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Symbolic computation, soliton solutions and 3d- plotting of KP- (2+1) dimentional non-linear evolution equation

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ABSTRACT : Most of the Non-Linear evolution equations give soliton solutions. The simplified version of Hirota's method, proved to be an effective and straightforward technique for finding soliton and multi soliton solutions for various non-linear evolution equations. In this paper, soliton and multisoliton solutions of Kadommstev Petviashvilli (KP) equation are obtained by using a symbolic manipulation package mathematica introduced by Hereman et. al. Symbolic manipulation package allows one to construct exact soliton solutions of non linear evolution and wave equations, provided the equations can be brought in bilinear form using Simplified version of Hirota method.We have illustrated the method in detail and shown graphical soliton solutions by 3D plotting using symbolic computer language maple. Graphs at different intervals of time clearly represent waves collision, dispersion and again retaining their original speed and shape after a collision. Mathematical results of solitons show that soliton solutions are just polynomials of exponentials.

Keywords : Hirota Method, Bilinearization, Non-Linear waves, symbolic computation

I. INTRODUCTION

Nonlinear evolution wave equations (NEEs) are partial differential equations (PDEs) involving first or second order derivatives with respect to time. Such equations have been intensively studied for the past few decades [1-3] and several new methods to solve nonlinear PDEs either numerically or analytically are now available. Hirota's bilinear method is a powerful tool for obtaining a wide class of exact solutions of soliton equations. In this paper, we have used symbolic manipulation packages mathematica to find soliton and multi soliton solutions of KP equation [4]. The program file hirota.m in the software package is used to test for the existence of solitary wave and soliton solutions of non-linear partial differential equations of bilinear form. It also explicitely construct one, two and three- soliton solutions of well known partial differential equations via Hirota's method [5-8]. Hirota's method allows one to construct exact soliton solutions of nonlinear evolution equation and wave equations, provided the equations can be brought in bilinear form.

II. SIMPLIFIED VERSION OF HIROTA METHOD

A. Bi-Linearization

The KP equation is given by:

$$(u_t - 6uu_x + u_{xxx})_x + \alpha u_{yy} = 0 \qquad ...(1)$$

First we transform this equation into bi-linear form using dependent variable transformation.

The bi-linearizing transformation for KP is

$$u(t, x, y) = 2\frac{\partial}{\partial x}\log f$$

The bilinearizing transformation for integrable non-linear evolution can be obtained from Painleve Analysis [9].

Simplifying, $u = 2\frac{\partial}{\partial x}\left(\frac{f_x}{f}\right)$

$$\Rightarrow \qquad u_t = 2\frac{\partial}{\partial x}\left[\frac{f\,f_{xt} - f_x\,f_t}{f^2}\right],$$

$$uu_x = 2\frac{\partial}{\partial x}\left[\left(\frac{f_{xx}}{f}\right)^2 - 2\frac{f_{xx}\,f_x^2}{f^3} + \left(\frac{f_x}{f}\right)^4\right],$$

$$u_{xxx} = 2\frac{\partial}{\partial x}\left[\frac{f_{xxxx}}{f} - 4\frac{f_{xxx}\,f_x}{f^2} + 12\frac{f_{xx}\,f_x^2}{f^3} - 3\left(\frac{f_{xx}}{f}\right)^2 - 6\left(\frac{f_x}{f}\right)^4\right]$$

and $$u_{yy} = 2\frac{\partial^2}{\partial x^2}\left[\frac{f\,f_{yy} - f_y^2}{f^2}\right]$$

Substituting the values in KP equation (1), we get

$$\frac{\partial^2}{\partial x^2}\left[\begin{array}{l}\left\{\frac{f\,f_{xt} - f_x\,f_t}{f^2}\right\} - 6\left\{\left(\frac{f_{xx}}{f}\right)^2 - 2\frac{f_{xx}\,f_x^2}{f^3} + \left(\frac{f_x}{f}\right)^4\right\} + \\ \left\{\frac{f_{xxxx}}{f} - 4\frac{f_{xxx}\,f_x}{f^2} + 12\frac{f_{xx}\,f_x^2}{f^3} - 3\left(\frac{f_{xx}}{f}\right)^2 - 6\left(\frac{f_x}{f}\right)^4\right\} \\ +\alpha\left\{\frac{f\,f_{yy} - f_y^2}{f^2}\right\}\end{array}\right] = 0$$

Integrating both sides and set constant of integration to be zero and on rearranging, we get

$$f_{xxxx}f-4f_{xxx}f_x+3\left(f_{xx}\right)^2+f_{xt}f-f_xf_t+\alpha\left(f\,f_{yy}-f_y{}^2\right)=0 \quad ...(2)$$

This is the bilinearized form of KP equation.

B. Transformation to Hirota Bi-linear form

By using Hirota D-operator we can write Bi-Linear form of KP to Hirota Bi-Linear form.

