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On lin's principle of minimum information

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ABSTRACT : Lind and Solana's principle of minimum information are discussed. New generalized Lin's cross entropies are introduced and its application of Lin's principle of minimum information are examined.

Keywords : Minimum Information, cross-entropy, Maximum Like hood, Csiszer's measure.

INTRODUCTION

Let $g(x, \theta)$ be a known Probability density function of continuous random variate defined over the interal $[X_0, X_{n+1}]$. However, the parameter θ is known and to estimate it. We draw a random sample $X_1, X_2, \ldots, X_n$ from the population and rearrange its members, so that

$$X_0 < X_1 < X_2 < \ldots\ldots < X_i < X_{i+1} < X_n < X_{n+1} \quad \ldots(1)$$

The usual method for estimation of θ is based on [1] principle of maximum likehood according to which estimate θ by maximizing likehood function;

$$L \cong g(x_1, \theta)\, g(x_2, \theta) \ldots\ldots g(x_n, \theta) \quad \ldots(2)$$

Recenly, [5] have suggested a two-stage method of estimating θ. In the first stage we assume θ to be known and choose a function $f(x, \theta)$ which satisfies

$$f(x, \theta)dx = \frac{1}{n+1}, \quad i = 0, 1, 2, \ldots, n \quad \ldots(3)$$

So that the probability in each of the $(n + 1)$ intervals defined by the n sample points are $1/(n + 1)$ each and which is such that

$$f(x, \theta)\ In = \frac{f(x,\theta)}{g(x,\theta)}\, dx \quad \ldots(4)$$

is minimum *i.e.*, cut of all the density functions satisfying constraints (3) we choose that function which is 'closest' to given $g(x, \theta)$ in the sencse that it minimizes [2] measure of cross-entropy of $f(x, \theta)$ from $g(x, \theta)$. Thus, first stage determines $f(x, \theta)$ for any given θ. The object of the IInd stage is to choose θ to minimize (4) for the function $f(x, \theta)$ determined by the first stage.

The first stage gives,

$$f(x, \theta) = \frac{g(x,\theta)}{k_i},\ X_1 < X < X_{i+1} \quad \ldots(5)$$

$$\text{Where, } k_i = (n+1)\int_{X_i}^{X_{i+1}} g(x, \theta)\, dx,\ i = 0, 1, 2, \ldots, n \quad \ldots(6)$$

the second stage gives that we chose θ to minimize

$$\sum_{i=0}^{n}\int_{X_i}^{X_{i+1}} \frac{g(x,\theta)}{k_i},\ \log\frac{1}{k_i}\, dx = -\frac{1}{n+1}\sum_{i=0}^{n}\log k_i \quad \ldots(7)$$

In other words, we chose θ by minimizing

$$\sum_{i=0}^{n}\log\int_{X_i}^{X_{i+1}} g(x, \theta)\, dx \quad \ldots(8)$$

[7] This can be compared with the principle of maximum likehood which suggests that we chose θ to minimize

$$\sum_{i=0}^{n}\log g(x, \theta)\, dx \quad \ldots(9)$$

[6] introduce a new directed divergence, which overcomes the difficulty of absolute continuity. This new divergence measure denoted by $k(P, Q)$ between two probability distributions P and Q is defined as

$$k(P, Q) = \sum_{X\in X} P(x)\log\frac{p(x)}{1/2P(x)+1/2Q(x)}$$

[8] introduced a new general class of diveragence measures as

$$M(P, Q, a) = P(x)\ \log\ \frac{p(x)}{[aP(x)+bQ(x)]}$$

$$0 < a,\ b < 1 \text{ and } a + b = 1$$

CSISER'S CLASS OF MEASURE OF CROSS-NTROPY

[4] gave the classes of measures of cross-entropy,

$$\int g(x,\theta)\phi\left[\frac{f(x,\theta)}{g(x,\theta)}\right]dx \text{ and } \int f(x,\theta)\phi\left[\frac{g(x,\theta)}{f(x,\theta)}\right]dx \quad \ldots(10)$$

Where $\phi(.)$ is twice-differentiable convex function for which $\phi(1)=0$

For different function $\phi(.)$, (10) can represent a variety of measures of cross-entropy. Using these for the first stage, we get

$$\phi'\ \frac{f(x,\theta)}{g(x,\theta)} = \text{Const.}$$

$$\text{or } \left[\frac{g(x,\theta)}{f(x,\theta)}\right] - \frac{g(x,\theta)}{f(x,\theta)}\ \phi'\left[\frac{g(x,\theta)}{f(x,\theta)}\right] = \text{Const.} \quad \ldots(11)$$

