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A New Class of Generalized Laguerre Polynomials

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ABSTRACT : In the present paper, a new class of generalized Laguerre polynomials, $L_{n,m,s}^{(\alpha,\beta,\gamma)}$ in three variables and three parameters has been introduced. A number of bilateral and trilateral generating functions for the new class have been obtained. Several special cases have also been discussed.

I. INTRODUCTION

Laguerree (Rainville 1960) introduced first Laguerre polynomials $L_n(x)$ and $L_n^{(\alpha)}(x)$, Many generalisations of these polynomials have been obtained by mathematicians. S.F. Ragab (1991) [2] introduced the generalized Laguerre polynomials $L_n^{(\alpha,\beta)}(x,y)$ with two parameters and two variables as

$$L_n^{(\alpha,\beta)}(x,y)=\frac{(\alpha+1)_n(\beta+1)_n}{(n!)^2}\sum_{p=0}^{n}\sum_{q=0}^{n-p}\frac{(-n)_{p+q}\,x^q y^p}{(\alpha+1)_q(\beta+1)_p\,q!\,p!} \quad \text{... (1.1)}$$

Sahoo, H.C. (2007) introduced a generalisation of Laguerre polynomials in two variables and two parameters as follows [3].

$$L_{m,n}^{(\alpha,\beta)}(x,y)=\frac{(\alpha+1)_m(\beta+1)_n}{m!\,n!}\sum_{p=0}^{m}\sum_{q=0}^{n}\frac{(-m)_p(-n)_q\,x^p y^q}{(\alpha+1)_p(\beta+1)_q\,p!\,q!} \quad \text{...(1.2)}$$

Padhiary L. K. (2013) [1]. introduced Laguerre polynomials $L_n(x,y,z)$ of three variables and degree n defined by

$$L_n(x,y,z)=\sum_{p=0}^{n}\sum_{q=0}^{n-p}\sum_{r=0}^{n-p-q}\frac{(-n)_{p+q+r}\,x^r y^q z^p}{(r!)^2(q!)^2(p!)^2} \quad \text{...(1.3)}$$

In the present paper we have introduced a new interesting generatlisation of Laguerre polynomials of three variables and three parameters denoted by $L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)$. We have discussed some special cases of this genralised class. We have also found a number of bilateral and trilateral generating functions for this new class. Many special cases have also been discussed.

2. We introduce Laguerre polynomials of three variables and three parameters as follows:

$$L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)=\frac{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}{n!\,m!\,s!}\sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{s}\frac{(-n)_p(-m)_q(-s)_r}{(\alpha+1)_p(\beta+1)_q(\gamma+1)_r}\times\frac{x^p y^q z^r}{p!\,q!\,r!} \quad \text{.....(2.1)}$$

Where m, n, s are three non negative integers.
Here α, β, γ are parameters m, n, s are indices and x, y, z are independent variables. The polynomials (2.1) can also be expressed as.

$$L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)=\sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{s}\frac{(-1)^{p+q+r}(\alpha+1)_n(\beta+1)_m(\gamma+1)_s\,x^p y^q z^r}{p!\,q!\,r!(n-p)!(m-q)!(s-r)!(\alpha+1)_p(\beta+1)_q(\gamma+1)_r} \quad \text{...... (2.2)}$$

These polynomials are indeed polynomials of three variables x, y, and z.

For polynomials of three variables the triple hypergeometric series was introduced by many mathematicians. The newly introduced polynomial can be expressed in Shrivastava's