Let us consider D_tD_x applied on the product $f.f$,

$$D_tD_x\{f.f\}=\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)\left\{f(x,y,t).f\left(x',y',t'\right)\right\}\Big|_{x'=x,\,y'=y,\,t'=t}$$

$$=f_{xt}f+f.f_{xt}-f_t.f_x-f_x.f_t=2\left(f_{xt}f-f_x.f_t\right)$$

Now consider D_x^4, we get

$$D_x{}^4\{f.f\}=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^4\left\{f(x,y,t).f\left(x',y',t'\right)\right\}\Big|_{x'=x,\,y'=y,\,t'=t}$$

$$=f_{xxxx}f-4f_{xxx}f_x+6f_{xx}f_{xx}-4f_xf_{xxx}+f.f_{xxxx}$$

$$=2\left(f_{xxxx}f-4f_{xxx}f+3f_{xx}{}^2\right)$$

Also,

$$D_y{}^2\{f.f\}=\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^2\left\{f(x,y,t).f\left(x',y',t'\right)\right\}\Big|_{x'=x,\,y'=y,\,t'=t}$$

$$=f_{yy}f-f_yf_y-f_yf_y+f\,f_{yy}=2\left(f_{yy}f-f_y{}^2\right)$$

Note that if we multiply the equation (2) by 2 and Substitute these values, we get the Hirota bilinear form

$$\left(D_xD_t+D_x{}^4+\alpha\,D_y{}^2\right)f.f=0 \quad ...(3)$$

This is the Hirota bilinear form of KP equation.

C. The Hirota Perturbation and Soliton Solutions

Substituting $f=1+\varepsilon f_1+\varepsilon^2 f_2+$.......... in (3), we get

$$F\left(D_t,D_x,D_y\right)\{1.1\}+\varepsilon\,F\left(D_t,D_x,D_y\right)\{f_1.1+1.f_1\}$$
$$+\varepsilon^2F\left(D_t,D_x,D_y\right)\{f_2.1+f_1.f_1+1.f_1\}+\;...\;=0.$$

Where we have set

$$F\left(D_t,D_x,D_y\right)=\left(D_xD_t+D_x{}^4+\alpha\,D_y{}^2\right)f.f \quad ...(4)$$

and $f_1=e^{\eta_1}$ with $\eta_1=k_1x+l_1y+w_1t+\eta_1^0$

Make the coefficients of ε^m, m = 0, 1, 2, ... to vanish.

The coefficient of ε^0 is $F(D_t, D_x, D_y)\{1, 1\} = 0$,

Since $F(0, 0, 0)\{1\} = 0$.

By the coefficient of ε^1, we have

$$F\left(D_t,D_x,D_y\right)\{f_1.1+1.f_1\}=0$$

$$\Rightarrow\quad F\left(D_t,D_x,D_y\right)\{f_1.1\}+F\left(D_t,D_x,D_y\right)\{1.f_1\}=0$$

$$\Rightarrow\quad F\left(\partial_t,\partial_x,\partial_y\right)f_1+F\left(\partial_t,\partial_x,\partial_y\right)f_1=0$$

$$\Rightarrow\quad 2F\left(\partial_t,\partial_x,\partial_y\right)=0$$

$$\Rightarrow\quad F\left(\partial_t,\partial_x,\partial_y\right)=0 \quad ...(5)$$

(1) One Soliton Solution : Now to construct one-soliton solution of KP we take,

$$f=1+\varepsilon f_1$$

where $f_1=e^{\eta_1}$ and $\eta_1=k_1x+l_1y+w_1t+\eta_1^0$.

Note that $f_j = 0$ for all $f_j \geq 2$.

Now

$$F\left(D_t,D_x,D_y\right)\{f_1.1+1.f_1\}=(D_xD_t+D_x^4+\alpha D_y^2)\{f_1.1+1.f_1\}$$

$$=(D_xD_t+D_x^4+\alpha\;D_y^2)\{f_1.1\}+(D_xD_t+D_x^4+\alpha\;D_y^2)\{1.f_1\}$$

Thus,

$$F\left(D_t,D_x,D_y\right)\{f_1.1+1.f_1\}=2\left(\frac{\partial^2}{\partial x\partial t}+\frac{\partial^4}{\partial x^4}+\alpha\;\frac{\partial^2}{\partial y^2}\right)f_1$$

$$=2\left(k_1w_1+k_1^4+\alpha\;l_1^2\right)\;e^{\eta_1} \quad ...(6)$$

From this equation we get the dispersion relation as follows

$$F\left(D_t,D_x,D_y\right)\{f_1.1+1.f_1\}=0$$

$$\Rightarrow\quad 2\left(k_1w_1+k_1^4+\alpha\;l_1^2\right)e^{\eta_1}=0$$

$$\Rightarrow\quad k_1w_1+k_1^4+\alpha\;l_1^2=0$$

$$\Rightarrow\quad w_1=-\frac{k_1^4+\alpha\;l_1^2}{k_1}$$

This is dispersion relation.

