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Mathematical Model for Drag Reduction due to Injection of Polymer Solutions into Laminar Flow in a Pipe

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ABSTRACT: The effect of a very thin layer of slightly viscoelastic fluid very close to the wall was studied analytically when a slightly viscoelastic fluid is being injected continuously and uniformly through the wall of the tube in which a purely viscous Newtonian fluid is flowing. The problem has been analyzed in two regions; one thin viscoelastic layer near the wall and the other, central core region of purely viscous fluid separating along with suitable matching conditions. Finally the wall shearing stress and total flux through a cross-section for a particular pressure gradient with and without the presence of the assumed thin layer of polymer additives has been presented.

Keywords: Drag Reduction, Polymer Injection, Laminar Flow, Viscoelastic Fluid, Wall Shear Stress.

I. INTRODUCTION

The discovery of gradual realization over a period of several years in early of 1960's was that extreme dilute polymer solutions could have turbulent frictional resistance properties much lower than those of pure solvent was a landmark event in the fluid dynamics community. By showing the possibility of greatly reduced frictional resistance, polymer solutions test results have added another tool to add the study of the fundamental of hydrodynamics. Experimentally it has been found that very small concentrations, of the order of a few parts-per-million, by weight of dissolved air polymer substance can reduce the friction in turbulent flow to as low as one-forth that of pure solvent. The application to this effect is attributive in many industrial situations involving momentum transfer (for example in firehouses, storm sewers and naval vessels) because of lowering of energy dissipation. Ousterhout [14] had noticed that when certain gums were used to suspend sand in the high pressure sand-water mixture employed in the oil-well technique, the friction was greatly decreased. Bentwich [3] develops the general solution for the axial two phase viscous flow in the long circular pipe. The interphase between the two phases is taken to be either circular cylinder or an arc segment of cylindrical surface. The expressions for the velocity distributions obtained may be of use in calculating the amount of water that should be added to the oil-pipelines, in order to reduce pumping costs. Astarita [1] measured velocity profile in pipe flow. Most of the previous velocity profile measurements were made with an impact tube, which resisters a pressure difference between the impact tube and the pipe wall. For normal fluids the pressure differences corresponds very closely to the velocity head or stagnation pressure $\rho u^2/g$, but for drag reducing polymer solutions, the pressure difference is the sum of the velocity head effects and normal stress difference effect caused by viscoelasticity of the fluid. The theory of Tomita [19] indicates a diameter effects on friction coefficients at constant Reynold number. Certain methods of plotting experimental data tend to minimize the diameter effects and can be used to form a more general picture of the friction reduction phenomena. Febula and Hoyt [5] demonstrated that additives were effective on rough surfaces and many subsequent workers have used commercial pipe of nominal roughness in their experiments. Poreh [16] has developed a theoretical model based on the assumption that the effect of the relative roughness size in similar for flows with or without polymers. The model appears to be successful in qualitatively describing the available experimental results. Virk [21] have introduced the three-zone concepts of the boundary layer in drag reducing polymer flow. Virk suggested a Newtonian viscous sub layer, an interactive zone or 'elastic' sub layer, which follows the ultimate or maximum achievable drag reduction time and outer region. Gills [6] used this technique of utilizing the maximum value of friction reduction obtained in pipe flow of drag reducing polymers to predict the minimum skin friction obtainable on a flat plate. Deshmukh *et al.* [4] have shown that the end grafting the drag reduction effectiveness and biodegradation resistance considerably in guargum.