$F^{(3)}$ [x, y, z] functions as follows :

$$L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z) = F^{(3)}\left[\begin{matrix} -::\ -;-;-:-n;-m;-s; \\ -::-;-;-:\alpha+1;\beta+1;\gamma+1; \end{matrix} x,y,z\right]$$

where $F^3(x, y, z)$ function is the unification of Lauricalla's fourteen triple hypergeometric functions F_1, F_2 .... F_{14} and extended F_k function of Sharma (1970) [4] and three additional functions H_A, H_B and H_C of Srivastava. This unified $F^3(x, y, z)$ function was introduced by Srivastava, H. M. and Manocha H.L. (1984) [5] as

$$F^3\left[\begin{matrix} (a)::(b);(b')(b''):(c);(c');(c''); \\ (d)::(e);(e');(e''):(f);(f');(f''); \end{matrix} x,y,z\right]$$

$$= \sum_{m,n,p=0}^{\infty} \frac{[a]_{m+n+p}[b]_{m+n}[b']_{n+p}[b'']_{p+m}}{[d]_{m+n+p}[e]_{m+n}[e']_{n+p}[e'']_{p+m}} \times \frac{[c]_m[c']_n[c'']_p}{[f]_m[f']_n[f'']_p} \times \frac{x^m y^{-n} z^p}{m!n!p!}$$

3. Generating Functions : In this section we obtain some bilateral and trilateral generating functions for $L_{n,m,s}^{(\alpha,\beta,\gamma)}$ (x, y, z)

(i) By series manipulation method we obtain generating functions for the polynomials ....(2.1) as follows :

$$\sum_{n=0}^{\infty} \frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t^n}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{s} \frac{(-n)_p(-m)_q(-s)_r\, x^p y^q z^r t^m}{(\alpha+1)_p(\beta+1)_q(\gamma+1)_r\, p!q!r!n!m!s!}$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{m}\sum_{r=0}^{s} \frac{(-n-p)_p(-m)_q(-s)_r\, x^p y^q z^r t^{n+p}}{(\alpha+1)_p(\beta+1)_q(\gamma+1)_r\, p!q!r!(n+p)!m!s!}$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{m}\sum_{r=0}^{s} \frac{(-1)^p(n+p)^1(-m)_q(-s)_r\, x^p y^q z^r t^{n+p}}{n!(\alpha+1)_p(\beta+1)_q(\gamma+1)_r\, p!q!r!} \times \frac{1}{(n+p)!m!s!}$$

$$= \sum_{n=0}^{\infty}\frac{t^n}{n!}\sum_{p=0}^{\infty}\frac{(-xt)^p}{(\alpha+1)_p\,p!}\sum_{q=0}^{m}\sum_{r=0}^{s}\frac{(-m)_q(-s)_r\, y^q z^r}{(\beta+1)_q(\gamma+1)_r\, q!r!m!}$$

$$= e^t\,{}_0F_1(-;\alpha+1;-xt)\,\frac{L_{m,s}^{(\beta,\gamma)}(y,z)}{(\beta+1)_m(\gamma+1)_s}$$

Thus we obtain a very important bilateral generating function.

$$\sum_{n=0}^{\infty} \frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t^n}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s} = e^t\,{}_0F_1(-;\alpha+1;-xt)\,\frac{L_{m,s}^{(\beta,\gamma)}(y,z)}{(\beta+1)_m(\gamma+1)_s} \quad ....(3.1)$$

We can also obtain another two sets of bilateral generating functions as

$$\sum_{m=0}^{\infty} \frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t^m}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s} = e^t\,{}_0F_1(-;\beta+1;-yt)\,\frac{L_{n,s}^{(\alpha,\beta)}(x,z)}{(\alpha+1)_n(\gamma+1)_s} \quad ....(3.2)$$

$$\sum_{s=0}^{\infty} \frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t^s}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s} = e^t\,{}_0F_1(-;\gamma+1;-zt)\,\frac{L_{n,s}^{(\alpha,\beta)}(x,y)}{(\alpha+1)_n(\gamma+1)_s} \quad ....(3.3)$$

(ii) We now obtain some double series generating functions as follows :