Now from the coefficient of ε^2, we get

$$F\left(D_t,D_x,D_y\right)\{f_2.1+f_1.f_1+1.f_1\}=0$$

$$\Rightarrow\quad F\left(D_t,D_x,D_y\right)\{f_2.1+1.f_2\}=-F\left(D_t,D_x,D_y\right)\{f_1.f_1\}$$

$$\Rightarrow 2F\left(\partial_t,\partial_x,\partial_y\right)f_2=-F\left(D_t,D_x,D_y\right)\left\{e^{\eta_1}.e^{\eta_1}\right\}$$

$$\Rightarrow 2F\left(\partial_t,\partial_x,\partial_y\right)f_2=0\;\Rightarrow F\left(\partial_t,\partial_x,\partial_y\right)f_2=0$$

Therefore we may set $f_2 = 0$

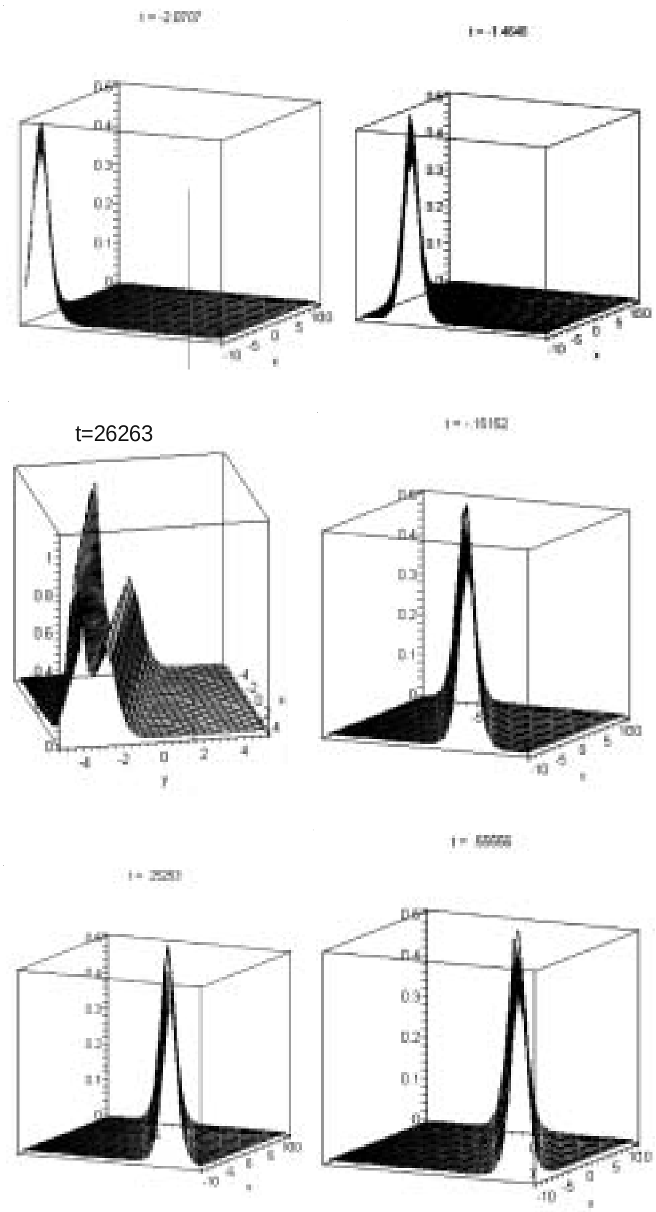
Similarly, we can prove that $f_3 = f_4 = ... = 0$. Thus for one Soliton solution $f_j = 0$ for all $j \geq 2$.

Finally without any loss of generality, we may set $\varepsilon = 1$, so $f = 1 + e^{h_1}$, therefore one-soliton solution of KP equation is given by

$$u(t, x, y) = -\frac{k_1^2}{\left(1 + e^{\eta_1}\right)^2}$$

where $\eta_1 = k_1 x + l_1 y + w_1 t + \eta_1^0$

Graphs for this equation represent one soliton travelling with constant speed retaining the same shape after collision and is plotted with the help of symbolic software maple.



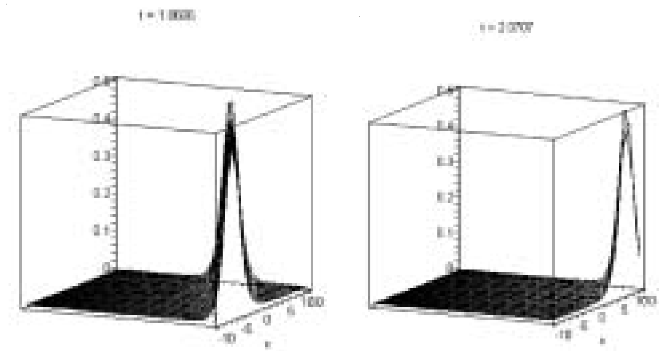


Fig. 1. One soliton solution for KP equation at different time intervals.

Values of the parameters is taken as $k1 = 1$, $l1 = 2$, $h_1^0 = 0$

(2) Two Soliton Solution: In order to construct two Soliton solution, we take

$$f_1 = e^{\eta_1} + e^{\eta_2}, \textit{ where } \eta_1 = k_1 x + l_1 y + w_t t + \eta_i^0,\ i = 1, 2$$

Now we have the coefficient of ε^1 as

$$F\left(D_t, D_x, D_y\right)\left\{ f_1.1 + 1.f_1 \right\} = 0$$

$$\Rightarrow \quad 2\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha\frac{\partial^2}{\partial y^2}\right) f_1 = 0$$

$$\Rightarrow \quad 2\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha\frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_1} + e^{\eta_2}\right\} = 0$$

$$\Rightarrow \quad \left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha\frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_1}\right\} = 0$$

and $$\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha\frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_2}\right\} = 0$$

Soving these equations, we get

$k_i w_i + k_i^4 + \alpha\, l_i^2 = 0$ for $i = 1, 2$.

$$\Rightarrow \quad w_i = -\frac{k_i^4 + \alpha\, l_i^2}{k_i},\ i = 1, 2$$

This gives the dispersion Relation.