Hinch [7] used physical arguments in connection with existing polymer elongation model to hypothesize a drag reduction mechanism and concluded that the degree to which a molecule will elongate depends on the strain rate of the fluid surrounding the polymer. Zakin and Huston [23] emphasized the need for a good solvent and pointed out that in order to achieve drag reduction the relaxation time of a stretched polymer molecule must be greater than the time scale of the flow. In recent theoretical studies, Rabin and Zielinska [18] examined the effect of polymer molecules on the vorticity distribution in elongation flows and pointed out that as long as the flow is Newtonian, vortex stretching occurs over all wave numbers. Tiederman *et al.* [20] and Luchik and Tiederman [9] focused on the time structure of the bursting process. They reported that the polymer did not affect the general shape of the conditionally sampled burst signals. However there was a reduction in the bursting rate and on increase in streak spacing. Wei and Willmarth [22] measured the fluctuating u- and v- velocity signals in the turbulent channel flow with or without drag reducing polymer injection with different injection conditions i.e. different polymer concentration were injected at different flow rates such that the total amount of polymer in the channel remained constant. They pointed out that for certain polymer concentrations there was a large negative Reynolds stress (≈ 0.2) in the near-wall region. Panda *et al.* [17] established a model of transportation of coarse coal and calculated yield stress with respect to addition of coarse coal particle to a fine particle slurry by analyzing to the lamina slurry flow. Arney *et al.* [2] gave a theory which is based on the concentric core-annular flow model and lead to a Reynolds number a friction factor which reduces a large body of experimental data onto one curve; with the best results in the high Reynolds number flow regime. They also predicted that pressure drop versus flow rate data could be reduced using the Reynolds number and friction factor but the technique fails to account for other effect which serves to increase the frictional losses such as an eccentric core or the presence of bends, elbows, tees and other common fittings. The major change in frictional coefficient which was obtained shows clearly that the drag reduction effect can be very efficient when the polymer is kept close to the wall. Lieu and Jovanovi [26] studied the effect of polymers on drag reduction in a turbulent channel flow and predicted that their model can be used in linearized equations to capture the effect of velocity and polymer stress fluctuations on the turbulent viscosity and drag. Tanga *et al.* [27] proposed a new drag correlation combining the existed drag correlations for low-Re flows and single-sphere flows, which fits the entire data set with an average relative deviation of 4%. Rodriguez *et al.* [28] studied drag-reduction phenomenon in dispersed oil-water flow in a 26-mm-i.d. twelve meter long horizontal glass pipe experimentally. They found drag reduction and slip ratio at oil volume fractions between 10 and 45% and high mixture Reynolds numbers, and with water as the dominant phase. Steiros *et al.* [29] studied the power consumption and drag of turbines with fractal and rectangular blades in a stirred tank. Power number decreases from rectangular to fractal impellers by over 10%, increasingly so with fractal iteration number. They suggested that this decrease was not caused by the wake interaction of the blades, nor solely by the wake interaction with the walls either. Pressure measurements on the blades' surface showed that fractal blades had lower drag than the rectangular ones, opposite to the wind tunnel experiment results. Khadom and Abdul-Hadi [24] studied the influence of polyacrylamide (PAM) as drag reducing polymer on flow of Iraqi crude oil in pipe lines and observed upto maximum 40.64% drag reduction with 50 ppm of PAM polymer flowing in straight pipes of 0.0508 m internal diameter.

In this paper the problem has been analyzed in the two regions; one thin viscoelastic layer near the wall and the other central core region of purely viscous fluid separately along with suitable matching conditions. Finally we have presented the numerical results for wall shearing stress and the total flux through a cross-section for a particular pressure gradient with or without the presence of the assumed thin layer of polymer additives of the so called dissolved layer of the polymer coated on the walls.

