,3,4 and 5 during a cycle consisted of the following:

$$\pi_i=\frac{1}{T}[SR\text{-}OC\text{-}HC(RW)\text{-}HC(OW)\text{-}DC\text{-}IP_i+IE_i] \quad (30)$$

Substituting values from equations (18) to (29) in equation (30), we get total profit per unit. Putting $\mu_1=v_1T$, $\mu_2=v_2T$ and value of S_1 and T from equation (14) and (17) in equation (30), we get profit in terms of tr and p for the five cases as under:

$$\pi_1=\frac{1}{T}[SR\text{-}OC\text{-}HC(RW)\text{-}HC(OW)\text{-}DC\text{-}IP_1+IE_1] \quad (31)$$

$$\pi_2=\frac{1}{T}[SR\text{-}OC\text{-}HC(RW)\text{-}HC(OW)\text{-}DC\text{-}IP_2+IE_1] \quad (32)$$

$$\pi_3=\frac{1}{T}[SR\text{-}OC\text{-}HC(RW)\text{-}HC(OW)\text{-}DC\text{-}IP_3+IE_1] \quad (33)$$

$$\pi_4=\frac{1}{T}[SR\text{-}OC\text{-}HC(RW)\text{-}HC(OW)\text{-}DC\text{-}IP_4+IE_1] \quad (34)$$

$$\pi_5=\frac{1}{T}[SR\text{-}OC\text{-}HC(RW)\text{-}HC(OW)\text{-}DC\text{-}IP_5+IE_2] \quad (35)$$

The optimal value of t_r* and p* (say), which maximizes π_i can be obtained by solving equation (31), (32), (33), (34) and (35) by differentiating it with respect to t_r and p and equate it to zero i.e. $\frac{\partial \pi_i (t_r ,p)}{\partial t_r}=0, \ \frac{\partial \pi_i (t_r ,p)}{\partial p}=0, \ i=1,2,3,4,5$ (36)

provided it satisfies the condition

$$\begin{vmatrix} \frac{\partial \pi_i^2 (t_r ,p)}{\partial {t_r}^2} & \frac{\partial \pi_i^2 (t_r ,p)}{\partial t_r \partial p} \\ \frac{\partial \pi_i^2 (t_r ,p)}{\partial p \partial t_r} & \frac{\partial \pi_i^2 (t_r ,p)}{\partial p^2} \end{vmatrix} > 0, \ i=1,2,3,4,5. \quad (37)$$

IV. NUMERICAL EXAMPLE

Case I: Considering A= Rs.100, W = 65, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x_1 = Rs. 2, y_1=0.04, x_2 = Rs. 6, y_2=0.08, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.01 in appropriate units. The optimal value of t_r*=0.0513, p* = Rs. 50.7003, Profit*=Rs.11775.0326 and Q*=77.6454.

Case II: Considering A= Rs.100, W = 65, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x_1= Rs. 2, y_1=0.04, x_2 = Rs. 6, y_2=0.08, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.07 in appropriate units. The optimal value of t_r*=0.0497, p* = Rs. 50.5995, Profit*=Rs.11834.9853 and Q*=77.2761.

Case III: Considering A= Rs.100, W = 65, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x_1=Rs.2, y_1=0.04, x_2 = Rs. 6, y_2=0.08, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.12 in appropriate units. The optimal value of t_r*=0.0458, p* = Rs. 50.5325, Profit*=Rs.11889.9319 and Q*=76.3281.

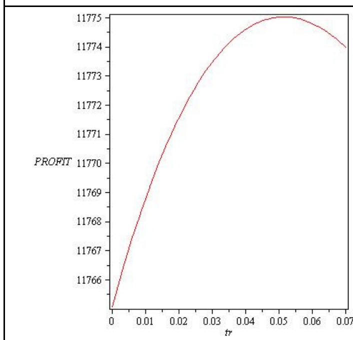
Case IV: Considering A= Rs.100, W = 65, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x_1=Rs.2, y_1=0.04, x_2 = Rs. 6, y_2=0.08, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.20 in appropriate units. The optimal value of t_r*=0.0345, p* = Rs. 50.4623, Profit*=Rs.11987.5446 and Q*=73.5453.

Case V: Considering A= Rs.100, W = 65, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x_1=Rs. 2, y_1=0.04, x_2 = Rs. 6, y_2=0.08, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.30 in appropriate units. The optimal value of t_r*=0.0136, p* = Rs. 50.4505, Profit*=Rs.12128.5746 and Q*=68.3694.

