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Some Results in Pseudo Compact Tichonov Spaces

Archana Agrawal[1] and Manoj Kumar Shukla[2]
[1]Asst.Professor, Department of Mathematics,
St. Aloysius College (autonomous), Jabalpur (Madhya Pradesh), India
[2]Professor, Department of Mathematics,
Institute for Excellence in Higher Education, Bhopal (Madhya Pradesh), India

(Corresponding author: Archana Agrawal)
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ABSTRACT: In this paper we have proved some results in pseudo compact Tichonov spaces with two continuous mappings but weekly ** commuting and fixed points results in this space which generalize many earlier results.

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I. INTRODUCTION

Edelstein [2] established the existence of a unique fixed point of a self map T of a compact metric space satisfying the Banach inequality which is the generalization of Banach fixed point theorem. Jain and Dixit [3], Pathak [6], Khan and Sharma [4] find some valuable results in pseudo compact Tichonov spaces. Many authors as Namdeo and Fisher [5], Popa and Telci [7], Sahu [8] also established some important results in compact metric space. Recently Bharadwaj *et al* [1], proved some fixed point theorems in compact spaces and pseudo compact Tichonov spaces. In this paper some fixed point results in pseudo compact Tichonov spaces with two continuous mappings but weakly $**$ commuting are proved, which are generalizing the results of Edelstein [2] and Bharadwaj *et al* [1].

II. PRELIMINARIES

Definition 2.1 Let F be a self mapping. A spaces X is called a fixed point spaces if every continuous mapping F of F into itself, has a fixed point, in the sense that $F(x_0) = x_0$
Definition 2.2 A class $\{G_i\}$ of open subset of F is said to be an open cover of X , if each point in X belongs to one G_i that is $\cup_i (G_i) = X$.

A subclass of an open cover which is at least an open cover is called a sub cover. A compact space is that space in which every open cover is called a sub-cover.
Definition 2.3 A Topological space X is said to be pseudo-compact space, if every real valued continuous function on X is bounded.

It may be noted that every compact space is pseudo compact but converse is not necessarily true. However, in a metric space notation 'compact' and 'pseudo compact' coincide. By Tichonov space we means a completely regular Hausdorff space.
Definition 2.4 Let (X, μ) be a compact Tichonov space. Two mappings F and G are said to be weakly $**$ commuting if they commute at greatest lower bound of the set.

III. MAIN RESULTS

Theorem 3.1 Let X be a pesudo compact Tichonov space and μ be a non negative real valued continuous function over $X \times X$ satisfying:
[3.1.1] $\mu(x,x) = 0$ and $\mu(x,y) = \mu(x,z) + \mu(z,y)\ \forall\, x,y,z \in X$
[3.1.2] F and G be two continuous mappings on X satisfying:

$$\mu(Fx,Gy) < \emptyset\left[\mu(x,y), \frac{\mu(x,Fx)\mu(y,Fx)+\mu(x,Gy)\mu(y,Gy)}{\mu(x,Fx)+\mu(y,Fx)+\mu(x,Gy)+\mu(y,Gy)}, \frac{\mu(x,Fx)\mu(y,Gy)+\mu(x,Gy)\mu(y,Fx)}{\mu(x,Fx)+\mu(y,Fx)+\mu(x,Gy)+\mu(y,Gy)}, \frac{\mu(x,Fx)\mu(x,Gy)+\mu(y,Fx)\mu(y,Gy)}{\mu(x,Fx)+\mu(y,Fx)+\mu(x,Gy)+\mu(y,Gy)}\right]$$

Then F and G have coincide point in X.