The coefficient of ε^2 is

$$F\left(D_t, D_x, D_y\right)\ \left\{ f_2.1 + f_1 \cdot f_1 + 1.f_1 \right\} = 0$$

$$\Rightarrow \quad F\left(D_t, D_x, D_y\right)\ \left\{ f_2.1 + 1.f_2 \right\} = -F\left(D_t, D_x, D_y\right)\left\{ f_1 \cdot f_1 \right\}$$

$$\Rightarrow 2\,F\left(\partial_t, \partial_x, \partial_y\right) f_2 = -F\,(D_t, D_x, D_y)\ \{(e^{\eta_1} + e^{\eta_2})\}\ (e^{\eta_1}.\ e^{\eta_2}\}$$

$$= -2F(D_t, D_x, D_y)\ \{e^{\eta_1}.\ e^{\eta_2}\}$$

$$= -F(D_t, D_x, D_y)\ \{e^{\eta_1}.\ e^{\eta_2} + e^{\eta_2}.\ e^{\eta_1}\}$$

$$\because \begin{bmatrix} F\left(D_t, D_x, D_y\right)\left\{e^{\eta_1}.e^{\eta_1}\right\} = 0 \\ \textit{and} \quad F\left(D_t, D_x, D_y\right)\left\{e^{\eta_2}.e^{\eta_2}\right\} = 0 \end{bmatrix}$$

Or $$F\left(\partial_t, \partial_x, \partial_y\right) f_2 = -F\left(D_t, D_x, D_y\right)\ \left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$= -\left[D_x D_t + D_x^4 + \alpha . D_y^2\right] \left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$= -\begin{bmatrix} \left(k_1 w_1 - w_2 k_1 - w_1 k_2 + k_2 w_2\right) \\ +\left(k_1^4 - 4k_1^3 k_2 + 6k_1^2 k_2^2 - 4k_1 k_2^3 + k_2^4\right) \\ +\alpha\left(l_1^2 - 2l_1 l_2 + l_2^2\right) \end{bmatrix} \left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$= -\begin{bmatrix} \left(k_1 - k_2\right)\left(w_1 - w_2\right) \\ +\left(k_1 - k_2\right)^4 \\ +\alpha\left(l_1 - l_2\right)^2 \end{bmatrix}$$

$$= -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\} \qquad ...(7)$$

where we have set $F\left(p_i\right) = k_i w_i + k_i^4 + \alpha\ l_i^2$,

Therefore,

$$F\left(\partial_t, \partial_x, \partial_y\right) f_2 = -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

If we take $f_2 = A_{12}\ e^{\eta_1}.e^{\eta_2}$, we get

$$F\left(\partial_t, \partial_x, \partial_y\right) A_{12}\ e^{\eta_1}.e^{\eta_2} = -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$\Rightarrow A_{12}\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha \frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_1}.e^{\eta_1}\right\} = -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$\Rightarrow \quad A_{12}\begin{bmatrix} \left(k_1 w_1 + w_2 k_1 + w_1 k_2 + k_2 w_2\right) \\ +\left(k_1^4 + 4k_1^3 k_2 + 6k_1^2 k_2^2 + 4k_1 k_2^3 + k_2^4\right) \\ +\alpha\left(l_1^2 + 2l_1 l_2 + l_2^2\right) \end{bmatrix}\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$= -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$\Rightarrow A_{12}\begin{bmatrix} \left(k_1 + k_2\right)\left(w_1 + w_2\right) \\ +\left(k_1 + k_2\right)^4 \\ +\alpha\left(l_1 + l_2\right)^2 \end{bmatrix}\left\{e^{\eta_1}.e^{\eta_2}\right\} = -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$\Rightarrow \quad A_{12}\ F\left(p_1 + p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\} = -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\}$$

$$\Rightarrow \qquad A_{12} = -\frac{F\left(p_1 - p_2\right)}{F\left(p_1 + p_2\right)} \qquad ...(8)$$

Thus, $f = 1 + \varepsilon\ f_1 + \varepsilon^2\ f_2$ become

$f = 1 + e^{\eta_1} + e^{\eta_2} + A_{12} e^{\eta_1 + \eta_2}$, $\eta_i = k_i w_i + k_i^4 + \alpha\ l_i^2 + \eta_i^0, i = 1, 2.$
here without any loss of generality, we have taken $\varepsilon = 1$.