Now we define the classes of measures of new cross-entropy

$$\int g(x,\theta)\phi\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right] dx$$

$$\text{and } \int f(x,\theta)\phi\left[\frac{2g(x,\theta)}{g(x,\theta)+f(x,\theta)}\right]dx \qquad \text{...(12)}$$

$$g(x,\ \theta)\frac{2[f(x,\theta)+g(x,\theta)-f(x,\theta)]}{[f(x,\theta)+g(x,\theta)]^2}\ \phi'$$

$$\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right] = \text{Const.}$$

$$\text{and } \phi\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right]\frac{2[g(x,\theta)]^2}{[f(x,\theta)+g(x,\theta)]^2}$$

$$\phi'\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right] = \text{Const.} - \frac{2f(x,\theta)+g(x,\theta)}{[f(x,\theta)+g(x,\theta)]^2}$$

$$= \text{Const.} \qquad \text{...(13)}$$

Whatever be the function θ(.), this gives (5) and (6) so that the first stage gives the same result for all csiszer's measure of cross-entropy. For the second stage we have to choose θ to minimize

$$\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}} g(x,\theta)\phi\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right]dx$$

$$\text{or } \sum_{i=1}^{n}\int_{X_i}^{X_{i+1}} f(x,\theta)\phi\left[\frac{2g(x,\theta)}{g(x,\theta)+f(x,\theta)}\right]dx$$

$$\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}} g(x,\theta)\phi\left[\frac{2}{1+\frac{g(x,\theta)}{f(x,\theta)}}\right]dx$$

$$\text{or } \sum_{i=0}^{n}\int_{X_i}^{X_{i+1}} f(x,\theta)\phi\left[\frac{2}{1+\frac{f(x,\theta)}{g(x,\theta)}}\right]dx$$

$$\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}} g(x,\theta)\phi\left[\frac{2}{1+k_i}\right]dx \text{ or } \sum_{i=0}^{n}\int_{X_i}^{X_{i+1}} \frac{g(x,\theta)}{k_i}\phi\left[\frac{2k_i}{1+k_i}\right]dx$$

$$\sum_{i=1}^{n}\phi\left[\frac{2}{1+k_i}\right]\int_{X_i}^{X_{i+1}} g(x,\theta)\,dx$$

$$\text{or } \sum_{i=0}^{n}\phi\left[\frac{1}{k_i}\right]\phi\left[\frac{2k_i}{1+k_i}\right]\int_{X_i}^{X_{i+1}} g(x,\theta)\,dx$$

$$\sum_{i=1}^{n}\phi\left[\frac{2}{1+k_i}\right].\frac{k_i}{n+1} \quad \text{or} \quad \sum_{i=0}^{n}\phi\left[\frac{2k_i}{1+k_i}\right].\frac{k_i}{n+1}$$

$$\frac{1}{n+1}\sum_{i=1}^{n}k_i\phi\left[\frac{2}{k_i+1}\right] \quad \text{or} \quad \frac{1}{n+1}\phi\sum_{i=1}^{n}\left[\frac{2k_i}{k_i+1}\right] \qquad \text{...(14)}$$

$$F(\theta) = \sum_{i=1}^{n}\phi\left[\frac{2}{1+k_i}\right]k_i$$

$$\frac{dF}{d\theta} = \sum_{i=1}^{n}\left[\phi\left(\frac{2}{k_i+1}\right)\frac{dk_i}{d\theta} - \frac{dk_i}{d\theta}\phi'\left(\frac{2}{(k_i+1)}\right)\right.$$

$$\left.\frac{2k}{(k_i+1)^2}\right]$$

$$= \sum_{i=1}^{n}\left[\phi\left(\frac{2}{k_i+1}\right) - \phi'\left(\frac{2}{k_i+1}\right)\frac{2k_i}{(k_i+1)^2}\right]\frac{dk_i}{d\theta}$$

$$\frac{1}{n+1}\frac{dF}{d\theta} = \sum_{i=1}^{n}\left[\phi\left(\frac{2}{k_i+1}\right) - \phi'\left(\frac{2}{k_i+1}\right)\frac{2k_i}{(k_i+1)^2}\right]$$

$$\left[\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx\right]$$

$$\frac{1}{n+1}\frac{d^2F}{d\theta^2} = \sum_{i=1}^{n}\left[\phi'\left(\frac{2}{k_i+1}\right)\left(\frac{-2}{(k_i+1)^2}\right)\frac{dk_i}{d\theta} - \phi'\left(\frac{2}{k_i+1}\right)\right.$$