Proof: Let us construct a function $T : X \rightarrow R \; as \; T(x) = \mu(Fx, Gx)$ for all $x \; in \; X$.As F and G are continuous therefore T is also. By the compactness of X there exist u in X such that $Tu = \inf\{Tx: x \in X\}$ then

$$\mu(Fu, Gu) < \emptyset\left[\mu(u,u), \frac{\mu(u,Fu)\mu(u,Fu)+ \mu(u,Gu)\mu(u,Gu)}{2\{\mu(u,Fu)+ \mu(u,Gu)\}}, \frac{\mu(u,Fu)\mu(u,Gu)+ \mu(u,Gu)\mu(u,Fu)}{2\{\mu(u,Fu)+ \mu(u,Gu)\}}, \frac{\mu(u,Fu)\mu(u,Gu)+ \mu(u,Gu)\mu(u,Fu)}{2\{\mu(u,Fu)+ \mu(u,Gu)\}}\right]$$

$$\mu(Fu, Gu) < \emptyset\left[0, \frac{\mu^2(u,Fu) + \mu^2(u,Gu)}{2\{\mu(u,Fu) + \mu(u,Gu)\}}, \frac{2(u,Fu)\mu(u,Gu)}{2\{(u,Fu) + \mu(u,Gu)\}}, \frac{2\mu(u,Fu)\mu(u,Gu)}{2\{\mu(u,Fu) + \mu(u,Gu)\}}\right]$$

There arise two cases:

Case1. $\mu\,(Fu\,, Gu) < \frac{\mu^2(u,Fu)+\mu^2(u,Gu)}{2\{\mu(u,Fu)+ \mu(u,Gu)\}}$

$$\mu\,(Fu\,, Gu) < \frac{\mu^2(u,Fu)+\mu^2\,(u,Gu)}{2\{\mu(Fu,Gu)\}}$$

$$\Rightarrow \quad \mu\,(Fu\,, Gu) < \frac{\mu^2(u,Fu)+\mu^2\,(u,Gu)}{\mu(Fu,Gu)}$$

$$\mu^2\,(Fu, Gu) < \mu^2(u,Fu) + \mu^2\,(u,Gu)$$

$[\mu(u,Fu) + \mu(u,Gu)]^2 = \mu^2\,(Fu,Gu) < \mu^2(u,Fu) + \mu^2(u,Gu)$,which is contradiction.

Case 2: $\mu\,(Fu\,, Gu) < \frac{2\mu(u,Fu)\mu\,(u,Gu)}{2\{\mu(u,Fu)+ \mu(u,Gu)\}}$

$$[\mu(u,Fu) + \mu(u,Gu)]^2 = \mu^2\,(Fu,Gu) < \mu(u,Fu)\,\mu(u,Gu)$$

Which again contradiction. Therefore by **case 1 and 2** $\mu(Fu\,, Gu) = 0$.Thus u is coincide point of F and G.

Theorem 3.2 Let X be a pesudo compact Tichnov space and μ be a non negative real valued continuous function over $X \times X$ satisfying **[3.1.1] .** F and T be two continuous mappings on X satisfying:

[3.2.1] $\mu\,(Fx\,, Ty) <$

$$\emptyset\left[\mu(x,y), \frac{\mu\,(x,Fx)\mu(y,Fx) + \mu(x,Ty)\mu(y,Ty)}{\mu\,(x,Fx) + \mu(y,Fx) + \mu(x,Ty) + \mu(y,Ty)}, \frac{\mu\,(x,Fx)\mu(y,Ty) + \mu(x,Ty)\mu(y,Fx)}{\mu\,(x,Fx) + \mu(y,Fx) + \mu(x,Ty) + \mu(y,Ty)}, \frac{\mu\,(x,Fx)\mu(x,Ty) + \mu(y,Fx)\mu(y,Ty)}{\mu\,(x,Fx) + \mu(y,Fx) + \mu(x,Ty) + \mu(y,Ty)}\right]$$

[3.2.2] ***F*** and T are weakly ** commuting mappings. Then F and T have unique common point in X.