II. MATHEMATICAL MODEL

As proposed in the introduction above, the two dimensional axisymmetric flow of viscous fluid flowing in a pipe has been considered here. A thin peripheral layer of viscoelastic fluid near the wall develops due to injection of viscoelastic fluid through the walls. The governing equations of motion and continuity for viscous core region may be written in form:

$$u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{\rho}\frac{(\tau_{rr} - \tau_{\theta\theta})}{r} \quad (1)$$

$$u_r \frac{\partial u_z}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{\rho}\frac{\partial \left(r\,\tau_{rz}\right)}{\partial r} \quad (2)$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} = 0 \quad (3)$$

and the constitutive equations representing the fluid in absorbed layer of polymer of high molecular weight in the sensitive region are given by:

$$\tau_{rr} - \lambda\left[u_r \frac{\partial \tau_{rr}}{\partial r} - 2\tau_{rr}\frac{\partial u_r}{\partial r}\right] = -2\mu\frac{\partial u_r}{\partial r} \quad (4)$$

$$\tau_{rz} - \lambda\left[u_r \frac{\partial \tau_{rz}}{\partial r} - \tau_{rr}\frac{\partial u_z}{\partial r} - \tau_{rz}\frac{\partial u_r}{\partial r}\right] = -\mu\frac{\partial u_z}{\partial r} \quad (5)$$

$$\tau_{zz} - \lambda\left[u_r \frac{\partial \tau_{zz}}{\partial r} - 2\tau_{rz}\frac{\partial u_r}{\partial r}\right] = 0 \quad (6)$$

$$\tau_{\theta\theta} - \lambda\left[u_r \frac{\partial \tau_{\theta\theta}}{\partial r} - 2\frac{u_r}{r}\tau_{\theta\theta}\right] = 2\mu\frac{u_r}{r} \quad (7)$$

NORMALIZATION

The following non-dimensional quantities have been used:

$$z^* = \frac{z}{b}, \quad r^* = \frac{r}{b}, \quad \frac{a}{b} = \alpha, \quad u_r^* = \frac{u_r}{U_0}, \quad u_z^* = \frac{u_z}{U_0}, \quad p^* = \frac{p}{\rho U_0^2}, \quad \tau_{ij}^* = \frac{\tau_{ij}}{\rho U_0^2}, \quad \mathrm{Re} = \frac{\rho U_0 b}{\mu} \quad (8)$$

Transforming the governing Eqs. (1) to (7) with the help of the non-dimensional quantities we obtain the following system of equations (omitting the stars)

$$u_r\frac{\partial u_r}{\partial r} = -\frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{(\tau_{rr} - \tau_{\theta\theta})}{r} \quad (9)$$

$$u_r\frac{\partial u_z}{\partial r} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial (r\,\tau_{rz})}{\partial r} \quad (10)$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} = 0 \quad (11)$$

and the equations representing the fluid in absorbed layer are given by:

$$\tau_{rr} - K\left[u_r\frac{\partial \tau_{rr}}{\partial r} - 2\tau_{rr}\frac{\partial u_r}{\partial r}\right] = -\frac{2}{R_e}\frac{\partial u_r}{\partial r} \quad (12)$$

$$\tau_{rz} - K\left[u_r\frac{\partial \tau_{rz}}{\partial r} - \tau_{rr}\frac{\partial u_z}{\partial r} - \tau_{rz}\frac{\partial u_r}{\partial r}\right] = -\frac{1}{R_e}\frac{\partial u_z}{\partial r} \quad (13)$$

$$\tau_{zz} - K\left[u_r\frac{\partial \tau_{zz}}{\partial r} - 2\tau_{rz}\frac{\partial u_r}{\partial r}\right] = 0 \quad (14)$$

$$\tau_{\theta\theta} - K\left[u_r\frac{\partial \tau_{\theta\theta}}{\partial r} - 2\frac{u_r}{r}\tau_{\theta\theta}\right] = \frac{2u_r}{R_e} \quad (15)$$

where a and b are the radius of inner viscous core region and radius of pipe respectively. R_e is the Reynolds number and K is viscoelastic parameter.

The Boundary Conditions are:

$$u_{z1} = 0 \quad at \quad r = 1$$
$$u_{z1} = u_z \quad at \quad r = \alpha \qquad (16)$$
$$\frac{\partial u_{z1}}{\partial r} = 0 \quad at \quad r = 0$$

where the subscript refers for viscous region.