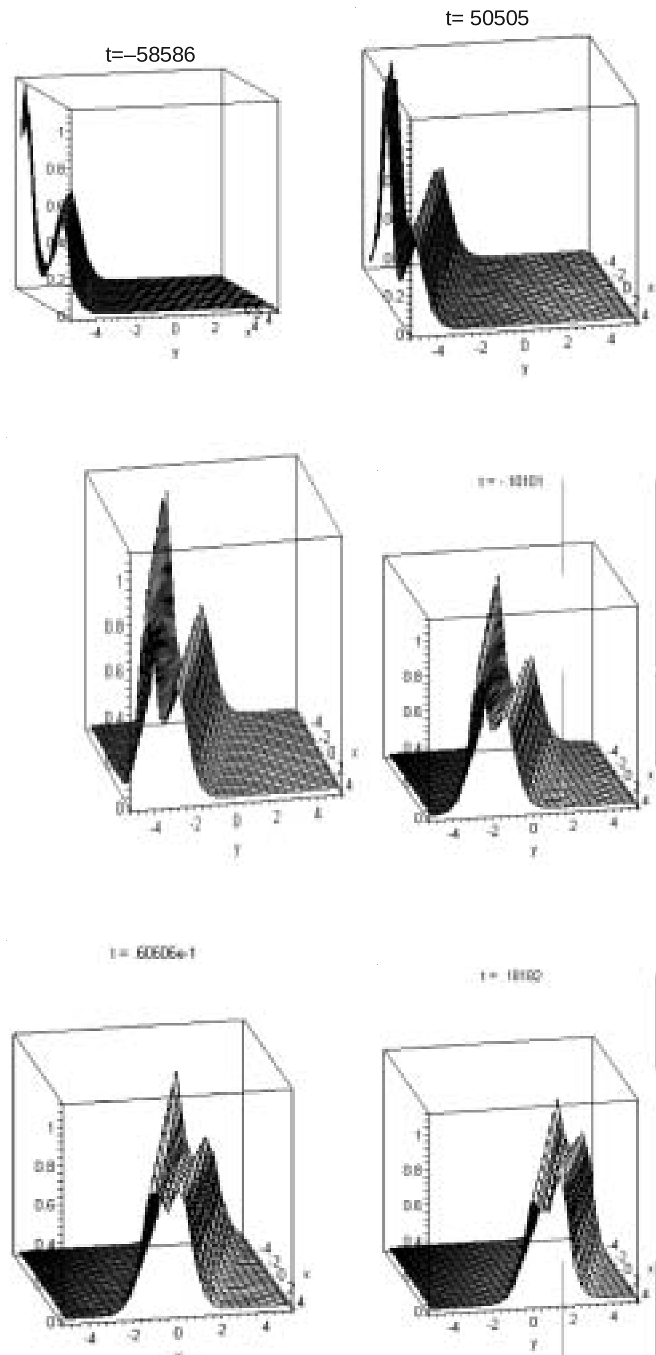
Now since $u\left(x, y, t\right) = 2\frac{\partial^2}{\partial x^2}\log f$, Therefore substituting the values of f in above equation, then

The two Soliton solution with the aid of symbolic computation is

$$u(x, y, t) = \frac{2\left\{k_1^2 e^{\eta_1} + k_2^2 e^{\eta_2} + \left[\left(k_1 - k_2\right)^2 + A_{12}\left(\left(k_1 + k_2\right)^2 + k_1^2 e^{\eta_2} + k_2^2 e^{\eta_1}\right)\right] e^{\eta_1 + \eta_2}\right\}}{\left(1 + e^{\eta_1} + e^{\eta_2} + A_{12} e^{\eta_1 + \eta_2}\right)^2}$$

Here $\eta_i = k_i w_i + k_i^4 + \alpha\ l_i^2 + \eta_i^0$ and A_{12} is given by (8)

Graphs for this equation represent two solitons travelling with constant speed retaining the same shape and is plotted with the help of symbolic software maple.



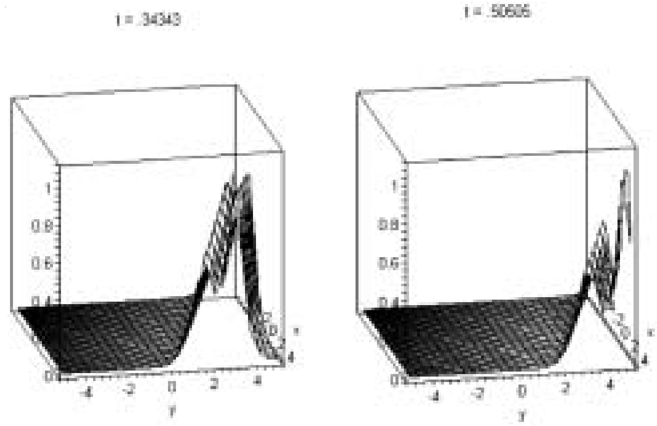


Fig 2. Two solitons of KP equation at different time intervals.

Values of the parameters used is $k1 = 1$, $k2 = 3/2$, $l1 = 2$, $l2 = 5/2$, $\eta_1^0 = 0$, $\eta_2^0 = 0$

(3) Three Soliton Solution: Now in a similar way we construct the three soliton solution of KP equation.

We take $f = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3$

where $f_1 = e^{\eta_1} + e^{\eta_2} + e^{\eta_3}$ with $\eta_i = k_i w_i + k_i^4 + \alpha\, l_i^2 + \eta_i^0$

Note that $f_j = 0$ for $j \geq 0$.