Proof: Let us construct a function $G : X \rightarrow R$ as $G(x) = \mu(FTx, Tx) for \; all \; x \; in \; X$. As F and T are continuous therefore G is also. By the compactness of X there exist u in X such that $= \inf\{Gx: x \in X\}$. Now we affirm that u is fixed point of F.

$$\text{G(Fu)} = \mu(FTFu\,, TFu)$$

$$\mu(FTFu\,, TFu) \quad < \emptyset[\mu\,(TFu, Fu), \frac{\mu(TFu\,, FTFu)\mu(Fu\,, FTFu) + \mu(TFu\,, TFu)\mu(Fu, TFu)}{\mu(TFu, FTFu) + \mu(Fu, FTFu) + \mu(TFu\,, TFu) + \mu(Fu, TFu)},$$
$$\frac{\mu(TFu\,, FTFu)\mu(Fu\,, TFu) + \mu(TFu\,, TFu)\mu(Fu, FTFu)}{\mu(TFu\,, FTFu) + \mu(Fu\,, FTFu) + \mu(TFu\,, TFu) + \mu(Fu, TFu)}, \frac{\mu(TFu\,, FTFu)\mu(TFu\,, TFu) + \mu(Fu\,, FTFu)\mu(Fu, TFu)}{\mu(TFu\,, FTFu) + \mu(Fu\,, FTFu) + \mu(TFu\,, TFu) + \mu(Fu, TFu)}]$$

$$\mu(FTFu\,, TFu) < \emptyset\,[\,\mu(TFu\,, Fu), \frac{\mu(TFu, FTFu)\mu(Fu, FTFu)}{2\mu(TFu, FTFu)}, \frac{\mu(TFu, FTFu)\mu(Fu, FTFu)}{2\mu(TFu, FTFu)}, \frac{\mu(Fu, FTFu)\mu(Fu, TFu)}{2\mu(TFu, Fu)}]$$

$$\mu(FTFu\,, TFu) < \emptyset[\mu(TFu, Fu), \frac{\mu(Fu, FTFu)}{2}, \frac{\mu(Fu\,, FTFu)}{2}, \frac{\mu(Fu, FTFu)}{2}]$$

$$\mu(FTFu\,, TFu) < \emptyset[\mu(TFu, Fu), \mu(Fu, FTFu), \mu(Fu, FTFu), \mu(Fu, FTFu)]$$

T(Fu) $<$ T(u), which contradiction . Therefore Fu = u.

Now , since $FTu = TFu = Tu$.

Let $Tu \neq u$, then $\mu(Tu, u) = \mu(FTu, Fu)$ and

$$\mu(FTu, Fu) < \emptyset[\mu(u,u), \frac{\mu(u,Fu)\mu(u,Fu) + \mu(u,Tu)\mu(u,Tu)}{\mu(u,Fu) + \mu(u,Fu) + \mu(u,Tu) + \mu(u,Tu)}, \frac{\mu(u,Fu)\mu(u,Tu)+ \mu(u,Tu)\mu(u,Fu)}{\mu(u,Fu)+\mu(u,Fu)+ \mu(u,Tu)+\mu(u,Tu)}, \frac{\mu(u,Fu)\mu(u,Tu)+ \mu(u,Fu)\mu(u,Tu)}{\mu(u,Fu)+\mu(u,Fu)+ \mu(u,Tu)+\mu(u,Tu)}]$$

$$< \emptyset[0, \frac{\mu^2(u,Gu)}{2\mu(u,Gu)}, 0,0]$$

$\mu(Gu, u) < \mu(Gu, u)$. This is contradiction, therefore $Gu = u$.

Hence $F \; and \; T$ have a common fixed point u.