III. METHOD OF SOLUTION

In order to solve the governing non-linear partial differential equations given above, the variables are expanded in ascending powers of K, the viscoelastic parameter in the form:

$$f = f^{(0)} + K f^{(1)} + K^2 f^{(2)}$$

Introducing similar expressions for u_{z1}, τ_{ij} etc. in Eqs. (9) to (15) and equating the terms of like powers of K, we obtain a set of differential equations for zeroth, first order etc. for determining $u_z^{(0)}, u_z^{(1)}$ and hence $\tau_{rz}^{(0)}, \tau_{rz}^{(1)}$ etc. as follows:

Zeroth Order Equations:

$$\tau_{rr}^{(0)} = \frac{2}{r^2 R_e} \quad (17)$$

$$\tau_{rz}^{(0)} = -\frac{1}{R_e}\frac{\partial u_z^{(0)}}{\partial r} \quad (18)$$

$$\tau_{zz}^{(0)} = 0 \quad (19)$$

$$\tau_{\theta\theta}^{(0)} = \frac{2}{r R_e} \quad (20)$$

$$-\frac{1}{r^3} = -\frac{\partial p^{(0)}}{\partial r} - \frac{2(r+1)}{r^3 R_e} \quad (21)$$

$$\frac{1}{r}\frac{\partial u_z^{(0)}}{\partial r} = -\frac{\partial p^{(0)}}{\partial z} + \frac{1}{R_e}\left[\frac{\partial u_z^{(0)}}{\partial r} + r\frac{\partial^2 u_z^{(0)}}{\partial r^2}\right] \quad (22)$$

First Order Equations:

$$\tau_{rr}^{(1)} = 0 \quad (23)$$

$$\tau_{rz}^{(1)} = -\frac{1}{R_e}\frac{\partial u_z^{(1)}}{\partial r} - \frac{1}{r^2 R_e}\left[r\frac{\partial^2 u_z^{(0)}}{\partial t^2} + 3\frac{\partial u_z^{(0)}}{\partial r}\right] \quad (24)$$

$$\tau_{zz}^{(1)} = -\frac{2}{r^2 R}\frac{\partial u_z^{(0)}}{\partial r} \quad (25)$$

$$\tau_{\theta\theta}^{(1)} = -\frac{6}{r^3 R_e} \quad (26)$$

$$0 = -\frac{\partial p^{(1)}}{\partial r} \quad (27)$$

$$\frac{1}{r}\frac{\partial u_z^{(1)}}{\partial r} = -\frac{\partial p^{(1)}}{\partial z} + \frac{\partial\left(r\,\tau_{rz}^{(1)}\right)}{\partial r} \quad (28)$$

After solving the Eqs. (17) to (28) we get

$$u_z^{(0)} = L_1 \log r.e^{R_e.r} + \frac{\partial p}{\partial z}\left[\frac{r^2-1}{2} + \frac{2}{R_e}(r-1) + \frac{2}{R_e^2}(r-1)\right] \quad (29)$$

$$\tau_{rz}^{(0)} = -\frac{1}{R_e}\frac{\partial u_z^{(0)}}{\partial r} \tag{30}$$

$$u_z^{(1)} = \frac{\partial p^{(1)}}{\partial z}\left\{\frac{r^2-1}{2}\right\} + \frac{\partial p^{(0)}}{\partial z}\left\{\frac{r^4}{4} + 2r^3\left(\frac{1}{3}+\frac{1}{R_e^3}\right) + r\left(\frac{5}{2}+\frac{6}{R_e^2}-\frac{6}{R_e^3}\right) - \frac{8}{R_e^3} + \frac{6}{R_e^2} - \frac{3}{2R_e} - \frac{7}{12}\right\}$$
$$- L_1 \exp(R_e)\left[\frac{1}{R_e}+\frac{1}{R_e^2}-\frac{1}{R_e^4}+2\right]$$
$$+ L_3\left[\frac{(\log r)\ \exp(r R_e)}{(\log \alpha)\ \exp(\alpha R_e)} L_2 + \exp(rR_e)\left\{\frac{2}{R_e}-\frac{1}{R_e^4}+\frac{r}{R_e^2}-\frac{1}{R_e r^2}+\frac{2}{r}\right\}\right] \tag{31}$$