Here let us consider only the coefficients of $\varepsilon^m,\ m = 1,2,3,4,$ since other coefficients vanish automatically. From the coefficient of ε^1, we have

$$F\left(D_t, D_x, D_y\right)\left\{ f_1.1 + 1.f_1 \right\} = 0$$

$$\Rightarrow \quad 2\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha \frac{\partial^2}{\partial y^2}\right) f_1 = 0$$

$$\Rightarrow \quad 2\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha \frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_1} + e^{\eta_2} + e^{\eta_3}\right\} = 0$$

$$\Rightarrow \quad \left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha \frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_1}\right\} = 0$$

$$\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha \frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_2}\right\} = 0$$

and
$$\left(\frac{\partial^2}{\partial x \partial t} + \frac{\partial^4}{\partial x^4} + \alpha \frac{\partial^2}{\partial y^2}\right)\left\{e^{\eta_3}\right\} = 0$$

Soving these equations, we get

$$k_i w_i + k_i^4 + \alpha\, l_i^2 = 0, \qquad i = 1,2,3.$$

$$\Rightarrow \quad w_i = -\frac{k_i^4 + \alpha\, l_i^2}{k_i}, i = 1,2,3.$$

This gives the dispersion Relation.

From the coefficient of ε^2, we have

$$F\left(D_t, D_x, D_y\right)\ \left\{ f_2.1 + f_1.f_1 + 1.f_1 \right\} = 0$$

$$F\left(D_t, D_x, D_y\right)\ \left\{ f_2.1 + 1.f_2 \right\} = -F\left(D_t, D_x, D_y\right)\left\{ f_1.f_1 \right\}$$

$$2\, F\left(\partial_t, \partial_x, \partial_y\right) f_2$$

$$= -F\left(D_t, D_x, D_y\right)\left\{\left(e^{\eta_1} + e^{\eta_2} + e^{\eta_3}\right).\left(e^{\eta_1} + e^{\eta_2} + e^{\eta_3}\right)\right\}$$

$$= -F\left(D_t, D_x, D_y\right)\left\{2\left(e^{\eta_1}.e^{\eta_2} + e^{\eta_2}.e^{\eta_3} + e^{\eta_1}.e^{\eta_3}\right)\right\}$$

$$F\left(\partial_t, \partial_x, \partial_y\right)$$

$$= -F\left(D_t, D_x, D_y\right)\left\{\left(e^{\eta_1}.e^{\eta_2} + e^{\eta_2}.e^{\eta_3} + e^{\eta_1}.e^{\eta_3}\right)\right\}$$

$$= -F\left(p_1 - p_2\right)\left\{e^{\eta_1}.e^{\eta_2}\right\} - F\left(p_2 - p_3\right)\left\{e^{\eta_2}.e^{\eta_3}\right\}$$

$$-F\left(p_1 - p_3\right)\left\{e^{\eta_1}.e^{\eta_3}\right\} \qquad \text{...(9)}$$

We see that f_2 should be of the form

$$f_2 = A_{12} e^{\eta_1}.e^{\eta_2} + A_{23} e^{\eta_2}.e^{\eta_3} + A_{13} e^{\eta_1}.e^{\eta_3}$$

We substitute this equation into (9) and use

$$k_i w_i + k_i^4 + \alpha\, l_i^2 = 0, \qquad i = 1,2,3.$$

We get A_{ij} where $i, j = 1, 2, 3\ i < j$ as

$$A_{ij} = -\frac{F\left(p_i - p_j\right)}{F\left(p_i + p_j\right)}$$

$$= -\frac{k_i w_j + k_j w_i + 4k_i^3 k_j - 6k_i^2 k_j^2 + 4k_i k_j^3 + 6 l_i l_j}{k_i w_j + k_j w_i + 4k_i^3 k_j + 6k_i^2 k_j^2 + 4k_i k_j^3 + 6 l_i l_j}$$

Now for the coefficient of ε^3, we have

$$F\left(D_t, D_x, D_y\right)\ \left\{ f_3.1 + f_2.f_1 + f_1.f_2 + 1.f_3 \right\} = 0$$

$$\Rightarrow \quad F\left(D_t, D_x, D_y\right)\ \left\{ f_3.1 + 1.f_3 \right\}$$

$$= -F\left(D_t, D_x, D_y\right)\ \left\{ f_1.f_2 + f_2.f_1 \right\}$$

$$\Rightarrow \quad F\left(\partial_t, \partial_x, \partial_y\right) f_3 = -F\left(D_t, D_x, D_y\right)$$

$$\left\{\left(A_{12} e^{\eta_1}.e^{\eta_2} + A_{23} e^{\eta_2}.e^{\eta_3} + A_{13} e^{\eta_1}.e^{\eta_3}\right)\left(e^{\eta_1} + e^{\eta_2} + e^{\eta_3}\right)\right\}$$

On simple calculations, we get

$$F\left(\partial_t, \partial_x, \partial_y\right) f_3$$

$$= -\{(A_{12}F(p_3 - p_2 - p_1) + A_{23}F(p_1 - p_2 - p_3) + A_{13}F(p_2 - p_1 - p_3)\}\left(e^{\eta_1+\eta_2+\eta_3}\right)$$