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t_1^n t_2^m}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{s}\frac{(-n)_p(-m)_q(-s)_r x^p y^q z^r t_1^n t_2^m}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s\,p!q!r!n!m!s!}$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{s}\frac{(-n-p)_p(-m-q)_q(-s)_r x^p y^q z^r t_1^{n+p} t_2^{m+q}}{(\alpha+1)_p(\beta+1)_q(\gamma+1)_r\,p!q!r!(n+p)!(m+q)!s!}$$

$$=\sum_{n,m,p,q=0}^{\infty}\sum_{r=0}^{s}\frac{(-1)^{p+q}(-s)_r x^p y^q z^r t_1^{n+p} t_2^{m+q}}{n!m!(\alpha+1)_p(\beta+1)_q(\gamma+1)_r\,p!q!r!s!}$$

$$=\sum_{n=0}^{\infty}\frac{t_1^n}{n!}\sum_{m=0}^{\infty}\frac{t_2^m}{m!}\sum_{p=0}^{\infty}\frac{(-xt_1)^p}{(\alpha+1)_p\,p!}\sum_{q=0}^{\infty}\frac{yt_2}{(\beta+1)_q\,q!}\times\sum_{r=0}^{s}\frac{(-s)_r z^r}{(\gamma+1)_r\,r!s!}$$

$$= e^{t_1}\,e^{t_2}\,{}_0F_1(-;\alpha+1;-xt)\,{}_0F_1(-;\beta+1;-yt_2)\,\frac{L_s^{(\gamma)}(z)}{(\gamma+1)_s}$$

Hence we obtain the most important trilateral generating function for the polynomial, $L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)$ as

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t_1^n t_2^m}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}$$

$$= e^{t_1+t_2}\,{}_0F_1(-;\alpha+1;-xt_1)\,{}_0F_1(-;\beta+1;-yt_2)\times\frac{L_s^{(\gamma)}(z)}{(\gamma+1)_s} \qquad \text{........(3.4)}$$

In a similar manner another two sets of trilateral generating functions involving double series expansion are derived as

$$\sum_{n=0}^{\infty}\sum_{s=0}^{\infty}\frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t_1^n t_2^s}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}$$

$$= e^{t_1+t_2}\,{}_0F_1(-;\alpha+1;-xt_1)\,{}_0F_1(-;\gamma+1;-zt_2)\times\frac{L_m^{(\beta)}(y)}{(\beta+1)_m} \qquad \text{..........(3.5)}$$

and

$$\sum_{m=0}^{\infty}\sum_{s=0}^{\infty}\frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t_1^m t_2^s}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}$$

$$= e^{t_1+t_2}\,{}_0F_1(-;\beta+1;-yt_1)\,{}_0F_1(-;\gamma+1;-zt_2)\times\frac{L_n^{(\alpha)}(x)}{(\alpha+1)_n} \qquad \text{..........(3.6)}$$

(iii) To obtain generating function involving triple series expansions for Laguerre polynomials, we proceed as follows:

$$\sum_{n,m,s=0}^{\infty}\frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)\,t_1^n t_2^m t_3^s}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s}$$

$$=\sum_{n,m,s=0}^{\infty}\sum_{p=0}^{n}\sum_{q=0}^{m}\sum_{r=0}^{s}\frac{(-n)_p(-m)_q(-s)_r x^p y^q z^r t_1^n t_2^m t_3^s}{(\alpha+1)_n(\beta+1)_m(\gamma+1)_s\,p!q!r!m!n!s!}$$

$$= \sum_{n,m,s=0}^{\infty} \sum_{p,q,r=0}^{\infty} \frac{(-n-p)_p (-m-q)_q (-s-r)_r x^p y^q z^r t_1^{n+p} t_2^{m+q} t_3^{s+r}}{(\alpha+1)_p (\beta+1)_q (\gamma+1)_r p!q!r!(n+p)!(m+q)!(s+r)!}$$

$$= \sum_{n,m,s,p,q,r=0}^{\infty} \frac{(-1)^{p+q+r} x^p y^q z^r t_1^{n+p} t_2^{m+q} t_3^{s+r}}{(\alpha+1)_p (\beta+1)_q (\gamma+1)p!q!r!n!m!s!}$$