The second order conditions given in equation (37) are also satisfied. The graphical representation of the concavity of the profit function is also given.

Case I

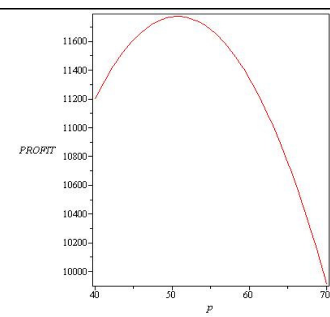
t_r and Profit



Graph 1

Case I

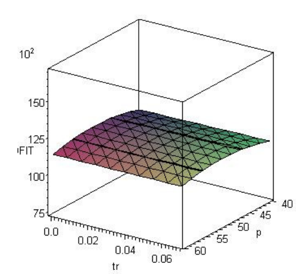
p and Profit



Graph 2

Case I

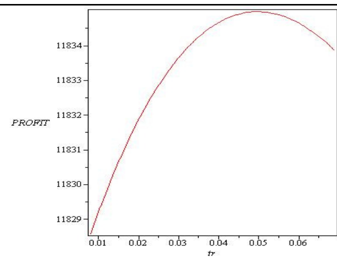
t_r, p and Profit



Graph 3

Case II

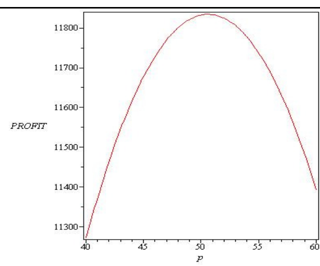
t_r and Profit



Graph 4

Case II

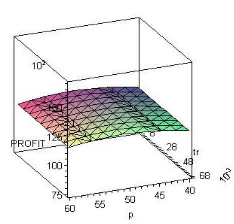
p and Profit



Graph 5

Case II

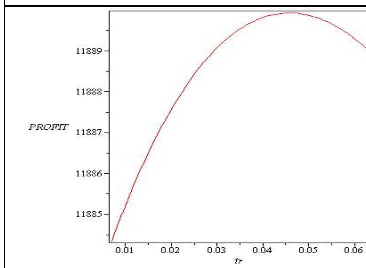
t_r, p and Profit



Graph 6

Case III

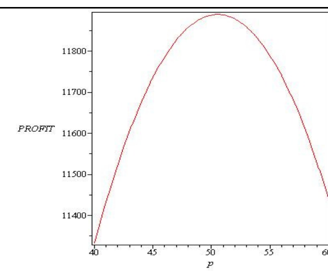
t_r and Profit



Graph 7

Case III

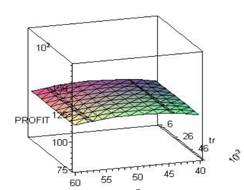
p and Profit



Graph 8

Case III

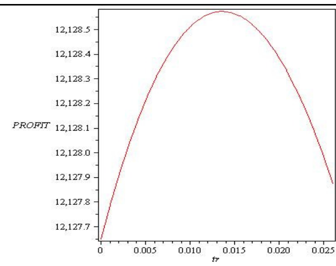
t_r, p and Profit



Graph 9

Case IV

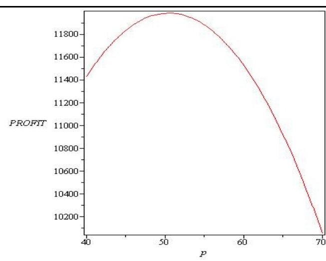
t_r and Profit



Graph 10

Case IV

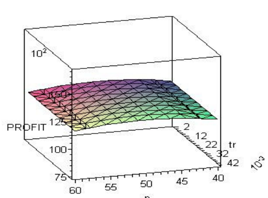
p and Profit



Graph 11

Case IV

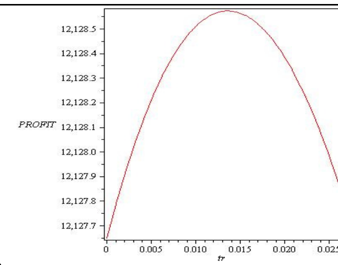
t_r, p and Profit



Graph 12

Case V

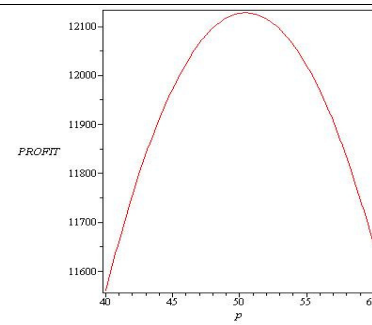
t_r and Profit



Graph 13

Case V

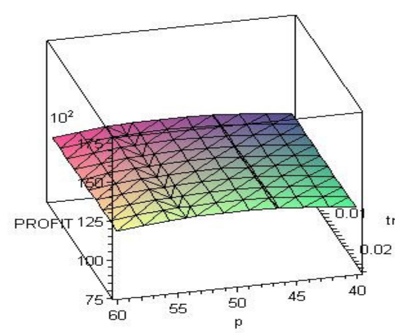
p and Profit



Graph 14

Case V

t_r, p and Profit



Graph 15

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1.
Case I ($0 \leq M \leq t_r$)