Hence f_3 is of the form $f_3 = Be^{\eta_1+\eta_2+\eta_3}$

If we substitute f_3 in the above equation, we find that

$$B = -\frac{A_{12}F(p_3 - p_2 - p_1) + A_{23}F(p_1 - p_2 - p_3) + A_{13}F(p_2 - p_1 - p_3)}{F(p_1 + p_2 + p_3)}$$

Now substituing the values of f_1, f_2 and f_3 in $f = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3$, we have $f = 1 + e^{\eta_1} + e^{\eta_2} + e^{\eta_3} + A_{12}e^{\eta_1+\eta_2} + A_{23}e^{\eta_2+\eta_3} + A_{13}e^{\eta_1+\eta_3} + Be^{\eta_1+\eta_2+\eta_3}$

$\eta_i = k_i w_i + k_i^4 + l_i^2 + \eta_i^0 \; i = 1,2,3$

Here without any loss of generality, we have taken $\varepsilon = 1$.

Now since $u(x,y,t) = 2\frac{\partial^2}{\partial x^2}\log f$

Therefore on substituting the values of f in above equation and with the help of symbolic computation, we get the three Soliton solution of KP as $u(x,y,t) = 2\frac{R(x,y,t)}{S(x,y,t)}$, where

$$\begin{aligned}
R(x,y,t) &= k_1^2e^{\eta_1} + k_2^2e^{\eta_2} + k_3^2e^{\eta_3} \\
&+ e^{2\eta_1+\eta_2+\eta_3}\left[A_{12}A_{13}(k_2 - k_3)^2 + B(k_2 + k_3)^2\right] \\
&+ e^{\eta_1+\eta_2+2\eta_3}\left[A_{13}A_{23}(k_1 - k_2)^2 + B(k_1 + k_2)^2\right] \\
&+ e^{\eta_1+2\eta_2+\eta_3}\left[A_{12}A_{23}(k_1 - k_3)^2 + B(k_1 + k_3)^2\right] \\
&+ e^{\eta_1+\eta_2}\left[(k_1 - k_2)^2 + A_{12}\left(k_1^2e^{\eta_2} + k_2^2e^{\eta_1} + (k_1 + k_2)^2\right)\right] \\
&+ e^{\eta_1+\eta_3}\left[(k_1 - k_3)^2 + A_{13}\left(k_1^2e^{\eta_3} + k_3^2e^{\eta_1} + (k_1 + k_3)^2\right)\right] \\
&+ e^{\eta_2+\eta_3}\left[(k_2 - k_3)^2 + A_{23}\left(k_2^2e^{\eta_3} + k_3^2e^{\eta_2} + (k_2 + k_3)^2\right)\right] \\
&+ e^{\eta_1+\eta_2+\eta_3}\begin{bmatrix} A_{12}\left(k_1^2 + k_2^2 + k_3^2 + 2k_1k_2 - 2k_1k_3 - 2k_2k_3\right) \\ +A_{13}\left(k_1^2 + k_2^2 + k_3^2 + 2k_1k_3 - 2k_1k_2 - 2k_2k_3\right) \\ +A_{23}\left(k_1^2 + k_2^2 + k_3^2 + 2k_2k_3 - 2k_1k_2 - 2k_1k_3\right) \\ +B\left(k_1^2 + k_2^2 + k_3^2 + 2k_1k_2 + 2k_1k_3 + 2k_2k_3\right) \end{bmatrix} \\
&+ Be^{\eta_1+\eta_2+\eta_3}\left[A_{12}k_3^2e^{\eta_1+\eta_2} + A_{13}k_2^2 + e^{\eta_1+\eta_3} + A_{23}k_1^2e^{\eta_2+\eta_3}\right]
\end{aligned}$$

and $S(x,y,t) = [1 + e^{\eta_1} + e^{\eta_2} + e^{\eta_3} + A_{12}e^{\eta_1+\eta_2} + A_{23}e^{\eta_2+\eta_3} + A_{13}e^{\eta_1+\eta_3} + Be^{\eta_1+\eta_2+\eta_3}]^2$ for $\eta_i = k_iw_i + k_i^4 + \alpha\, l_i^2 + \eta_i^0$, $i = 1,2,3$.

Graphs for this equation represent three solitons travelling with constant speed retaining the same shape and is plotted with the help of symbolic software maple.