$$\tau_{rz}^{(1)} = \left[\frac{1}{R_e}\frac{\partial u_z^{(1)}}{\partial r} - \frac{1}{rR_e}\frac{\partial^2 u_z^{(0)}}{\partial r^2}\right] + \frac{1}{r^2}\tau_{rz}^{(0)} \tag{32}$$

Where,

$$L_1 = \frac{\partial p^{(0)}}{\partial z}\left[\frac{(\alpha^2-1)}{4\mu} - \frac{(\alpha^2-1)}{2} - \frac{2(\alpha-1)}{R_e} - \frac{2(\alpha-1)}{2}\right]$$

$$L_2 = \frac{(\log \alpha)\ \exp(\alpha R_e)}{2R_e^4(1-R_e)^2(1+R_e^2)(1+R_e)}\left[\begin{array}{l} 2R_e^5\left\{(1-R_e^2)(1+R_e^2)(1+R_e)\right\} \\ + R_e^3\left[6(2-R_e^2)(1+R_e^2)(1-R_e)\right] \\ + R_e^2\left[(6+R_e)(1+R_e^2)(1-R_e)\right] \\ - \left[(R_e)(1+R_e^2)(R_e^2-2)\right] \end{array}\right]$$

$$L_3 = \frac{\partial p^{(1)}}{\partial z}\left\{\frac{\alpha^2-1}{2}\right\} + \frac{\partial p^{(0)}}{\partial z}\left\{\frac{\alpha^4}{4} + 2\alpha^3\left(\frac{1}{3}+\frac{1}{R_e^3}\right) + \alpha\left(\frac{5}{2}+\frac{6}{R_e^2}-\frac{6}{R_e^3}\right) - \frac{8}{R_e^3} + \frac{6}{R_e^2} - \frac{3}{2R_e} - \frac{7}{12}\right\}$$
$$- L_1 \exp(R_e)\left[\frac{1}{R_e} - \frac{1}{2_{R_e}} - \frac{1}{4_{R_e}} + 2\right] / L_2$$
$$+ \left[\exp(\alpha R_e)\left\{\frac{2}{R_e}-\frac{1}{R_e^4}+\frac{\alpha}{R_e^2}-\frac{1}{R_e\alpha^2}+\frac{2}{\alpha}\right\}\right] \tag{33}$$

Finally we obtain stress on the wall from the expression

$$\tau_{rz}\Big|_w = \tau_{rz}^{(0)}\Big|_w + K\tau_{rz}^{(1)}\Big|_w \tag{34}$$

Total flow across the section has been obtained from the formula

$$Q = \int_0^{\alpha} 2\pi r\, u_z\, dr \tag{35}$$

$$Q = 2\pi \int_0^{\alpha} r\, u_z^{(0)}\, dr + K\, 2\pi \int_0^{\alpha} r\, u_z^{(1)}\, dr \quad (36)$$

Again, idealizing the problem of purely viscous fluid flowing in the pipe the governing equations of continuity and motion can be written as follows:

$$\frac{\partial u_{z1}}{\partial z} = 0 \quad (37)$$

$$\mu \left[\frac{\partial^2 u_{z1}}{\partial r^2} + \frac{1}{r} \frac{\partial u_{z1}}{\partial r} \right] = \frac{\partial p}{\partial r} \quad (38)$$