Sensitivity Analysis

Para-meter	%	t_r	p	Profit	Q
a	+20%	0.0725	60.6245	17160.1483	86.5273
	+10%	0.0636	55.6581	14342.3121	82.2808
	-10%	0.0338	45.7547	9458.6865	72.4775
	-20%	0.0086	40.8274	7393.8518	66.6844
θ	+20%	0.0497	50.7061	11772.0687	77.2496
	+10%	0.0504	50.7032	11773.5486	77.4228
	-10%	0.0521	50.6974	11776.5209	77.8433
	-20%	0.0529	50.6945	11778.0134	78.0414
x_1	+20%	0.0414	50.7440	11761.9621	75.1960
	+10%	0.0463	50.7218	11768.4201	76.4079
	-10%	0.0562	50.6795	11781.7975	78.8591
	-20%	0.0612	50.6594	11788.7125	80.0983
x_2	+20%	0.0603	50.7395	11756.5013	79.8521
	+10%	0.0558	50.7201	11765.6983	78.7491
	-10%	0.0466	50.6801	11784.5107	76.4916
	-20%	0.0419	50.6595	11794.1392	75.3369
A	+20%	0.0815	50.7070	11714.2499	85.0870
	+10%	0.0667	50.7030	11743.9460	81.4406
	-10%	0.0349	50.6996	11807.7267	73.6029
	-20%	0.0176	50.7014	11842.3087	69.3383
M	+20%	0.0513	50.6966	11776.9263	77.6464
	+10%	0.0513	50.6984	11775.9785	77.6459
	-10%	0.0512	50.7022	11774.0885	77.6203
	-20%	0.0512	50.7041	11773.1462	77.6198
R	+20%	0.0539	50.7226	11731.3620	78.2803
	+10%	0.0526	50.7116	11753.1905	77.9629
	-10%	0.0500	50.6890	11796.8901	77.3278
	-20%	0.0482	50.6774	11818.7643	76.8868
ρ	+20%	0.0608	42.3697	9712.0191	79.9436
	+10%	0.0564	46.1562	10649.6506	78.8824
	-10%	0.0453	56.2548	13150.7991	76.1825
	-20%	0.0383	63.1984	14870.9239	74.4680