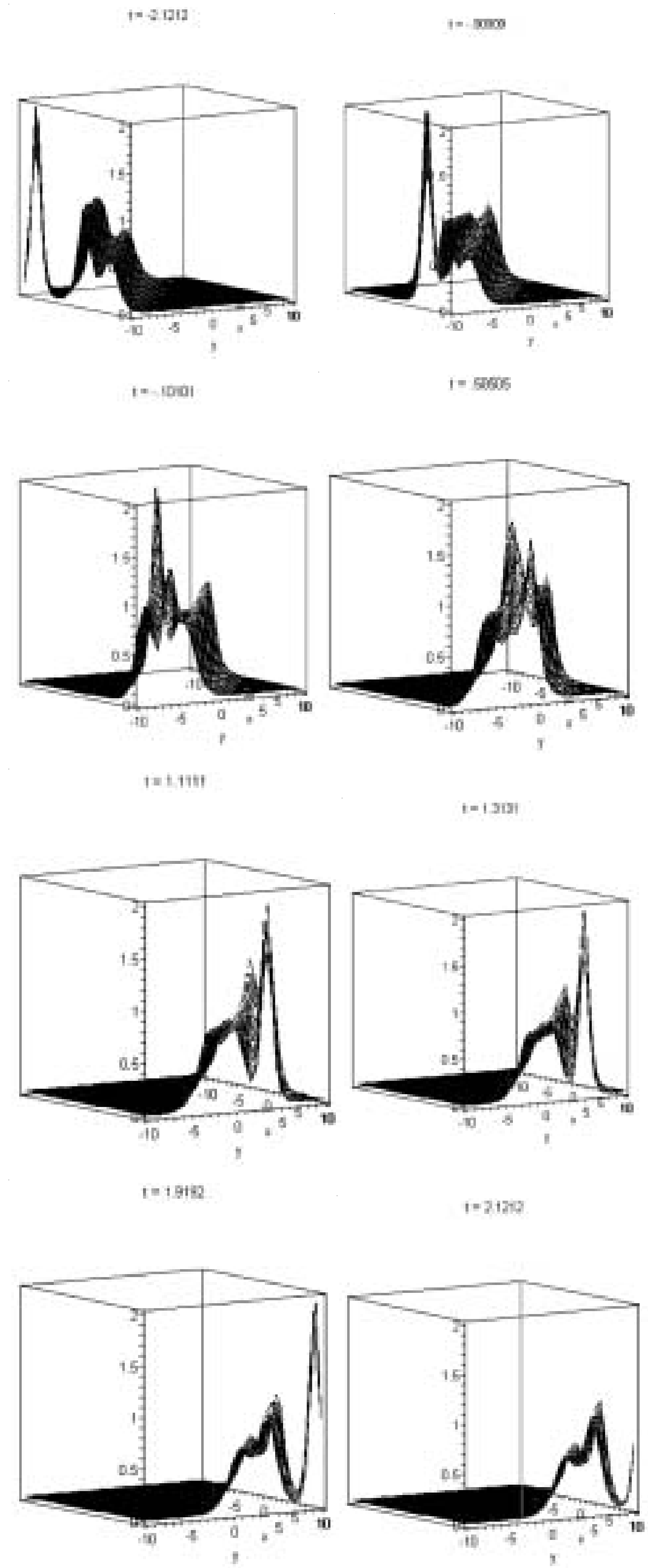


Fig 3. Three solition waves of KP equation at different time intervals

Values of the various parameters used are:

$k1 = 1$, $k2 = 3/2$, $k3 = 5/4$, $l1 = 2$, $l2 = 5/2$, $l3 = 7/4$, $\eta_1^0 = 0$, $\eta_2^0 = 0$, $\eta_3^0 = 0$.

(4) N-Soliton Solution: We have three soliton solution given as

$$f = 1 + e^{\eta_1} + e^{\eta_2} + e^{\eta_3} + A_{12}e^{\eta_1+\eta_2}$$

$$+ A_{23}e^{\eta_2+\eta_3} + A_{13}e^{\eta_1+\eta_3} + A_{12}A_{23}A_{13}e^{\eta_1+\eta_2+\eta_3}$$

$$\eta_i = k_i w_i + k_i^4 + \alpha\, l_i^2 + \eta_i^0, i = 1,2,3.$$

where

$$A_{ij} = -\frac{F(p_i - p_j)}{F(p_i + p_j)} = -\frac{k_i w_j + k_j w_i + 4k_i^3 k_j - 6k_i^2 k_j^2 + 4k_i k_j^3 + 6l_i l_j}{k_i w_j + k_j w_i + 4k_i^3 k_j + 6k_i^2 k_j^2 + 4k_i k_j^3 + 6l_i l_j}$$

and $i, j = 1, 2, 3$.

By writing $A_{ij} = \exp A_{ij}$, we may express f as

$$f = \sum_{\mu=0,1} \exp\left(\sum_{j=1}^{3} \mu_j \eta_j + \sum_{j>k}^{3} \mu_j \mu_k A_{jk}\right)$$

Where $\sum_{\mu=0,1}$ indicates the summation over all possible combinations of $\mu_1 = 0, 1, \mu_2 = 0, 1, ... \mu_N = 0, 1$ and $\sum_{j>k}^{3}$ means the summation over all possible combinations of 3 elements under the condition $j > k$. For example, the choice $\mu_1 = 1$, $\mu_2 = 0, \mu_3 = 0$ gives $\exp \eta_1$.

By employing the above notation, the N-soliton solution is expressed as $f = \sum_{\mu=0,1} \exp\left(\sum_{j=1}^{N} \mu_j \eta_j + \sum_{j>k}^{N} \mu_j \mu_k A_{jk}\right)$, Where $\sum_{\mu=0,1}$ indicates the summation over all possible combinations of $\mu_1 = 0, 1, \mu_2 = 0, 1, ...\mu_N = 0, 1$ and $\sum_{j>k}^{N}$ means the summation over all possible combinations of N elements under the condition $j > k$.

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