$$\frac{2k_i}{(k_i+1)^2}\frac{dk_i}{d\theta} - \phi'\left(\frac{2}{k_i+1}\right)\frac{-4k_i}{(k_i+1)^3}\frac{dk_i}{d\theta}$$

$$\left.-\phi''\left(\frac{2}{k_i+1}\right)\frac{-2}{(k_i+1)^2}\frac{dk_i}{d\theta}.\frac{2k_i}{(k_i+1)^2}\right]$$

$$\left[\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx\right] + \sum_{i=1}^{n}\left[\phi\left(\frac{2}{k_i+1}\right) - \phi'\left(\frac{2}{k_i+1}\right)\right.$$

$$\left.\frac{2k_i}{(k_i+1)^2}\right]\left[\int_{X_i}^{X_{i+1}}\frac{\partial^2 g}{\partial\theta^2}dx\right]$$

$$\frac{1}{n+1}\frac{d^2F}{d\theta^2} = \sum_{i=1}^{n}\left[-\phi'\left(\frac{2}{k_i+1}\right).\frac{2}{\left(k_i+1\right)^2} - 2\phi'\left(\frac{2}{k_i+1}\right)\right.$$

$$\frac{1}{\left(k_i+1\right)^2} + \frac{4k_i}{\left(k_i+1\right)^3}\phi'\left(\frac{2}{k_i+1}\right) + \frac{4k_i}{\left(k_i+1\right)^3}$$

$$\left.\phi''\left(\frac{2}{k_i+1}\right)\frac{k_i}{\left(k_i+1\right)^4}\right]\left[\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx\right](n+1) +$$

$$\sum_{i=1}^{n}\phi\left(\frac{2}{k_i+1}\right)-\phi'\left(\frac{2}{k_i+1}\right)\frac{2k_i}{(k_i+1)^2}$$

$$\left[\int_{X_i}^{X_{i+1}}\frac{\partial^2 g}{\partial\theta^2}dx\right] \quad ...(15)$$

Now when $\phi(.)$ is convex, $\phi''(k_i) > 0$, and the first expression on the RHS > 0 again $g(x, \theta)$ is a concave function of θ, therefore the second term will be > 0 as well. If $\phi'(k_i) < 0$, so that is if $\phi'(.)$ is a descending function at k_i, $F(\theta)$, will be a convex function of θ and its minimum will be its inclusive minimum.

Now Let

$$G(\theta) = \sum_{i=0}^{n}\phi\left(\frac{2k_i}{k_i+1}\right)$$

$$G'(\theta) = \sum_{i=1}^{n}\phi'\left(\frac{2k_i}{k_i+1}\right)\frac{2[k_i+1-k_i]}{(k_i+1)^2}\frac{dk_i}{d\theta}$$

$$\frac{G'(\theta)}{n+1} = \sum_{i=1}^{n}\phi'\left(\frac{2k_i}{k_i+1}\right)\frac{2}{(k_i+1)^2}\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx$$

$$\frac{G'(\theta)}{n+1} = \sum_{i=1}^{n}\left[\phi'\left(\frac{4k_i}{k_i+1}\right)\frac{4}{(k_i+1)^2}-\frac{4}{(k_i+1)^3}\phi'\right.$$

$$\left.\left(\frac{2k_i}{k_i+1}\right)\right](n+1)\left[\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx\right]^2+\sum_{i=1}^{n}\frac{2k_i}{(k_i+1)^2}$$

$$\phi'\left(\frac{2k_i}{k_i+1}\right)\int_{X_i}^{X_{i+1}}\frac{\partial^2 g}{\partial\theta^2}dx \quad ...(16)$$

HAVRADA AND CHARVAT MEASURES OF CROSS ENTROPY

These measures are defined by either

$$\frac{\int f(x,\theta)\left[\left(\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)}$$

or
$$\frac{\int g(x,\theta)\left[\left(\frac{2g(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)} \quad ...(17)$$