$$= \sum_{p,q,r=0}^{\infty} \sum_{n=0}^{\infty} \frac{t_1^n}{n!} \sum_{m=0}^{\infty} \frac{t_2^m}{m!} \sum_{s=0}^{\infty} \frac{t_3^s}{s!} \frac{(-xt_1)^p (-yt_2)^q (-zt_3)^r}{p!q!r!(\alpha+1)_p (\beta+1)_q (\gamma+1)_r}$$

$$= e^{t_1} e^{t_2} e^{t_3} \sum_{p=0}^{\infty} \frac{(-xt_1)^p}{p!(\alpha+1)_p} \sum_{q=0}^{\infty} \frac{(-yt_2)^q}{q!(\beta+1)_q} \sum_{r=0}^{\infty} \frac{(-zt_3)^r}{r!(\gamma+1)_r}$$

$$= e^{t_1+t_2+t_3} {}_0F_1(-;\alpha+1;-xt) {}_0F_1(-;\beta+1;-yt_2) {}_0F_1(-;\gamma+1;-zt_3)$$

Hence we obtain the trilateral generating function for

$L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z)$ as

$$\sum_{n,m,s=0}^{\infty} \frac{L_{n,m,s}^{(\alpha,\beta,\gamma)}(x,y,z) t_1^n t_2^m t_3^s}{(\alpha+1)_n (\beta+1)_m (\gamma+1)_s}$$

$$= e^{t_1+t_2+t_3} {}_0F_1(-;\alpha+1;-xt) {}_0F_1(-;\beta+1;-yt_2) {}_0F_1(-;\gamma+1;-zt_3)$$

$$= e^{t_1+t_2+t_3} F^{(3)}\left[\begin{matrix} -::-;-;-:-;-;-; \\ -::-;-;-:\alpha+1;\beta+1;\gamma+1 \end{matrix} -xt_1, -yt_2, -zt_3\right] \quad \text{.......(3.7)}$$

4. **Special cases** : Putting $\alpha = 0$, $\beta = 0$ and $\gamma = 0$ in (2.2) we obtain the Laguerre polynomials in three variables

$L_{n,m,s}(x,y,z) = L_{n,m,s}^{(0,0,0)}(x,y,z)$

$$= \sum_{p=0}^{n} \sum_{q=0}^{m} \sum_{r=0}^{s} \frac{(-1)^{p+q+r} n!m!s! x^p y^q z^r}{(n-p)!(m-q!)(s-r)!(p!)^2 (q!)^2 (r!)^2} \quad \text{.........(4.1)}$$

Again if we put m = 0 = s (then q = r = 0) Thus we get from (4.1)

$$L_{n.0.0}^{(0,0,0)}(x,y,z) = \sum_{p=0}^{n} \frac{(-1)^p n! x^p}{(n-p)!(p!)^2} = L_n(x)$$

where $L_n(x)$ is the simple Laguerre polynomials
In a similar manner we establish the identity.

$$L_{n.m.0}^{(\alpha,0,0)}(x,y,z) = L_{n,m}(x,y) \quad \text{......... (4.2)}$$

as studied in the thesis of H. Sahoo (2007)
Now in the relation (2.2) if we put m = 0 = s. Then (q and r take values 0 only)

$L_{n,0,0}^{(\alpha,0,0)}(x,y,z)$

$$= \sum_{p=0}^{n} \frac{(-1)^p (\alpha+1)_n x^p}{(\alpha+1)_p (n-p)!p!} = L_n^{(\alpha)}(x) \quad \text{.........(4.3)}$$

Also in (2.2) if we put s = 0 (r taken the values 0 only)
we obtan the identity.