Let v is another fixed point of F and T, then by [3.2.1]

$$\mu(Fu,Tv) < \emptyset[\mu(u,v),\frac{\mu(u,Fu)\mu(v,Fu)+\mu(u,Tv)\mu(v,Tv)}{\mu(u,Fu)+\mu(v,Fu)+\mu(u,Tv)+\mu(v,Tv)},\frac{\mu(u,Fu)\mu(v,Tv)+\mu(u,Tv)\mu(v,Fu)}{\mu(u,Fu)+\mu(v,Fu)+\mu(u,Tv)+\mu(v,Tv)},\frac{\mu(u,Fu)\mu(u,Tv)+\mu(v,Fu)\mu(v,Tv)}{\mu(u,Fu)+\mu(v,Fu)+\mu(u,Tv)+\mu(v,Tv)}]$$

$$\mu(u,v) < \emptyset[\mu(u,v),\frac{\mu(u,u)\mu(v,u)+\mu(u,v)\mu(v,v)}{\mu(u,u)+\mu(v,u)+\mu(u,v)+\mu(v,v)},\frac{\mu(u,u)\mu(v,v)+\mu(u,v)\mu(v,u)}{\mu(u,u)+\mu(v,u)+\mu(u,v)+\mu(v,v)},\frac{\mu(u,u)\mu(u,v)+\mu(v,u)\mu(v,v)}{\mu(u,u)+\mu(v,u)+\mu(u,v)+\mu(v,v)}]$$

$$\mu(u,v) < \emptyset[\mu(u,v),0,\frac{\mu(u,v)}{2},0]$$

$\mu(u,v) < \mu(u,v)$ .This is contradiction.Thus u is unique fixed point of F and T.

Theorem 3.3 Let F be a continuous mapping of a compact metric space X into itself satisfying

[3.3.1] $$d(Fx,Fy) < \emptyset[d(x,y),d(x,Fx),d(y,Fx),\frac{d(x,Fx)d(y,Fx)+d(x,Fy)d(y,Fy)}{d(x,Fx)+d(y,Fx)+d(x,Fy)+d(y,Fy)},\frac{d(x,Fx)d(y,Fy)+d(x,Fy)d(y,Fx)}{d(x,Fx)+d(y,Fx)+d(x,Fy)+d(y,Fy)},\frac{d(x,Fx)d(x,Fy)+d(y,Fx)d(y,Fy)}{d(x,Fx)+d(y,Fx)+d(x,Fy)+d(y,Fy)}]$$

has unique fixed point in X.

Proof: Let us construct a function T on X as $Tx = d(x,Fx)$ for all x in X. Since F and metric d are continuous therefore T is also. By compactness of X there exist a point u in X such that $Tu = \inf\{Tx: x \in X\}$ ……….(3.3.2)

If $Tu \neq 0$, then $Fu \neq u$. Now

$$T(Fu) = d(Fu,F(Fu)) < \emptyset[d(u,Fu),d(u,Fu),d(Fu,Fu),\frac{d(u,Fu)d(Fu,Fu)+d(u,FFu)d(Fu,FFu)}{d(u,Fu)+d(Fu,Fu)+d(u,FFu)+d(Fu,FFu)},\frac{d(u,Fu)d(Fu,FFu)+d(u,FFu)d(Fu,Fu)}{d(u,Fu)+d(Fu,Fu)+d(u,FFu)+d(Fu,FFu)},\frac{d(u,Fu)d(u,FFu)+d(Fu,Fu)d(Fu,FFu)}{d(u,Fu)+d(Fu,Fu)+d(u,FFu)+d(Fu,FFu)}]$$

$$d(Fu,F(Fu)) < \emptyset[d(u,Fu),d(u,Fu),0,\frac{d(Fu,FFu)}{2},\frac{d(Fu,FFu)}{2},d(u,Fu)]$$

If $d(Fu,F(Fu)) < d(Fu,FFu)$, which is contradiction.Therefore

$$d(Fu,F(Fu)) < d(u,Fu)$$

$\Rightarrow$ $T(Fu) < T(u)$, which again contradiction to (3.3.2). Hence $u = Fu$, i.e. u is a fixed point of F.

By [3.3.1] can easily prove the uniqueness of u.

Remark: Theorem 3.3 generalizes the result of Edelstein [2] and Bharadwaj *et.al.* [1].

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