If $\alpha \to 1$, these gives Kullback-Leibler measures

$$\lim_{\alpha\to1}\frac{\int f(x,\theta)\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right]^{\alpha-1}\log\left(\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)}{\alpha+\alpha-1} \text{ or}$$

$$\lim_{\alpha\to1}\frac{\int g(x,\theta)\left[\frac{2g(x,\theta)}{f(x,\theta)+g(x,\theta)}\right]^{\alpha-1}\log\left(\frac{2g(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)dx}{\alpha+\alpha-1}$$

$$\int f(x,\theta)\log\left[\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right]dx$$

or
$$\int g(x,\theta)\log\left[\frac{2g(x,\theta)}{f(x,\theta)+g(x,\theta)}\right]dx \quad ...(18)$$

$$\lim_{\alpha\to1}\frac{\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)}$$

or
$$\lim_{\alpha\to1}\frac{\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\left[\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)}$$

$$\lim_{\alpha\to1}\frac{\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\left(\frac{2}{k_i+1}\right)^{\alpha-1}\log\left(\frac{2}{k_i+1}\right)dx}{\alpha+\alpha-1}$$

or
$$\lim_{\alpha\to1}\frac{\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}\log\left(\frac{2k_i}{k_i+1}\right)dx}{\alpha+(\alpha-1)}$$

$$\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\log\left(\frac{2}{k_i+1}\right)dx$$

or
$$\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\log\left(\frac{2k_i}{k_i+1}\right)dx \quad ...(19)$$

In the general case

$$\sum_{i=1}^{n}\frac{1}{(n+1)}\log\frac{2}{k_i+1} \quad \text{or} \quad \sum_{i=1}^{n}\frac{k_i}{(n+1)}\log\frac{2k_i}{k_i+1}$$

$$\frac{1}{(n+1)}\sum_{i=1}^{n}\log\frac{2}{k_i+1} \quad \text{or} \quad \frac{k_i}{(n+1)}\sum_{i=1}^{n}\log\frac{2k_i}{k_i+1}$$

$$\frac{\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)}$$

$$\frac{1}{(n+1)} = \int_{X_i}^{X_{i+1}}\frac{g(x,\theta)}{k_i}dx$$

We have to minimize

$$= \frac{1}{\alpha(\alpha-1)(n+1)}\sum_{i=1}^{n}\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]$$

More General Case

$$P(\theta) = \frac{1}{\alpha(\alpha-1)(n+1)}\sum_{i=0}^{n}\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]$$

$$\frac{dP}{d\theta} = \frac{1}{\alpha(n+1)}\sum_{i=0}^{n}\left(\frac{2}{k_i+1}\right)^{\alpha-2}\frac{-2}{(k_i+1)^2}\frac{dk_i}{d\theta}$$

$$= \frac{1}{\alpha(n+1)}\sum_{i=0}^{n}2^{\alpha-1}(k_i+1)^{-\alpha}(n+1)\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx$$

$$= \frac{-(2)^{\alpha-1}}{\alpha}\sum_{i=0}^{n}(k_i+1)^{-\alpha}\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx$$

$$\frac{d^2P}{d\theta^2} = 2^{\alpha-1}\sum_{i=0}^{n}(k_i+1)^{-\alpha-1}\int_{X_i}^{X_{i+1}}\left[\frac{\partial g}{\partial\theta}dx\right]^2$$

$$-\frac{2^{\alpha-1}}{\alpha}\sum_{i=0}^{n}(k_i+1)^{-\alpha}\int_{X_i}^{X_{i+1}}\frac{\partial^2 g}{\partial\theta^2}dx$$

$$= \frac{\sum_{i=1}^{n}\int_{X_i}^{X_{i+1}}\int g(x,\theta)\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)},$$

$$\frac{k_i}{(n+1)} = \int_{X_i}^{X_{i+1}} g(x,\theta)dx$$

We have to minimize

$$= \frac{1}{\alpha(\alpha-1)(n+1)}\sum_{i=1}^{n}\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]$$

Discuss of the more General Case

$$Q(\theta) = \frac{1}{\alpha(\alpha-1)(n+1)}\sum_{i=1}^{n}\left[\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right]$$

$$\frac{dQ}{d\theta} = \frac{1}{\alpha(\alpha-1)(n+1)}\left[\sum_{i=1}^{n}\left\{\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right\}\right.$$

$$\left.+\sum_{i=1}^{n}k_i\left\{(\alpha-1)\left(\frac{2k_i}{k_i+1}\right)^{\alpha-2}\frac{2}{(k_i+1)^2}\right\}\right]\frac{dk_i}{d\theta}$$

$$= \frac{1}{\alpha(\alpha-1)(n+1)}\left[\sum_{i=1}^{n}\left\{\left(\frac{2}{k_i+1}\right)^{\alpha-1}-1\right\}\right.$$

$$\left.+\sum_{i=1}^{n}k_i(\alpha-1)\left(\frac{2k_i}{k_i+1}\right)^{\alpha-2}\left(\frac{1}{(k_i+1)}\right)\right]\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx$$

$$\frac{d^2Q}{d\theta^2} = \frac{1}{\alpha(\alpha-1)}\sum_{i=1}^{n}k_i\left\{(\alpha-1)\left(\frac{2k_i}{k_i+1}\right)^{\alpha-2}\frac{2}{(k_i+1)^2}\right\}$$