$$L_{n,m,s}^{(\alpha,\beta,0)}(x,y,z) = L_{n,m}^{(\alpha,\beta)}(x,y) \quad \text{......... (4.4)}$$

which is the two variables, two parameters. Laguerre polynomials as defined in the thesis of H. Sahoo (2007). Using these identites we can reduce the generating functions satisfied by the three variables, three parameters, Laguerre polynomial $L_{n,m,s}^{(\alpha,\beta,\gamma)}$ (x, y, z) to the generating function for two variables, two parameters. Laguerre polynomials $L_{n,m}^{(\alpha,\beta)}$ (x,y) and one variable, one parameter, Laguerre polynomials $L_n^{(\alpha)}$ (x). Also we can deduce generating functions for simple Laguerre polynomials of one variable, two variables and three variables $L_n(x)$, $L_n(x, y)$ and $L_n(x,y,z)$ respectively as follows :

In the generating function (3.1) if we put s = 0 and $\gamma = 0$ then we get

$$\sum_{n=0}^{\infty} \frac{L_{n,m,0}^{(\alpha,\beta,0)}(x,y,z)\,t^n}{(\alpha+1)_n(\beta+1)_m(0+1)_0} = e^t\,{}_0F_1(-;\alpha+1;-xt)\,\frac{L_{m,0}^{(\beta,0)}(y,z)}{(\beta+1)_m(0+1)_0}$$

which implies that

$$\sum_{n=0}^{\infty} \frac{L_{n,m}^{(\alpha,\beta)}(x,y)\,t^n}{(\alpha+1)_n(\beta+1)_m} = e^t\,{}_0F_1(-;\alpha+1;-xt)\,\frac{L_m^{(\beta)}(y)}{(\beta+1)_m} \qquad \text{......... (4.5)}$$

which is a completely new bilateral generating relation for $L_{n,m}^{(\alpha,\beta)}$ (x, y)

In relation (3.2) taking same substiution s = 0 and $\gamma = 0$, we obtain the important and new generating function.

$$\sum_{m=0}^{\infty} \frac{L_{n,m}^{(\alpha,\beta)}(x,y)\,t^m}{(\alpha+1)_n(\beta+1)_m} = e^t\,{}_0F_1(-;\beta+1;-yt)\,\frac{L_n^{(\alpha)}(x)}{(\alpha+1)_n} \qquad \text{.......... (4.6)}$$

Similarly substituting m = 0 = s and $\beta = 0 = \gamma$ in the relation (3.1) we obtain very interesting result as follows:

$$\sum_{n=0}^{\infty} \frac{L_{n,0,0}^{(\alpha,0,0)}(x,y,z)\,t^n}{(\alpha+1)_n(0+1)_0(0+1)_0} = e^t\,{}_0F_1(-;\alpha+1;-xt)\,\frac{L_{0,0}^{(0,0)}(y,z)}{(0+1)_0(0+1)_0}$$

which is same as

$$\sum_{n=0}^{\infty} \frac{L_n^{(\alpha)}(x)\,t^n}{(\alpha+1)_n} = e^t\,{}_0F_1(-;\alpha+1;-xt) \qquad \text{........ (4.7)}$$

a well known generating function for $L_n^{(\alpha)}$ (x).

Now considering the double series generating relation (3.4) , (3.5), (3.6)
we can deduce the following generating relations by taking first s = 0 and $\gamma = 0$,
From (3.4) we get.

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{L_{n,m,0}^{(\alpha,\beta,0)}(x,y,z)\,t_1^n t_2^m}{(\alpha+1)_n(\beta+1)_m(0+1)_0} = e^{t_1+t_2}\,{}_0F_1(-;\alpha+1;-xt_1)\,{}_0F_1(-;\beta+1;-yt_2) \times \frac{L_0^{(0)}(z)}{(0+1)_0}$$

which gives the result

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{L_{n,m}^{(\alpha,\beta)}(x,y)\,t_1^n t_2^m}{(\alpha+1)_n(\beta+1)_m} = e^{t_1+t_2}\,{}_0F_1(-;\alpha+1;-xt_1)\,{}_0F_1(-;\beta+1;-yt_2) \qquad \text{......... (4.8)}$$