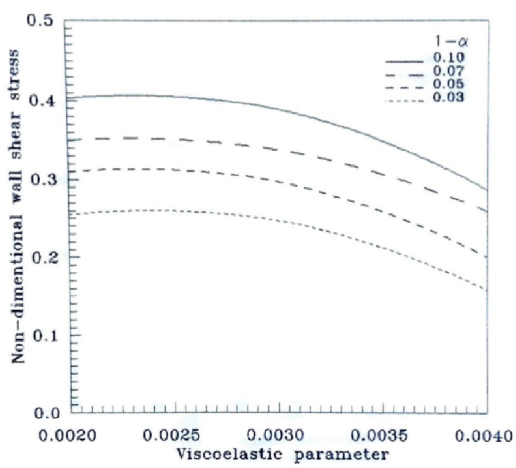
These equations can be solved to obtain the stress on the wall as follows:

$$\tau_{rz1}\Big|_w = \frac{\partial p}{\partial z} \frac{R_e}{2} \quad (39)$$

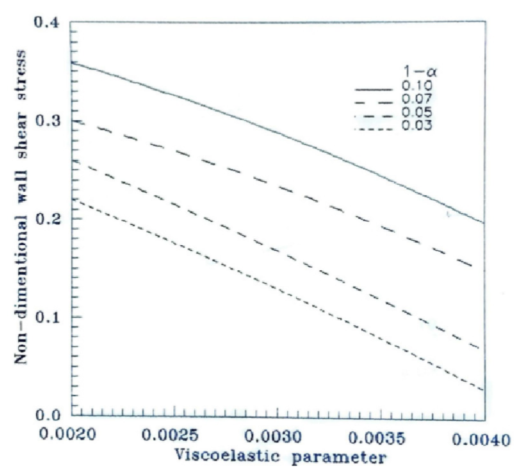
IV. RESULTS AND DISCUSSIONS

Taking into account the experimental results from literature [10–13, 15, 25] the problem of drag reduction has been analyzed in laminar flow in pipes. Assuming slow dissolution of polymers or injection of slightly viscoelastic fluid from the wall forming a thin peripheral layer near the wall, a two region flow model has been studied.

In figures 1(a) and 1(b) the effect of the viscoelastic parameter on the rate of decrease of wall shear stress can be seen. As the parameter *K* increases, rate of decrease in wall shearing stress also increases i.e. drag reduces. Thus, we conclude that the reduction in the drag laminar flows in presence of very small amount of high polymer (i.e. viscoelastic fluids) is also of the same order of magnitude as in turbulent flows [16]. The wall shearing stress decreases as the Reynolds number increases, but for a particular value of viscoelastic parameter, the wall shear stress increases as the thickness of the viscoelastic layer near the wall decreases. The wall shearing stress increases as the viscoelastic parameter increases. We further observe from figures 2(a) and 2(b) that the total flow rate increases as the Reynolds number increases. The total flow rate also increases as the thickness of the viscoelastic layer increases. These results agree well with the experimental results available in literature [7, 22, 25] i.e. larger amount of fluids can be transported with the same pressure gradients in presence of very small polymers of high molecular weight near the wall. Also the percentage of the drag reduction in laminar flow is of the same order as that in turbulent flows observed experimentally. Drag reduction and flow rate increases with the concentration of polymer of high molecular weight.