Table 2.
Case II ($t_r \le M \le \mu_1$)
Sensitivity Analysis

Para-meter	%	t_r	p	Profit	Q
a	+20%	0.0696	60.5251	17235.6924	85.6974
	+10%	0.0613	55.5580	14409.8691	81.6865
	-10%	0.0330	45.6530	9511.3840	72.3173
	-20%	0.0086	40.7248	7439.6058	66.6888
θ	+20%	0.0481	50.6052	11832.0740	76.8795
	+10%	0.0489	50.6024	11833.5277	77.0778
	-10%	0.0505	50.5966	11836.4471	77.4744
	-20%	0.0513	50.5937	11837.9130	77.6728
x_1	+20%	0.0398	50.6438	11822.0043	74.8219
	+10%	0.0447	50.6213	11828.4169	76.0362
	-10%	0.0546	50.5784	11841.7071	78.4922
	-20%	0.0596	50.5580	11848.5801	79.7338
x_2	+20%	0.0588	50.6384	11816.2946	79.5124
	+10%	0.0543	50.6191	11825.5697	78.4070
	-10%	0.0450	50.5795	11844.5479	76.1196
	-20%	0.0403	50.5590	11854.2645	74.9624
A	+20%	0.0801	50.6045	11773.8270	84.7830
	+10%	0.0653	50.6012	11803.6985	81.1288
	-10%	0.0332	50.5998	11867.9116	73.2004
	-20%	0.0158	50.6030	11902.7674	68.9023
M	+20%	0.0488	50.5791	11849.9118	77.0587
	+10%	0.0493	50.5892	11842.4040	77.1798
	-10%	0.0500	50.6101	11827.6556	77.3475
	-20%	0.0503	50.6211	11820.4147	77.4188
R	+20%	0.0525	50.6217	11791.2256	77.9619
	+10%	0.0511	50.6107	11813.0978	77.6190
	-10%	0.0482	50.5882	11856.8901	76.9083
	-20%	0.0464	50.5766	11878.8144	76.4663
ρ	+20%	0.0602	42.2682	9769.8524	79.8328
	+10%	0.0553	46.0550	10708.4383	78.6424
	-10%	0.0431	56.1544	13212.2091	75.6589
	-20%	0.0353	63.0988	14934.2118	73.7405

Table 3.
Case II ($\mu_1 \leq M \leq \mu_2$)
Sensitivity Analysis

Para-meter	%	t_r	p	Profit	Q
a	+20%	0.0632	60.4610	17308.6533	83.8144
	+10%	0.562	55.4925	14473.4174	80.3167
	-10%	0.0303	45.5847	9558.4661	71.7289
	-20%	0.0072	40.6548	7479.4846	66.4164
θ	+20%	0.0443	50.5383	11887.0900	75.9558
	+10%	0.0450	50.5354	11888.5090	76.1296
	-10%	0.0466	50.5296	11891.3587	76.5266
	-20%	0.0473	50.5267	11892.7895	76.7005
x_1	+20%	0.0359	50.5784	11877.1925	73.8712
	+10%	0.0408	50.5551	11883.4827	75.0868
	-10%	0.0508	50.5107	11896.5375	77.5703
	-20%	0.0557	50.4896	11903.2973	78.7887
x_2	+20%	0.0550	50.5706	11870.9666	78.5931
	+10%	0.0504	50.5518	11880.3761	77.4610
	-10%	0.0410	50.5129	11899.6408	75.1449
	-20%	0.0362	50.4928	11909.5104	73.9608
A	+20%	0.0766	50.5340	11828.0095	83.9456
	+10%	0.0616	50.5323	11858.2365	80.2361
	-10%	0.0291	50.5352	11923.3349	72.1971
	-20%	0.0114	50.5412	11958.7577	67.8191
M	+20%	0.0431	50.5064	11917.9330	75.6659
	+10%	0.0445	50.5189	11903.7985	76.0095
	-10%	0.0469	50.5471	11876.3301	76.5967
	-20%	0.0479	50.5627	11862.9904	76.8403
R	+20%	0.0489	50.5548	11846.0104	77.0894
	+10%	0.0474	50.5437	11867.9616	76.7212
	-10%	0.0440	50.5212	11911.9236	75.8854
	-20%	0.0421	50.5096	11933.9394	75.4178
ρ	+20%	0.0582	42.1997	9820.7358	79.3639
	+10%	0.0525	45.9872	10761.1376	77.9712
	-10%	0.0380	56.0886	13270.0054	74.4089
	-20%	0.0288	63.0345	14995.7322	72.1384