which is the bilateral generating function for two variables, two parameters.
Laguerre polynomials as obtained in the thesis of H. Sahoo.
In a similar manner we obtain the bilateral generating functions by the same substitutions from the trilateral function (3.5) and (3.6) as

$$\sum_{n=0}^{\infty} \frac{L_{n,m,0}^{(\alpha,\beta,0)}(x,y,z)\,t_1^n}{(\alpha+1)_n(\beta+1)_m(0+1)_0} = e^t\,{}_0F_1(-;\alpha+1;-xt)\,{}_0F_1(-;0+1;0)\,\frac{L_m^{(\beta)}(y)}{(\beta+1)_m}$$

which is equal to

$$\sum_{n=0}^{\infty} \frac{L_{n,m}^{(\alpha,\beta)}(x,y)\,t_1^n}{(\alpha+1)_n(\beta+1)_m} = e^{t_1}\,{}_0F_1(-;\alpha+1;-xt)\,\frac{L_m^{(\beta)}(y)}{(\beta+1)_m} \qquad \text{............ (4.9)}$$

From (3.6) we get similar type of generating relation.

Now we consider the trilateral triple series generating function which after suitable substitution reduces to the double series bilateral generating functions.

Let us take s = 0 and $\gamma = 0$ in (3.7)

Then we get

$$\sum_{n,m=0}^{\infty} \frac{L_{n,m,0}^{(\alpha,\beta,0)}(x,y,z)\, t_1^n t_2^m}{(\alpha+1)_n (\beta+1)_m (0+1)_0} = e^{t_1+t_2}\, {}_0F_1(-;\alpha+1;-xt_1)\, {}_0F_1(-;\beta+1;-yt_2)$$

which is same as

$$\sum_{n,m=0}^{\infty} \frac{L_{n,m}^{(\alpha,\beta)}(x,y)\, t_1^n t_2^m}{(\alpha+1)_n (\beta+1)_m} = e^{t_1+t_2}\, {}_0F_1(-;\alpha+1;-xt_1)\, {}_0F_1(-;\beta+1;-yt_2)$$

which was obtained by H. Sahoo (2007).

In the same generating function, if we put m = 0 = s and $\beta = 0 = \gamma$, we reduce the triple series trilateral generating function to linear generating function. From (3.7) we get by above substitutions.

$$\sum_{n=0}^{\infty} \frac{L_{n,0,0}^{(\alpha,0,0)}(x,y,z)\, t_1^n}{(\alpha+1)_n (0+1)_0 (0+1)_0} = e^{t_1}\, {}_0F_1(-;\alpha+1;-xt_1)$$

which implies that

$$\sum_{n=0}^{\infty} \frac{L_n^{(\alpha)}(x)\, t^n}{(\alpha+1)_n} = e^{t_1}\, {}_0F_1(-;\alpha+1;-xt) \qquad \text{......... (4.10)}$$

which is exactly same as that in Rainville (1960, P - 201). In a similar manner, if we put $\alpha = 0 = \beta = \gamma$ and m = 0 = s, the relation (3.7) reduces to

$$\sum_{n=0}^{\infty} \frac{L_n(x)\, t^n}{n!} = e^t\, {}_0F_1(-;1;-xt) \qquad \text{.......... (4.11)}$$

which coincids with relation (3) of Rainville (1960, P - 213)

REFERENCES

[1]. Padhiary, L. K. (2013). Thesis for Ph. D, submitted to F. M. University, Balasore.

[2]. Ragab S. F. (1991). On Laguerre Polynomials of two variables $L_n^{(\alpha,\beta)}$ (x, y); *Bull cal, Math. Soc.*, **83**, P.P. 253–262.

[3]. Sahoo, H. C. (2007). Special functions of several variables and their generating function.

[4]. Sharma, B.L. (1970). Some formula for special functions; *Proc. Cambridge Philos. Soc.*, **67** (1970) P.P. 613-618.

[5]. Srivastava, H.M. and Manocha, H.L. (1984). A treaties on genrating functions, Ellis Horwood series john wiley and sons, New York.