$$+\sum_{i=1}^{n}\left\{(\alpha-1)^2\left(\frac{2k_i}{k_i+1}\right)^{\alpha-2}\frac{2}{(k_i+1)^2}\left(\frac{1}{k_i+1}\right)\right.$$

$$\left.+(\alpha-1)\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}\frac{-1}{(k_i+1)^2}\right\}\Bigg]\frac{dk_i}{d\theta}\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx$$

$$+\frac{1}{\alpha(\alpha-1)}\left\{\sum_{i=1}^{n}\left[\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}-1\right]+(\alpha-1)\right.$$

$$\left.\sum_{i=1}^{n}\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}\frac{1}{(k_i+1)}\right\}\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx$$

$$= \frac{1}{\alpha(\alpha-1)}\left[\sum_{i=1}^{n}(\alpha-1)\left(\frac{2k}{k_i+1}\right)^{\alpha-2}\frac{2}{(k_i+1)^2}\right.$$

$$+\sum_{i=1}^{n}(\alpha-1)^2\left(\frac{2k}{k_i+1}\right)^{\alpha-2}\frac{2}{(k_i+1)^3}-\frac{(\alpha-1)}{(k_i+1)^2}$$

$$\left.\left(\frac{2k_i}{(k_i+1)^3}\right)^{\alpha-1}\right](n+1)\left(\int_{X_i}^{X_{i+1}}\frac{\partial g}{\partial\theta}dx\right)^2+\frac{1}{\alpha(\alpha-1)}$$

$$\left\{\sum_{i=1}^{n}\left[\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}-1\right]+(\alpha-1)\sum_{i=1}^{n}\left(\frac{2k_i}{k_i+1}\right)^{\alpha-1}\right.$$

$$\left.\frac{1}{(k_i+1)}\right\}\int_{X_i}^{X_{i+1}}\frac{\partial^2 g}{\partial\theta^2}dx$$

CONCLUSION

(*i*) In the first stage of application of Lind and Solana's principle of least information, we get results (5) and (6), for all Csiszer's measure of cross-entropy, whether we take cross-entropy, of $f(x, \theta)$ from $g(x, \theta)$ or of $g(x, \theta)$ from $f(x, \theta)$ or we use the symmetric measure,

$$\int\left[f(x,\theta)\phi\left(\frac{2g(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)\right.$$

$$\left.+g(x,\theta)\phi\left(\frac{2f(x,\theta)}{g(x,\theta)+f(x,\theta)}\right)\right]dx \quad ...(20)$$

(*ii*) In the second stage, the estimate of θ will depend on the measure of cross-entropy used and whether we take cross-entropy of $f(x, \theta)$ from $g(x, \theta)$ or of $g(x, \theta)$ from $f(x, \theta)$ or we use the symmetric measure (20).

(*iii*) In fact, in the second stage, some measures may lead to concave functions of θ for minimization or convex function of θ for maximization, and thus a great deal of care will have to be used in finding the global minimum or maximum.

(*iv*) Havrda and Charvat's measure [3]

$$\frac{\int g(x,\theta)\left[\left(\frac{2g(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)} \quad ...(21)$$

will give a convex function of θ when $\alpha < 1$ and so this measure can be conveniently used when $\alpha < 1$. Similarly,

$$\frac{\int f(x,\theta)\left[\left(\frac{2f(x,\theta)}{f(x,\theta)+g(x,\theta)}\right)^{\alpha-1}-1\right]dx}{\alpha(\alpha-1)}$$

can be used when $\alpha > 0$ ...(22)

(*v*) The approximation that match up to the value of (21) will correspond to the value of i-d for (22).

(*vi*) Hence, we receive an extensive range of approximations but the problem is of choosing the right one. The principles of monotonocity & invariance have been recommended by Lind & Solana for the above reason that can be used effectively to find whether our preference is limited by these otherwise; the choice is left to the users or decided by deliberations of computational expediency.

(*vii*) The theory of least information moves about neutrally in the approximation of arbitrary variables from data.

(*viii*) The MPS (Maximum Product of Spacings) was given by chang and Amin (2) intending to enlarge the geometric mean of spacings. Apart from this, Renneby (18) also observed that a good conjecture method should lesson the gap between the true distribution & the model with respect to a relevant metric. These give us quite alike but marked by different rationals for approximation from the one provided by PLI.

(*ix*) A few other papers that discuss the PLI are (6, 11, 12, 13, 14, 15, 16, 17).

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