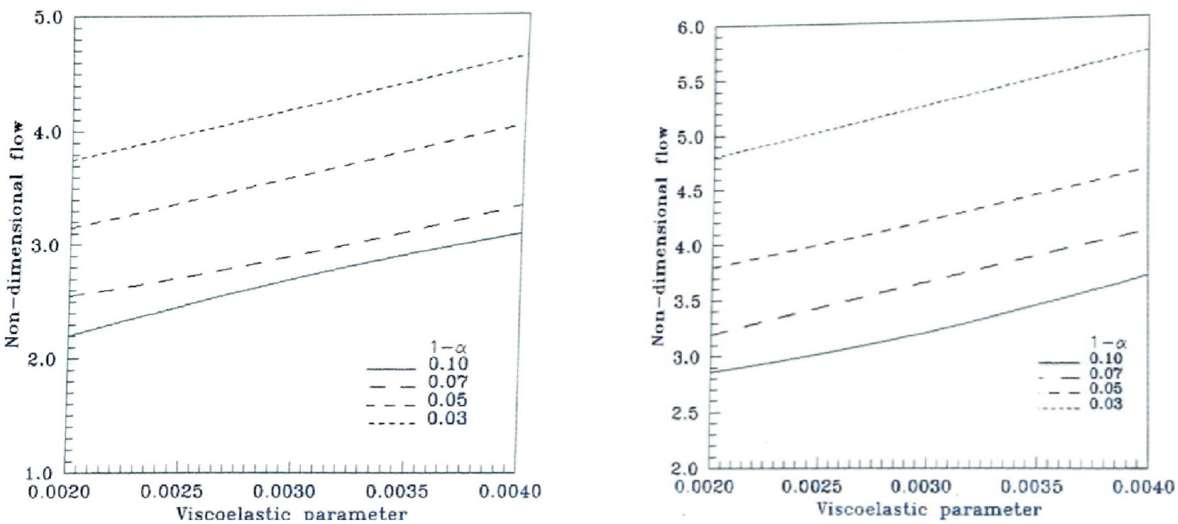


(a)



(b)

Fig. 1. (a). Variation of wall shear stress with viscoelastic Parameter *K* and thickness of polymer layer at Reynold's Number 5. (**b**) Variation of wall shear stress with Parameter *K* and thickness of polymer layer at Reynold's Number 10.



(a) (b)

Fig. 2. (a) Variation of Total flow with viscoelastic Parameter *K* and thickness of polymer layer at Reynold's Number 5. (b) Variation of Total flow with viscoelastic Parameter *K* and thickness of polymer layer at Reynold's Number 10.

REFERENCES

[1]. G. Austarita and L. Nicodema, (1966). 'Velocity distribution and normal stress in viscoelastic turbulent pipe flow, *AICHE Journal,* **12**(1966), 478-484.

[2]. M.S. Arney, R. Bai, E. Guevara, D.D. Joseph and K. Kiu, (1993). Friction factor and hold up studies for lubricated pipelining, *Int. J. Multiphase flow*, Vol. **19**, No. 6 (1993), 1061-1076.

[3]. M. Bentwich, (1964). Two phase viscous axial flow in pipe, Trans. of the ASME, Journal of Basic Engg. (1964) Paper No. 63 - WA – 99.

[4]. S.R. Deshmukh, P.N. Chaturvedi and R.P. Singh, (1982). The turbulent drag reduction by graft copolymers of Guar Gum and polyacrylamide. *Applied Polymer Science,* Vol. **30** (1982), 4013 -4018.

[5]. A.G. Fabula and J.W. Hoyt, (1964). The effect of additives on fluid friction. Proc. V^{th} Symp. on Naval Hydrodynamics, Bergen, Norway 1964, 947.

[6]. W.B. Gills, (1968). Similarity laws of friction reduced flows, *J. of Hydrodynamics*, **2** (1968), 34- 40.

[7]. J.W. Hoyt, (1972). The effect of additives on fluid friction, *J. of Basic Engg. Trans. of the ASME* **94** (1972), 258-263.

[8]. E.J. Hinche, (1977). Mechanical model for dilute polymer solution in a strong flow, *Phys. Fluids,* **20** (1977) 522 – 527.

[9]. T.S. Luchik and W.G. Tiederman, (1988). Turbulent structure in low concentrations drag reducing channel flow. *J. Fluid Mech.* **190** (1988), 241-251.

[10]. W.D. McComb, (1974). Drag reduction polymer and liquid column oscillation, *Nature,* **251** (1974) 598.