Table 4.
Case II ($\mu_2 \leq M \leq T$)
Sensitivity Analysis

Para-meter	%	t_r	p	Profit	Q
a	+20%	0.0448	60.4034	17445.8747	78.3496
	+10%	0.0413	55.4281	14589.5630	76.2691
	-10%	0.0223	45.5092	9639.7878	69.9607
	-20%	0.0025	40.5746	7546.4780	65.4928
θ	+20%	0.0329	50.4683	11984.8379	73.1480
	+10%	0.0336	50.4653	11986.1895	73.3218
	-10%	0.0351	50.4592	11988.9032	73.6944
	-20%	0.0358	50.4561	11990.2655	73.8684
x_1	+20%	0.0243	50.5130	11975.5543	71.0127
	+10%	0.0293	50.4872	11981.4643	72.2536
	-10%	0.0394	50.4382	11993.7919	74.7637
	-20%	0.0445	50.4150	12000.2093	76.0327
x_2	+20%	0.0440	50.4980	11967.8234	75.8904
	+10%	0.0392	50.4803	11977.6022	74.7059
	-10%	0.0294	50.4439	11997.6590	72.3847
	-20%	0.0244	50.4251	12007.9545	71.0481
A	+20%	0.0664	50.4531	11923.4101	81.4497
	+10%	0.0508	50.4562	11954.6599	77.5842
	-10%	0.0169	50.4721	12022.3535	69.1851
	-20%	0.0000	50.4790	12059.4626	65.0000
M	+20%	0.0260	50.4474	12041.2131	71.4418
	+10%	0.0304	50.4529	12013.9480	72.5312
	-10%	0.0379	50.4751	11961.9697	74.3850
	-20%	0.0410	50.4912	11937.1966	75.1493
R	+20%	0.0385	50.4872	11943.1291	74.5318
	+10%	0.0365	50.4733	11965.3208	74.0386
	-10%	0.0321	50.4511	12009.8046	72.9536
	-20%	0.0295	50.4397	12032.1058	72.3101
ρ	+20%	0.0520	42.1232	9907.1347	77.8576
	+10%	0.0439	45.9132	10852.5082	75.8643
	-10%	0.0231	56.0235	13375.6700	70.7264
	-20%	0.0097	62.9770	15112.1420	67.4065

From the table we observe that as parameter a increases/ decreases average total profit and order quantity increases/ decreases for all five cases.
From the table we observe that as parameter θ increases/ decreases there is very minor change in average total profit and order quantity for all five cases.
From the table we observe that as parameter x_1 increases/ decreases average total profit and order quantity decreases/ increases for all five cases.
From the table we observe that as parameters x_2, A, R and ρ increases/ decreases average total profit decreases/ increases and order quantity increases/ decreases for all five cases.

Table 5.
Case II (M> T)
Sensitivity Analysis

Para-meter	%	t_r	p	Profit	Q
a	+20%	0.0265	60.4053	17654.7592	70.2463
	+10%	0.0212	55.4243	14762.6012	69.7250
	-10%	0.0025	45.4872	9752.8237	65.6814
	-20%	0.0000	40.4659	7635.0903	65.0000
θ	+20%	0.0126	50.4566	12126.0857	68.1212
	+10%	0.0131	50.4536	12127.3290	68.2453
	-10%	0.0141	50.4474	12129.8225	68.4934
	-20%	0.0146	50.4443	12131.0727	68.6175
x_1	+20%	0.0056	50.5073	12117.9762	66.3858
	+10%	0.0096	50.4785	12123.2010	67.3770
	-10%	0.0176	50.4232	12134.0942	69.3627
	-20%	0.0216	50.3966	12139.7572	70.3572
x_2	+20%	0.0214	50.4840	12107.3212	70.2982
	+10%	0.0175	50.4674	12117.8701	69.3341
	-10%	0.0096	50.4332	12139.4417	67.3792
	-20%	0.0055	50.4155	12150.4791	66.3637
A	+20%	0.0401	50.4239	12059.2046	74.9400
	+10%	0.0271	50.4360	12093.1023	71.7159
	-10%	0.0000	50.4636	12165.8654	65.0000
	-20%	0.0000	50.3898	12204.0987	65.0000
M	+20%	0.0136	50.4473	12218.5724	68.3695
	+10%	0.0136	50.4489	12173.5735	68.3695
	-10%	0.0136	50.4521	12083.5758	68.3692
	-20%	0.0136	50.4538	12038.5770	68.3691
R	+20%	0.0176	50.4725	12083.0734	69.3584
	+10%	0.0156	50.4615	12105.8026	68.8640
	-10%	0.0114	50.4394	12151.3944	67.8249
	-20%	0.0090	50.4281	12174.2677	67.2307
ρ	+20%	0.0355	42.0950	10024.3757	75.2781
	+10%	0.0249	45.8930	10980.3017	71.7363
	-10%	0.0014	55.0206	13533.5378	65.3078
	-20%	0.0000	62.8968	15290.9818	65.0000

From the table we observe that as parameter M increases/ decreases average total profit also increases/ decreases for all five cases but for order quantity almost remains fixed for all five cases.

VI. CONCLUSION

In this paper, we have developed a two warehouse inventory model for deteriorating items with different deterioration rates under time and price dependent demand, and time varying holding cost under inflationary conditions. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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