[11]. W.D. McComb and L.H. Rabie, (1978). Drag-reducing polymer and turbulent bursts, *Nature,* **273** (1978) 653 – 656.

[12]. W.D. McComb and L.H. Rabie, (1979). Development of local turbulent drag reduction due to non- uniform polymer concentration. *Phys. Fluids* **22** (1979) 183-189.

[13]. W.D. McComb and L. H. Rabie, (1982). Local drag reduction due to injection of polymer solutions into turbulent flow in a pipe, *AICHE Journal,* Vol. **28**, (1982) No. 4, 547-557.

[14]. R.S. Ousterhout and C.D. Hall, (1960). Reduction of fraction loss in fracturing operations, *J. of Petroleum Technology,* **13** (1960), 217-225.

[15]. V. D. Popadic, (1975). Drag reductions in film flow, *AICHE Journal,* **21** (1975), 612-619.

[16]. M. Poreh, (1971). Flow of dilute polymer solutions in rough pipes, *J. of Hydrodynamics,* **4** (1971), 151-160.

[17]. D. Panda, A. Parida, J. S. Murthyn and A. K. Mitra, (1994). Transportation of coarse coal in find medium, *Int. J. Multiphase flow,* **20** (1994) 171-177.

[18]. Y. Rabin and C. J. Abell, (1989). Scale dependent enhancement and damping of vorticity disturbances by polymer in elongation flows, *Phys. Rev. Lect.* **63** (1989), 512.

[19]. Y. Tomita, (1966). A study of viscoelastic fluid flow, *Bulletin of ISME* **9** (1966), 730.

[20]. W. G. Tiederman, T. S. Luchik and D. G. Bogard, (1985). Wall layer structure and drag reduction, *J. Fluid Mech.,* **156** (1985), 419-429.

[21]. P. S. Virk, (1971). Drag reduction in rough pipes, *J. Fluid Mech.* **45** (1971), 225-232.

[22]. T. Wei and W. W. Willmarth, (1992). Modifying turbulent structure with drag-reducing polymer additives in turbulent channel flow, *J. Fluid Mech.* **245** (1992), 619-641.

[23]. J. L. Zarin and D. L. Hunston, (1980). Effect of polymer molecular variables on drag reduction, *J. Macromol. Sci. Phys.* **B18** (1980), 795.

[24]. Khadom A. A. and Abdul-Hadi A.A. (2014). Performance of polyacrylamide as drag reduction polymer of crude petroleum flow, *Ain Sham Eng. J.* (2014) vol. **5** issue 3 pp. 861-865.

[25]. McComb W. C. (1991). 'The Physics of Fluid Turbulence' Clarendon Press Oxford.

[26]. Y. Tanga, S.H.L. Kriebitzschb, E.A.J.F. Petersa, M.A. van der Hoefc, J.A.M. Kuipersa (2014). 'A New Drag Correlation from Fully Resolved Simulations of Flow past Monodisperse Static Arrays of Spheres' *AIChE Journal,* **61**(2), pp. 688-698.

[27]. B. K. Lieu and M. R. Jovanovi (2012). 'Analysis of drag reduction by polymers in a turbulent channel flow' Center for Turbulence Research Proceedings of the Summer Program 2012, pp. 65-74.
[28]. H. Rodriguez, H. K. B. Yamaguti, and M.S. de Castro, M.J. Da Silva, O.M.H. Rodriguez (2012). 'Drag Reduction Phenomenon in Viscous Oil-Water Dispersed Pipe Flow: Experimental Investigation and Phenomenological Modeling', *AIChE Journal*, vol. **58**(9), pp. 2900-2910.
[29]. K. Steiros, P.J.K. Bruce, O.R.H. Buxton and J.C. Vassilicos 'Power consumption and form drag of regular and fractal-shaped turbines in a stirred tank', *AIChE Journal*, vol. **63**(2) pp. 843-854.